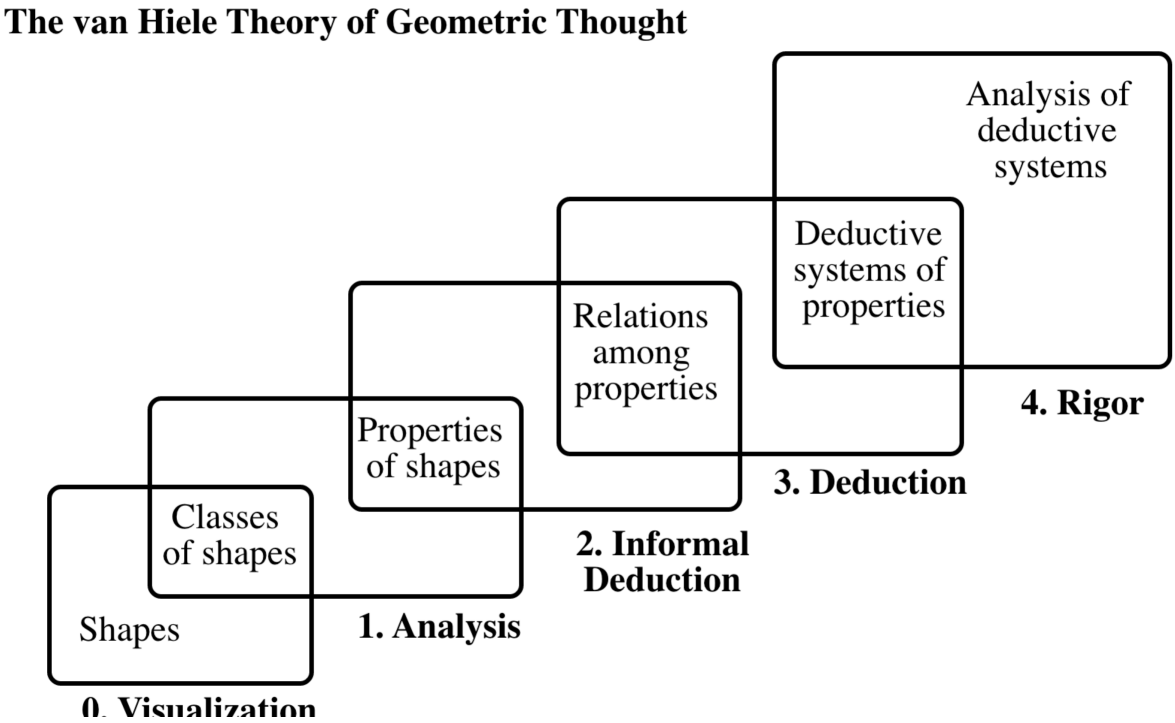
Understanding Van Hiele Levels for Geometry



We know that our brains develop in stages. From Freud and Erikson with their stages of development to Piaget with his teachings of scaffolding, they all knew that in some way we all learned in levels. In order to get to successfully get to the next level, you had develop the knowledge in the first level.

However, this isn't always the way that we approach teaching geometry in school.

As teachers, we have a limited amount of time to teach lessons. Not to mention the competition we have for students' attention while they are in our classrooms. Even our youngest students have favorite "YouTubers," video games, and TV shows. And this is all just piled on to the usual social distractions that we all experience without the technology.

Something that a lot of our lesson plans are missing is an understanding of the Van Hiele levels and how it plays into understanding geometry concepts. Often when our older students aren't grasping what we are teaching, it is simply because they aren't ready for it.

When they were younger, they didn't truly understand the first levels of learning. And this is where the Van Hiele Levels come into play.

What are the Van Hiele Levels?

This theory originated in 1957 by husband and wife team Dina Van Hiele-Geldof and Pierre van Hiele from the Utrecht University in the Netherlands. It helps to describe how students learn geometry. The Van Hiele levels have helped shaped curricula throughout the world, including a large influence in the standards of geometry in the US

How do they work?

Geometric reasoning starts as soon as we can start processing information and in early schooling. However, depending on the individual, the ages in each stage can vary, especially as they progress through school. Basically the level is dependent on the experiences that each student has, no matter what their age.

In Learning Mathematics in Elementary & Middle Schools, *Cathcart*, et al "*In general, most elementary school students are at levels 0 or 1; some middle school students are at level 2. State standards are written to begin the transition from levels 0 and 1 to level 2 as early as 5th grade "Students identify, describe, draw and classify properties of, and relationships between, plane and solid geometric figures." (5th grade, standard 2 under Geometry and Measurement) This emphasis on relationships is magnified in the 6th and 7th grade standards.*"

Level 0: Visualization

They can recognize shapes by their whole appearance, but not its exact properties. For example, students will think of a shape in terms of what it "looks like." A rectangle is a door or a triangle is a clown's hat. And the student may not be able to recognize the shape if it's rotated to a different standing point.

Level 1: Analysis (Description)

Students start to learn and identify parts of figures as well as see figures in a class of shapes. They can describe a shape's properties and are able to understand that shapes in a group have the same properties as well. A student in this level will know that parallelograms have opposite sides that are parallel and will be able to group them accordingly.

Level 2: Informal Deduction / Abstraction

A student in this level will start to recognize the relationship between properties of shapes. They will also be able to participate and understand informal deductive discussions about the shapes and their different characteristics.

Level 3: Formal Deduction

At this level students are able of more complex geometric concepts. They can think about properties are related, as well as relationships between axioms, theorems, postulates and definitions. According to John Van Del Walle, students should be able to "work with abstract statements about geometric properties and make conclusions more on logic than intuition."

Level 4: Rigor

Finally, students will reach the last level of learning geometric reasoning. Even in the absence of concrete examples, they should be able to compare geometric results in different axiomatic systems. Basically, they will see geometry in the abstract. Mostly, this is the level of college mathematic majors and how they think about geometry.

Some students may seamlessly pass through these stages, while others may be get a little left behind. And in the meantime, of course the curriculum keeps going, so without proper attention to the missing links or tutoring they won't ever be able to full catch up with the lessons.

Ways to Teach Geometry for Deeper Understanding Using the Van Hiele Levels

As it goes with most learning, the earlier the better. A head start in the early grades is the only way to make sure students have a better van Hiele level when they enter high school geometry. It's imperative that we improve basics of learning in elementary and middle school for abstract and relational levels. How can we incorporate this?

- Visual recognition in elementary school (grades 2-5)
- Drawing practice (for accuracy)
- Practice the relationships of different shapes (grades 6-8)
- Hands-on activities (with manipulatives), ideally with some level of inquiry / exploration
- Teaching students to explain, prove and show why

Geometry is a subject that builds upon itself. A student who is lost at the beginning is likely to be lost for the duration of their school career. That's why it's imperative to have underlying concepts taught throughout all education levels.

Here are some examples of what you'll see at each level, and the types of activities that will help students solidify their learning in order to pass on to the next level.

Level 0: VISUALIZATION

Understanding: The child can recognize and classify individual shapes based on classic examples.

Thought Process: The shape is a rectangle because it looks like a box.

Challenges: When figures are turned a different way or do not fit the "typical" visual presentation, the child may not think it qualifies (examples: The child may think that a very thin scalene triangle is not a triangle since it's not the classic equilateral shape they learned. He/she may think that a square that is turned is not actually a square, but instead is a "diamond."

Activities for Middle / High Schoolers stuck at Level 0:

- Practice transformations (to see that shapes are congruent when turned, etc.)
- Use geometry software to explore triangles, parallel and perpendicular lines, angles, quadrilaterals, and circles. When students can drag vertices and

display measures, they can really begin to grasp how the properties of each figure work.

Level 1: ANALYSIS

Understanding: The student can identify figures, or components of figures. He/she accepts basic, separate properties of geometry.

Thought Process: This set of angle measures is from an isosceles triangle because two of the measures are the same, and the sum is 180 degrees.

Challenges: The student may have difficulty progressing from one property to another. The sequence of steps may be foggy, and the ability to "see" where to begin in a more complex figure or a problem that requires two properties may be lacking.

Activities for Middle / High Schoolers at Level 1:

- Play games that allow students to convert words into diagrams and figures into words.
- Use hands-on inquiry activities as much as possible.
- Allow students to practice just identifying situations where a certain property may be at play. They need a lot of practice with recognizing figures and reinforcement of the properties before they can begin to apply the more abstract concepts
- Take time for clear, concise notes. During lecture time, be sure to focus in on new vocabulary terms. Build up the basics

Level 2: ABSTRACTION

Understanding: A student can draw conclusions and is able to order and connect properties. He/she understands conditions and can begin to USE deductive logic, but does not yet understand the meaning of deduction or how to formally show a proof.

Thought Process: This figure has two pairs of congruent sides, so it could be a rectangle or maybe a parallelogram…. Ah, but there are no right angles, and opposite pairs of angles are congruent, so it must be a parallelogram.

Challenges: The student may have trouble extending a definition beyond its most basic interpretation. It's difficult to explain or follow a deductive

reasoning sequence that represents a formal proof with many steps.

Activities for Students at Level 2:

- Incorporate challenges in which students have to use multiple properties layered upon one another to find missing measures or solve problems. Angle puzzles offer a fun and challenging task that fits this level of learner well,

- Try card sorts that require students to use a variety of theorems across a diverse mixture of input (words, diagrams, measures...) so students make connections between geometric properties and have to sometimes take multiple steps before they can successfully classify a figure. For additional practice, use games where students have to make decisions based on the properties of a figure.

Again, be sure to include a variety of input information. Use some cards with diagrams, some with word explanations and definitions, and some with measurements so that students get a blend of practice with all forms and can build mental connections.

Level 3: DEDUCTION

Understanding: The student has a grasp of reasoning & logic and can communicate this to others. He/she is able to construct formal proofs.

Thought Process: Since a implies b, and b implies c, then a implies c. In this case a is true, so I can conclude that c must be true as well.

Challenges: Students in phase 3 understand theorems, undefined terms, definitions, and axioms, but have a fixed view of them. They see axioms as concrete and have difficulty comprehending non-Euclidean geometry.

Activities for High School Students at Level 3:

- Explicitly teach your students logic, including conditional statements, conjectures, laws of syllogism and detachment, etc. Focus on the skills and understanding behind deductive reasoning. In your practice activities, be sure to include examples in all formats: some in logical language (p --> q, some in day-to-day examples about students / animals, etc, some in geometry terms, and some in algebra terms as a review. The variety of

examples will help students connect how logic works in geometry to how it works in other areas.

- When you dive into proofs, be sure that your students have the basics of logic and deduction down. Then, start with the postulates that they will need for their first proofs

Level 4: RIGOR

Understanding: The grasp of geometry has been extended to the level of a mathematician. The student has an understanding non-euclidian geometry and is able to compare the study of geometry to other areas.

(Most of our high school students won't get to this level yet.)

You can't just throw a baby into water and expect them to know how to swim. They need to see the water, touch the water, practice and experiment with and experience it. Then they slowly learn the more complex aspects of swimming such as floating, manipulating limbs to move, holding your breath under water and so on.

That is much like geometry. Students who have never had an actual understanding of the basics in geometry are going to be lost. They can easily drown in the onslaught of information that is geometry. No matter what level of math is taught, at some point a lesson that includes basic geometry is not only needed, it's crucial. Be sure to give your students the time, space, and solid practice that allows them to progress from one level to another!