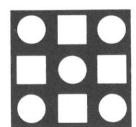
# The Problem Solver 8

Activities for Learning Problem-Solving Strategies

Alissa Scanlin Judy Goodnow Shirley Hoogeboom









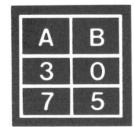


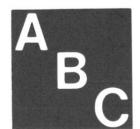












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> > ISBN: 0-88488-686-7 14 15 • MAL• 04 03

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### THE PROBLEM SOLVER PROGRAM

The Problem Solver is a step-by-step instructional program designed to help your students become competent and confident problem solvers. It utilizes an easy-to-learn, four-step method combined with ten useful problem-solving strategies. A wide range of reproducible problems are carefully sequenced to systematically expand your students' problem-solving abilities.

### CONTENTS AND ORGANIZATION OF THE PROBLEM SOLVER 8

### Section 1: Teaching the Strategies

The first part of this section describes the four-step method and the ten solution strategies. Sample problems are used to illustrate how the method and strategies can be implemented.

The second part of this section presents 48 teaching problems. For each problem there is a reproducible student page and a one-page teaching plan. The problems are organized in groups of two that focus on the same strategy. The symbol of that strategy is shown in the upper left-hand corner of both the student page and the teaching plan page.

### Section 2: Practice Problems

This section presents 72 reproducible problems similar to those in Section 1. But here, no strategies are recommended. The students must decide which strategy to use. There are two problems on each page, and they have purposely been mixed up so that they do not follow the order of strategies presented in the teaching section. Students record their work and solutions on a separate recording sheet. All ten strategy symbols are shown at the bottom of the recording sheet, and the students circle the appropriate symbol(s) to indicate which strategy or strategies they used.

### Section 3: Solutions

Here you will find a solution for every problem in Sections 1 and 2. For many problems, notes are also included describing the solution process. (Solutions for all the problems in Section 1 are also shown on their respective teaching plan pages.)

### **GENERAL TEACHING SUGGESTIONS**

### **Presenting the Problems**

Before you present any problems to the students, read through the descriptions of the four-step method and the ten solution strategies in Section 1. We recommend that you present the problems in the order they appear. The problems have been carefully sequenced so that they progress from easy to more challenging. The math skills required to solve even the most challenging problems are those which students have usually mastered by the end of the seventh grade.

Give each student a copy of the problem and a copy of the recording sheet. You may also want to make calculators available for them to use. Read the problem aloud to the students. Encourage the students to verbalize their thought processes as they choose strategies and find solutions. Develop an atmosphere in which the students feel comfortable expressing themselves. Let them know it's okay to make mistakes. In this setting, your students will become enthusiastic problem solvers and will begin to see problems as interesting challenges.

Use the teaching plans. The sequence of questions in each plan guides the students through the four-step method: FIND OUT, CHOOSE A STRATEGY, SOLVE IT, and LOOK BACK. You'll probably want to add some of your own questions to further clarify the process. The responses to the questions, shown in italics in the teaching plans, are only samples of the kinds of responses you want to draw from your students. Encourage them to risk giving incorrect answers. Remind them that it is more important for them to take an active role in solving the problem, and enjoy doing it, than it is to respond with the right answer.

Since students learn in different ways, some students may wish to use a strategy other than the focus strategy for solving a problem given in Section 1. Encourage students to use methods that are best for them, when they can demonstrate a legitimate solution process with another strategy. (The goal is to equip your students with techniques for approaching future problems, but you also want them to be flexible in applying them.) Some students may even discover additional strategies and use them to solve the problems. Encourage the students to design new symbols for those strategies.

### **Additional Practice**

For additional practice, you can give students the problem extensions which appear on many teaching plans in Section 1, and the practice problems from Section 2. Since a problem extension restates a given problem with different data or a different question, you may wish to have students solve it immediately after solving the original problem. Each practice problem is similar to a group of two problems in Section 1. When students have solved both problems in that group, you can give them a similar problem from Section 2. However, you may wish to wait until the students have solved several groups of problems using different strategies before giving them the practice problems. That will make choosing a strategy more of a challenge.

The table below lists problems in *The Problem Solver 7* which are similar to the problems in *The Problem Solver 8*. The problems for seventh grade are less difficult and can be used with all students.

provident p ylone	The Problem Solver 8				blem n Solv	
Problems	Strategy Focus		F	Proble	ems	
1, 2	Use or make a table	1,	2,	50,	58,	67
3, 4	Make an organized list	3,	4,	49,	56,	72
5, 6	Act out or use objects	5,	6,	52,	69,	75
7, 8	Make an organized list	7,	8,	51,	65,	78
9, 10	Use or look for a pattern	9,	10,	54,	71,	83
11, 12	Use or make a table	11,	12,	53,	70,	84
13, 14	Make a picture or diagram	13,	14,	55,	79,	88
15, 16	Guess and check	15,	16,	57,	68,	82
17, 18	Work backwards	17,	18,	60,	81,	91
19, 20	Make a picture or diagram	19,	20,	61,	93,	101
21, 22	Use logical reasoning	21,	22,	59,	80,	96
23, 24	Use or make a table	23,	24,	62,	90,	106
25, 26	Make a picture or diagram					
27, 28	Make an organized list	27,	28,	63,	87,	111
29, 30	Guess and check	29,	30,	73,	95,	118
31, 32	Make a picture or diagram					
33, 34	Use or look for a pattern	33,	34,	76,	104,	109
35, 36	Use logical reasoning	35,	36,	74,	100,	112
37, 38	Make it simpler	41,	42,	85,	99,	115
39, 40	Use logical reasoning	39,	40,	77,	105,	113
41, 42	Work backwards	43,	44,	94,	107,	119
43, 44	Make it simpler	45,	46,	98,	103,	108
45, 46	Use logical reasoning	25,	26,	64,	92,	116
47, 48	Brainstorm	47,	48,	97,	110,	120

If you wish to provide more practice for your students, the following materials on problem solving are available from Creative Publications:

Brainstorming: Activities for Creative Thinking

Problem Solver Projects, Grade 8

**ThinkerGames** 

Trivia Math: A Problem a Day

### **TEACHING THE STRATEGIES**

Part one of this section describes the four-step method and the ten solution strategies. Sample problems are used as examples to illustrate how the method and strategies can be implemented. Part two presents 48 teaching problems. For each problem there is a reproducible student page and a one-page teaching plan. The problems are organized in groups of two that focus on the same strategy. The symbol of that strategy is shown in the upper left-hand corner of both the student page and the teaching plan page. Please see pages vii-viii for some general teaching suggestions on presenting the problems to your students.

### What Is the Four-Step Method?

The four-step method is a systematic approach to problem solving that can be used for solving any problem.

The first step is to FIND OUT what the problem means and what question you must answer to solve it. To find out what the problem means, you must understand the words and phrases used and what's happening in the problem. You must be able to identify the important information and the unimportant information, and determine if any necessary information is missing and what you must do to get that information. In some cases, the problem may have to be broken down into smaller problems before the larger problem can be solved. You should understand the problem well enough to say it in your own words. And finally, you must be able to state the question you have to answer to solve it.

The second step is to CHOOSE A STRATEGY that will help solve the problem. You will often find there is more than one strategy that can be used. The idea, however, is to find the strategy or strategies that will help you the most with a particular problem.

The third step is to SOLVE IT. Work through the problem until you find the answer to the question, using the strategy you selected. It is important that you record your work in a way that lets you see at a glance what you've completed. As you work to find the answer, you may find that the strategy you selected is not as helpful as you thought it might be. In that case, you will want to try a different strategy.

The fourth step is to LOOK BACK. Reread the problem and check the solution to see that it meets the conditions stated in the problem and that it answers the question. To review your solution and ask yourself if it's logical and reasonable is a very important step in problem solving.

### What Are the Ten Solution Strategies?



### **ACT OUT OR USE OBJECTS**

It can be very helpful to act out a problem or to move objects around while trying to solve a problem. It allows you to develop visual images of both the data in the problem and the solution process. By taking an active role in finding the solution, you are more likely to remember the process you used and be able to use it again for solving similar problems. The dramatizations and objects need not be elaborate: small scraps of paper and colored chips or counters will usually work quite well. This strategy is especially helpful when you want to visualize relationships. For example:

**Problem 5:** Kane works as a salesclerk at the Threads Swimshop. His manager asked him to arrange a shirt, a pair of shorts, a pair of sandals, a swimsuit, a beach towel, a beach bag, suntan lotion, a magazine, a can of soda pop, and a pair of sunglasses in a display case. When Kane finished the job, the suntan lotion was behind the sunglasses and to the left of the beach bag. The beach towel was between the shirt and the suntan lotion. The sunglasses were to the right of the shorts and swimsuit. The shorts were not in front of the beach towel. The can of soda pop was to the right of the beach bag, and the sandals were to the left of the magazine. Where was each product in the display case?

Solution:

shirt	beach towel	suntan lotion	beach bag	can of soda
shorts	swimsuit	sunglasses	sandals	magazine



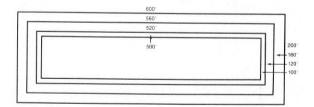
### MAKE A PICTURE OR DIAGRAM

It may be helpful to use an available picture or make one when trying to solve certain problems. The pictures or diagrams need not be well drawn. It is most important that they help you understand and manipulate the data in the problem. For example:

**Problem 13:** C 623 and C 279 are Cyborgs, who work together on interstellar developments for the Cosmos Construction Company. Their last job was to clear a small rectangular area 200 feet wide and 600 feet long for a new dance club. They used a special planet planer, a bulldozer-type machine, to clear loose rocks and smooth the surface. The planet planer covers an area 20 feet across. If C 623 begins at an outside corner, and works his way into the center, on which trip around will he clear his half of the area?

Solution: During the third trip

Area = 120,000 sq ft 1/2 area = 60,000 sq ft After 1st trip — 89,600 sq ft left (560  $\times$  160) After 2nd — 62,400 sq ft left (520  $\times$  120) Need to clear additional 2,400 sq ft





### **USE OR MAKE A TABLE**

A table is an orderly arrangement of data, such as numbers. Making tables helps you keep track of data, spot missing data, and identify data that is asked for in the problem. Because patterns often become obvious when data is organized in a table, this strategy is often used in conjunction with other strategies. In the example below, the table is used to keep track of data and could also be used for identifying a number pattern.

**Problem 11:** Professor Orion and Professor Polaris are working every night this month at the Mt. Olympus Observatory. On Monday night, during his shift from 8–10 p.m., Professor Orion notices 12 new stars. He finds 18 new stars on Tuesday, 30 on Wednesday, 36 on Thursday, and 48 on Friday. Professor Polaris, working the 10–12 p.m. shift, notices 18 new stars on Monday, 26 on Tuesday, 44 on Wednesday, 52 on Thursday, and 70 on Friday. If the professors continue to spot new stars at the same rate, during what day of the week will their combined total equal the number of days in a year?

Solution: Wednesday

Days	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
Orion	12	18	30	36	48	54	66	72	84	90
Polaris	18	26	44	52	70	78	96	104	122	130
Total	30	44	74	88	118	132	162	176	206	220

Days	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
Orion	102	108	120	126	138	144	156
Polaris	148	156	174	182	200	208	226
Total	250	264	294	308	338	352	382



### MAKE AN ORGANIZED LIST

Making an organized list helps you organize your thinking about a problem. Recording work in an organized list makes it easy to review what you have done and to identify important steps that must yet be completed. It also provides a systematic way of recording computations made with given data or recording combinations of given items. For example:

**Problem 27:** Lenny, the zookeeper's assistant, was exhausted. His new charges were eating up a storm. The tiny-toothed trilobat ate 3 ounces of food at each feeding. When Lenny fed the 3-horned hepplefoot, he had to bring 7 ounces of food, while the red-eyed rippersnout ate 29 ounces at each feeding. The fuzzy-necked fizzlenit ate 38 ounces at a feeding and the furry furkadoo ate 44 ounces at a time. After one particularly long day, Lenny made 71 separate feedings, a total of 1,773 ounces of food. How many times did each creature get fed, and how many ounces did each one eat altogether?

Solution: Trilobat: 13 — 39 oz, Hepplefoot: 14 — 98 oz, Rippersnout: 14 — 406 oz, Fizzlenit: 15 — 570 oz, Furkadoo: 15 — 660 oz

	Trilobat	Hepplefoot	Rippersnout	Fizzlenit	Furkadoo
1	3	7	29	38	44
2	6	14	58	76	88
3	9	21	87	114	132
4	12	28	116	152	176
5	15	35	145	190	220
6	18	42	174	228	264
7	21	49	203	266	308
8	24	56	232	304	352
9	27	63	261	342	396
10	30	70	290	380	440
11	33	77	319	418	484
12	36	84	348	456	528
13	*39	91	377	494	572
14	42	*98	*406	532	616
15	45	105	435	•570	*660



### **GUESS AND CHECK**

Guessing and checking is helpful when a problem presents large numbers or many pieces of data, or when the problem asks you to find one solution but not all possible solutions to a problem. When using this strategy, you guess the answer, test to see if it is correct, and make another guess if the previous one was incorrect. In this way, you gradually come closer and closer to a solution by making increasingly more reasonable guesses. You can also use this strategy to get started, and then find another strategy that can be used. Guessing and checking is particularly helpful when a problem presents so many pieces of data that making an organized list becomes a major task. For example:

**Problem 15:** Business was booming for the Sensational Events Company. During the past year they supplied 1295 entertainers for parties. They sent three times as many magicians as clowns, 25% as many fire-eaters as clowns, one third as many people in gorilla suits as magicians, and a number of jugglers equal to the combined number of clowns and magicians. How many magicians, clowns, fire-eaters, people in gorilla suits, and jugglers had Sensational Events supplied for parties during the past year?

Solution: Clowns—140, magicians—420, fire-eaters—35, people in gorilla suits—140, jugglers—560



### **USE OR LOOK FOR A PATTERN**

A pattern is a regular, systematic repetition. A pattern may be numerical, visual, or behavioral. By identifying the pattern, you can predict what will "come next" and what will happen again and again in the same way. Looking for patterns is a very important strategy for problem solving, and is used to solve many different kinds of problems. Sometimes you can solve a problem just by recognizing a pattern, but often you will have to extend a pattern to find a solution. Making a number table often reveals patterns, and for this reason is frequently used in conjunction with the "look for a pattern" strategy. For example:

**Problem 33:** "This is Dino, your dynamite D.J. from radio station KLOT," announced the familiar voice. "All you listeners out there, our Rock Trivia Contest begins today, and we're giving away a LOT of prizes. We're giving one first prize, two second prizes, three third prizes, and so on. Call in and be a winner. Good luck! Go for it!" How many winners will there be by the time the sixteenth-place prizes have been awarded?

Solution: 136

Prize	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Winners	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

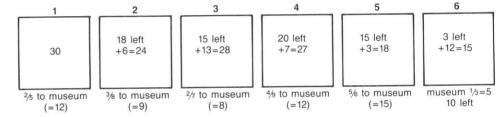


### WORK BACKWARDS

To solve certain problems you must make a series of computations, starting with data presented at the end of the problem and ending with data presented at the beginning of the problem. For example:

**Problem 18:** Maresa and her brother Andrew are on a dinosaur dig in New Mexico. Each day they dig for fossils, and then check in with the resident paleontologist at the end of the day. He helps them to separate the fossils, those for the museum and those they can keep. At the end of the first day ½ of the total goes to the museum. On the second day they find 6 new ones and then ¾ of the total goes to the museum. On the third day they find 13 new fossils and that night the museum keeps ½ of the total. On the fourth day they add 7 and ¼ goes to the museum. On the fifth day they find 3 and the museum keeps ½ of their total. They add 12 on the sixth day, and the museum keeps ½ of their collection. At the end of their dig Andrew and Maresa have 10 fossils to take home. How many new fossils did they collect for the museum?

Solution: 61





### USE LOGICAL REASONING

Logical reasoning is really used for all problem solving. However, there are types of problems that include or imply various conditional statements such as: "if...then," or "if...then...else," or "if something is true, then..." or "if something is not true, then..." The data given in the problems can often be displayed in a chart or matrix. This kind of problem requires formal logical reasoning as you step your way through the statements given in the problem. For example:

**Problem 21:** The planets in the Andromedia galaxy are working out a system of trade. The main resource of cold, dark Dox is ice. Hot, dry Sterp has a large quantity of warm sand, which the Doxians need. Aquam, a watery planet with little land, could also use the sand. Torf, which has acres of ancient forests, is a source of fuel for the other planets. So far the planets have worked out this exchange rate: 6 pounds of Doxian ice for 4 pounds of sand from Sterp; 9 pounds of Aquam water for 11 pounds of fuel from Torf; and 6 pounds of Aquam water for 8 pounds of Doxian ice. The residents of Sterp would also like to trade with Torf. Based on the current rates, what would be the exchange rate for fuel and sand?

Solution: 16 pounds of sand for 22 pounds of fuel

```
If
6 pounds ice = 4 pounds sand
9 pounds water = 11 pounds fuel
6 pounds water = 8 pounds ice

Then
24 pounds ice = 16 pounds sand (4 × 6 and 4 × 4)
18 pounds water = 22 pounds fuel (2 × 9 and 2 × 11)
18 pounds water = 24 pounds ice (3 × 6 and 3 × 8)

Then
24 pounds ice = 22 pounds fuel
and
22 pounds fuel = 16 pounds sand
```



### MAKE IT SIMPLER

You may find it helpful to make a problem simpler, especially when you begin to solve complex problems. Making a problem simpler may mean reducing large numbers to small numbers, or reducing the number of items given in a problem. The simpler representation of the problem, then, may suggest what operation or process you can use to solve the more complex problem. The simpler representation may even reveal a pattern that can be used to solve the problem. For example:

**Problem 38:** On a large planet in a distant galaxy lives Mr. Alphotec with his collection of 85,800 exotic animals. He has one-half as many auples as burfuls, one-third as many burfuls as curlaps, and one-fourth as many curlaps as drolops. Auples have soft patches of fur dotting their tubular bodies. Blue burfuls have gold spots all over their pyramid-shaped bodies. Curlaps have metallic green plates covering their egg-shaped bodies, and drolops have thick, pointed hairs all over their short, stubby bodies. If an auple is worth \$43.50, a drolop sells for \$12.50, a curlap is valued at \$22.50, and a burful is worth \$30.00, what is the value of Mr. Alphotec's collection?

Solution: \$1,400,100 (\$113,000 + \$156,000 + \$351,000 + \$780,000)



### **BRAINSTORM**

This strategy is often used when all else fails. When you cannot think of a similar problem that you have solved before, and cannot think of another strategy to use, brainstorming is a good strategy to try. Brainstorming means looking at a problem in new and inventive ways. There are always problems that stretch people beyond their experience and expertise. When you encounter problems that you cannot solve, try to open up your mind, allow for inspiration, be creative, be flexible, and keep on trying until a light goes on! For example:

**Problem 47:** The Beachcomber Cafe is having a contest. Winners get a free milkshake. The contest question is: When you multiply 5 times 115469 on the calculator, what do you find that is ordinarily found at the beach? You are a winner! What is your answer?

SHELLS

Solution: Shells

### How Can You Teach the Four-Step Method and Solution Strategies?

Use the teaching plans. Read the problem aloud. Then begin the sequence of questions given in the teaching plan for the problem. Name each step as you begin it. Encourage the students to exchange a variety of ideas and opinions after each question is raised. The questions will lead the students through the four steps and serve as a model for them to follow when they must develop their own questions for solving later problems. The fourstep method is illustrated below by the teaching plan for problem 20. Italicized responses following the guestions are examples of the kinds of responses you want to elicit from students.



### MAKE A PICTURE OR DIAGRAM

Teaching Plan

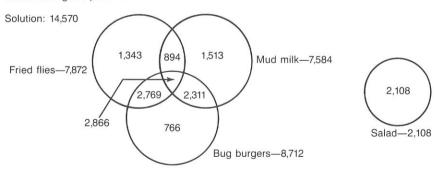
The demand for earthworm specialties at Elliot's Underground Gourmet Shop is growing. During the past month Elliot sold 7,584 glasses of mud milk, 7,872 orders of fried flies, and 8,712 bug burgers. He had 2,866 customers buy all three on the combination plate; 2,108 bought just the sawdust salad platter; 894 bought fried flies and a glass of mud milk; 2,769 bought a bug burger and some fried flies; and 2,311 bought mud milk and a bug burger. How many customers visited Elliot's Gourmet Shop in the past month?

- FIND OUT What is the question you have to answer? How many customers visited Elliot's Gourmet Shop in the past month?
  - How many glasses of mud milk were sold? 7,584 Orders of fried flies? 7,872 Bug burgers?
  - How many customers bought the combination plate? 2,866 The sawdust platter? 2,108 Fried flies and mud milk? 894 A bug burger and fried flies? 2,769 A bug burger and mud

### CHOOSE A STRATEGY

• Is there a special kind of diagram that can help you organize all the information in the problem? Yes, we can make a Venn diagram.

- SOLVE IT If you make a Venn diagram, how many circles will you have to draw? 4 What will you label each of the circles? Fried flies, mud milk, bug burgers, and salad Do the circles intersect? Three of the circles intersect. Why or why not? Three intersect to show that more than one kind of food was bought by a customer; one circle is for the customers who bought only salad.
  - How many fried flies were sold? 7.872 Where will you write this number? *Under the label* outside the circle How many fried flies with mud milks were sold? 894 Where will you write this number? In the intersection between fried flies and mud milk How many fried flies with bug burgers were sold? 2,769 Where will you write this number? In the intersection between fried flies and bug burgers How many fried flies with mud milk and bug burgers were sold? 2,866 Where will you write this number? In the center intersection between all 3 circles Then how many fried flies were sold without mud milk or without bug burgers? 1,343 Where does this number go? Outside the intersections in the fried flies circle
  - (Have the students finish their diagrams.) What was the total number of customers at Elliot's during the past month?



LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

EXTEND IT • Elliot's Outlet sold 582 fried flies, 669 burgers, 934 mud milks, 323 salads, and 295 spider soups. They sold 320 flies with burgers; 508 burgers with milk; 411 milk with flies; 416 flies with burgers and milk; 301 salads with milk. How many customers did the outlet have?

When you have completed the four steps with the students, you can give them the problem extension to solve as further practice.

After your students have solved one or more groups of two problems, you can give them similar practice problems from Section 2. The practice problems related to a given group are identified in the teaching plan for the second problem in that group.

Problem Number \_\_\_\_\_

- **FIND OUT** What is the question you have to answer?
  - · What information does the problem give you?

CHOOSE A STRATEGY

















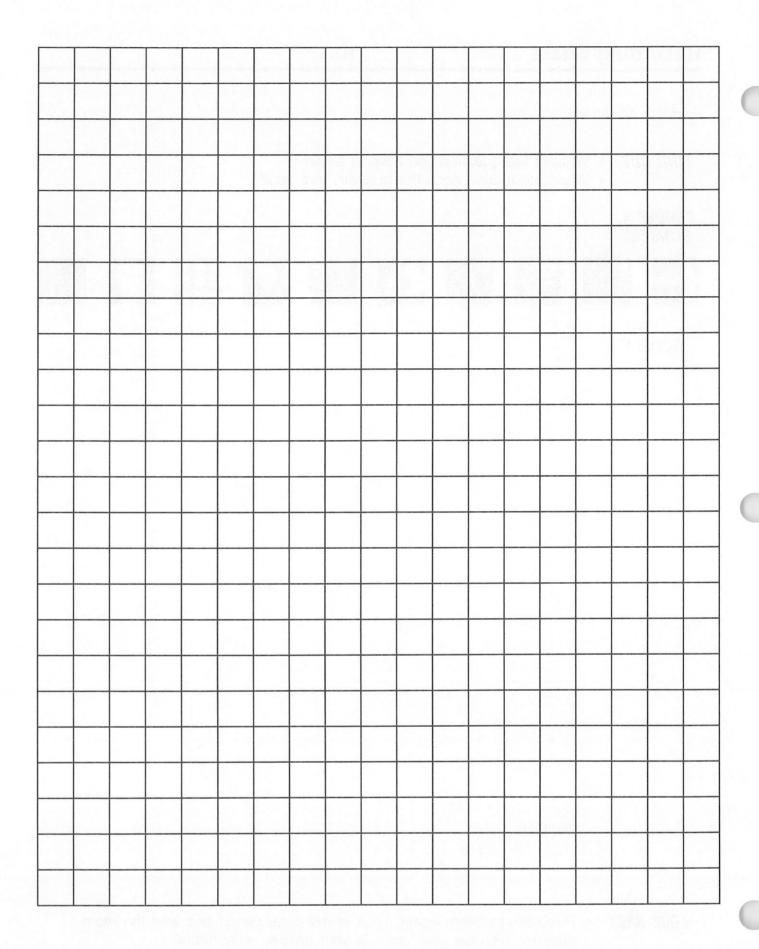




SOLVE IT

Solution:

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?



Yesterday afternoon Jason fed all of the fish in his fish tanks. He has five different fish, and each one is on a different feeding schedule. He feeds the glass fish every 2 days, the black molly every 3 days, and the catfish every 5 days. He feeds the eel every  $2\frac{1}{2}$  days, and the angel fish every  $1\frac{1}{2}$  days. On how many days in the next 30 days will Jason feed at least 4 fish?

- **FIND OUT** What is the question you have to answer?
  - What are the feeding schedules of Jason's fish?
  - · When did Jason last feed all of the fish?

- **CHOOSE A** Do you need to keep track of what fish are fed on each day?
- **STRATEGY** Is there a way to organize and lay out the information to help you see when different fish are fed on the same day?

- **SOLVE IT** When you make the table, what will you use as the column headings? What will you keep track of in the rows? How many rows do you need?
  - How often does Jason have to feed the catfish? Then on what days will he feed the catfish?
  - How often does Jason have to feed the angelfish? Do you have to record the angelfish feeding times in a different way than the catfish feeding times? On what days will Jason feed the angelfish, and will he feed it in the morning or the afternoon?
  - Continue to fill in your table. How many fish will Jason feed on the second
  - On how many days in the next 30 days will Jason feed at least 4 fish?

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Catfish					X														
Molly			×																
Angel		×/						/		/									
Eel			X/																
Glass		X																	

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

Yesterday afternoon Jason fed all of the fish in his fish tanks. He has five different fish. and each one is on a different feeding schedule. He feeds the glass fish every 2 days. the black molly every 3 days, and the catfish every 5 days. He feeds the eel every 21/2 days, and the angel fish every 11/2 days. On how many days in the next 30 days will Jason feed at least 4 fish?

- FIND OUT What is the question you have to answer? On how many days in the next 30 days will Jason feed at least 4 fish?
  - What are the feeding schedules of Jason's fish? Glass fish-every 2 days; black mollyevery 3 days; catfish—every 5 days; eel—every 21/2 days; angel fish—every 11/2 days
  - · When did Jason last feed all of the fish? Yesterday afternoon

### CHOOSE A STRATEGY

- Do you need to keep track of the days and what fish are fed on each day? Yes
- Is there a way to organize and lay out the information to help you see when different fish are fed on the same day? Yes, we can make a table.

### SOLVE IT

- When you make the table, what will you use as the column headings? The numbers of the days What will you keep track of in the rows? The 5 fish How many rows do you need? 5
- · How often does Jason have to feed the catfish? Every 5 days Then on what days will he feed the catfish? 5, 10, 15, 20, 25, 30
- How often does Jason have to feed the angelfish? Every 1½ days Do you have to record the angelfish feeding times in a different way than the catfish feeding times? Yes, we have to record the angelfish schedule in half days, rather than in whole days. On what days will Jason feed the angelfish, and will he feed it in the morning or the afternoon? 2, morning; 3. afternoon; 5. morning (See completed table below.)
- (Have students continue filling in their tables.) How many fish will Jason feed on the second day? 2
- On how many days in the next 30 days will Jason feed at least 4 fish?

Solution: 4 (15th, 18th, 20th, 30th)

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Catfish		_	-	-	×		Ė	-		X		-		· ·	×		-		-	×					×				118	X
Molly			X			X			X			X			X			X			X			X			X			X
Angel		X/	X	7	X	X	7	X	X		X	X	/	X	X		X	X		×/	X		×/	X	$\vee$	×/	X		×/	/x
Eel		/	X	7	X	1	/	X		X	1		X/		X			X/		1			X/		X			<b>X</b> /		1
Glass		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT • On the fourth day of the 30-day period, Jason bought a goldfish which he has to feed every 31/2 days. He fed the goldfish the afternoon that he bought it. On how many days did he feed 4 or more fish?

King Midas is well known for his golden touch, but Millicent Metals hasn't gotten such good press. She also changed many of the things she touched, but not into gold. Every 12th object she touched turned to platinum. Every 16th object became silver, every 9th object turned to copper, and every 7th object changed to brass. She kept count of the objects she touched, and found that the 36th object turned into a combination of platinum and copper. If Millicent touched 150 objects in all, how many of them changed to combinations of 2 metals, and how many changed to combinations of 3 metals?

- **FIND OUT** What is the question you have to answer?
  - What happened to many of the things Millicent touched?
  - What did every 12th object become? Every 16th object? Every 9th object? Every 7th object?
  - What happened to the 36th object Millicent touched? Why?
  - How many objects did Millicent touch in all?

### CHOOSE A STRATEGY

- Do you need to keep track of each object Millicent touched, or just the objects that turned to metal?
- Is there a way to organize and lay out the information, to see which objects became more than one metal?

- **SOLVE IT** How can you set up the table to help you see which objects changed to which metal, and which objects turned into more than one metal?
  - What do you want to use for row labels? How many rows do you need?
  - What is the first object that became silver? The next object? The last object?
  - Continue to fill in your table to find out how many objects were changed into a combination of 2 metals or a combination of 3 metals.

### **Objects Turned to Metal**

Silver	16	32		
Platinum	12	24		
Copper				
Brass				

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

King Midas is well known for his golden touch, but Millicent Metals hasn't gotten such good press. She also changed many of the things she touched, but not into gold. Every 12th object she touched turned to platinum. Every 16th object became silver, every 9th object turned to copper, and every 7th object changed to brass. She kept count of the objects she touched, and found that the 36th object turned into a combination of platinum and copper. If Millicent touched 150 objects in all, how many of them changed to combinations of 2 metals, and how many changed to combinations of 3 metals?

- FIND OUT What is the question you have to answer? If Millicent touched 150 objects, how many of them changed to combinations of 2 metals, and how many changed to combinations of 3 metals?
  - What happened to many of the things Millicent touched? They turned to metal.
  - What did every 12th object become? Platinum Every 16th object? Silver Every 9th object? Copper Every 7th object? Brass
  - What happened to the 36th object Millicent touched? It turned into a combination of platinum and copper. Why? Because 36 is a multiple of 12 and 9; every 12th item Millicent touched turned to platinum, and every 9th item she touched turned to copper.
  - How many objects did Millicent touch in all? 150

### CHOOSE A STRATEGY

- Do you need to keep track of each object Millicent touched, or just the objects that turned to metal? Just the objects that turned to metal
- Is there a way to organize and lay out the information, to see which objects became more than one metal? Yes, we can make a table.

### SOLVE IT

- How can you set up the table to help you see which objects changed to which metal, and which objects turned into more than one metal? We can record which objects turned to each metal, and then look for objects that were changed to more than one metal.
- What do you want to use for row labels? The names of the metals How many rows do you need? 4
- What is the first object that became silver? 16th The next object? 32nd The last object?
- (Have students continue to fill in their tables.) How many objects changed into a combination of 2 metals or a combination of 3 metals?

Solution: Combination of 2 metals—9 objects; combination of 3 metals—1 object

### **Objects Turned to Metal**

Silver	16 32 48 64 80 96 (12) 128 (144)
Platinum	12 24 (36)(48) 60 (72) (84) (96)(108) 120 132 (144)
Copper	9 18 27 (36) 45 54 (63) (72) 81 90 99 (108) 117 (126) 135 (144)
Brass	7 14 21 28 35 42 49 56 (63) 70 77 (84) 91 98 105 (112) 119 (126) 133 140 147

2 metals: 36, 48, 63, 72, 84, 96, 108, 112, 126; 3 metals: 144

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

**EXTEND IT** • How many of the 150 objects would have changed into combinations of metals if every 6th object became copper, every 14th became brass, every 18th became silver, and every 21st became platinum?

PRACTICE • Similar Practice Problems: 50, 66, 76

Laura is in charge of lighting the Rock Palace for the upcoming Willy Nilly Wild Ones concert. Each light fixture supplies exactly 1,000 watts of power to light the bulbs in the fixture. Laura can use any combination of 150-watt, 100-watt, 75-watt, or 60-watt bulbs, but the total number of watts must be 1,000. How many different combinations of bulbs could Laura use in a light fixture?

- **FIND OUT** What is the question you have to answer?
  - What must the total number of watts be for each combination of bulbs?
  - What sizes of light bulbs can Laura use?

- **CHOOSE A** What is one possible combination of bulbs that totals 1,000 watts?
- STRATEGY How can you systematically record all possible combinations of bulbs that Laura could use?

- **SOLVE IT** Make a list. What do you want to keep track of in the first column of your list? Second column? Third column? Fourth column?
  - What do you want to keep track of in each row of your list?
  - What is the largest number of 150-watt bulbs Laura can use? What is their total wattage? Does Laura need to use any 100-watt bulbs? How many? Any 75-watt bulbs? Any 60-watt bulbs?
  - Is there any other combination that could be made with 6 150-watt bulbs?
  - Is there a combination that could be made with 5 150-watt bulbs? What is their total wattage? Does Laura need to use any 100-watt bulbs? How many? Any 75-watt bulbs? How many?
  - Is there any other combination that could be made with 5 150-watt bulbs?
  - Continue to fill in your list to find out how many different combinations of bulbs Laura could use.

e de la constante de la consta	150w	100w	75w	60w
Γ	6	1	0	0
	5	1	2	0
	4	4	0	0
	4	1	4	0

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?

Laura is in charge of lighting the Rock Palace for the upcoming Willy Nilly Wild Ones concert. Each light fixture supplies exactly 1,000 watts of power to light the bulbs in the fixture. Laura can use any combination of 150-watt, 100-watt, 75-watt, or 60-watt bulbs, but the total number of watts must be 1,000. How many different combinations of bulbs could Laura use in a light fixture?

### FIND OUT

- What is the question you have to answer? How many different combinations of bulbs could Laura use in a light fixture?
- What must the total number of watts be for each combination of bulbs? 1,000 watts
- What sizes of light bulbs can Laura use? 150 watts, 100 watts, 75 watts, 60 watts

### CHOOSE A STRATEGY

- What is one possible combination of bulbs that totals 1,000 watts? 4 150-watt bulbs, and 4 100-watt bulbs
- How can you systematically record all possible combinations of bulbs that Laura could use? We can make an organized list.

### SOLVE IT

- Make a list. What do you want to keep track of in the first column of your list? 150-watt bulbs Second column? 100-watt bulbs Third column? 75-watt bulbs Fourth column? 60-watt bulbs
- What do you want to keep track of in each row of your list? The number of each size bulb in the combination
- What is the largest number of 150-watt bulbs Laura can use? 6 What is their total wattage? 900 Does Laura need to use any 100-watt bulbs? Yes How many? 1 Any 75-watt bulbs? No Any 60-watt bulbs? No
- Is there any other combination that could be made with 6 150-watt bulbs? No
- Is there a combination that could be made with 5 150-watt bulbs? Yes What is their total wattage? 750 Does Laura need to use any 100-watt bulbs? Yes How many? 1 Any 75-watt bulbs? Yes How many? 2
- Is there any other combination that could be made with 5 150-watt bulbs? No
- (Have students complete their lists. Encourage them to use a systematic way of finding and recording their combinations so as not to miss any.) How many different combinations of bulbs could Laura use in a light fixture?

Solution: 29

150w	100w	75w	60w		150w	100w	75w	60w
6	1	0	0	1	1	4	6	0
5	1	2	0		1	4	2	5
4	4	0	0		1	1	6	5
4	1	4	0		1	1	2	10
4	1	0	5		0	10	0	0
3	4	2	0		0	7	4	0
3	1	6	0		0	7	0	5
3	1	2	5		0	4	8	0
2	7	0	0		0	4	4	5
2	4	4	0		0	4	0	10
2	4	0	5		0	1	12	0
2	1	8	0		0	1	8	5
2 2 2	1	4	5		0	1	4	10
2	1	0	10		0	1	0	15
1	7	2	0					
	(conti	nued)	'			'		1

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?

**EXTEND IT** • If the light fixture takes a maximum of 10 bulbs, how many combinations would Laura be able to use?

Arturo is selling ads for his school paper, The Gab Gazette. Burger Binge, Cobbies' Fish & Chips, and Doolittle Donuts all want to buy ads, but Arturo has only one page left to fill. The sizes of his ads are  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ , and  $\frac{1}{16}$  of a page. Before Arturo sells the ads to the restaurants, he has to figure out all of the different combinations of his ads that would fill one page. How many combinations are there?

- **FIND OUT** What is the question you have to answer?
  - What is Arturo doing?
  - How many different sizes of ads does Arturo have? What are the sizes?
  - How much space does Arturo have left to fill?

### CHOOSE A STRATEGY

- If you begin with a ½-page ad, what can you add to this to make 1 full page? Are there other combinations, beginning with a  $\frac{1}{2}$ -page ad?
- How can you systematically record all the possible combinations of ads that would fill one page?

- **SOLVE IT** What do you want to keep track of in the first column of your list? Second column? Third column? Fourth column?
  - What do you want to keep track of in each row of your list?
  - What is the largest number of  $\frac{1}{2}$ -page ads you could use? What is the total space covered by those ads? Do you have to use any  $\frac{1}{4}$ -page ads,  $\frac{1}{8}$ -page ads, or  $\frac{1}{16}$ -page ads with the  $\frac{1}{2}$ -page ads?
  - How many different combinations can you make, using one  $\frac{1}{2}$ -page ad?
  - Finish your organized list to find out how many different combinations of Arturo's ads would fill one page.

1/2	1/4	1/8	16
2	0	0	0
1	2	0	0
1	1	2	0
1	1	1	2

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?



### MAKE AN ORGANIZED LIST

### Teaching Plan

Arturo is selling ads for his school paper, The Gab Gazette. Burger Binge, Cobbies' Fish & Chips, and Doolittle Donuts all want to buy ads, but Arturo has only one page left to fill. The sizes of his ads are ½, ¼, ¼, and ¼6 of a page. Before Arturo sells the ads to the restaurants, he has to figure out all of the different combinations of his ads that would fill one page. How many combinations are there?

### FIND OUT

 What is the question you have to answer? How many different combinations of Arturo's ads would fill a page?

What is Arturo doing? He is selling ads for his school paper.

- How many different sizes of ads does Arturo have? 4 What are the sizes? 1/2 page, 1/4 page, 1/8 page, and 1/16 page
- How much space does Arturo have left to fill? 1 page

### CHOOSE A STRATEGY

- If you begin with a ½-page ad, what can you add to this to make 1 full page? Another ½page ad Are there other combinations, beginning with a 1/2-page ad? Yes, with two 1/4-page ads.
- · How can you systematically record all the possible combinations of ads that would fill one page? We can make an organized list.

### SOLVE IT

- What do you want to keep track of in the first column of your list? 1/2-page ads Second column? 1/4-page ads Third column? 1/8-page ads Fourth column? 1/16-page ads
- What do you want to keep track of in each row of your list? The number of each size ad in
- What is the largest number of 1/2-page ads you could use? 2 What is the total space covered by those ads? 1 page Do you have to use any 1/4-page ads, 1/8-page ads, or 1/16-page ads with the 1/2-page ads? No

How many different combinations can you make, using one ½-page ad?

(Have students finish their organized lists.) How many different combinations of Arturo's ads would fill one page?

Solution: 35	1/2	1/4	1/8	1/16	1/2	1/4	1/8	1/16
	2	0	0	0	0	2	0	8
	1	2	0	0	0	1	6	0
	1	1	2	0	0	1	5	2
	1	1	1	2	0	1	4	4
	1	1	0	4	0	1	3	6
	1	0	4	0	0	1	2	8
	1	0	3	2	0	1	1	10
	1	0	2	4	0	1	0	12
	1	0	1	6	0	0	8	0
	1	0	0	8	0	0	7	2
	0	4	0	0	0	0	6	4
	0	3	2	0	0	0	5	6
	0	3	1	2	0	0	4	8
	0	3	0	4	0	0	3	10
	0	2	4	0	0	0	2	12
	0	2	3	2	0	0	1	14
	0	2	2	4	0	0	0	16
1	0	2	1	6				
		(conti	nued)					

LOOK BACK

- Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?
- **EXTEND IT** How many different combinations would fill one page if the sizes of Arturo's ads were ½, 1/3, 1/4, 1/6, and 1/12 of a page?
- PRACTICE Similar Practice Problems: 49, 63, 80



### ACT OUT OR USE OBJECTS

Name

Kane works as a salesclerk at the Threads Swimshop. His manager asked him to arrange a shirt, a pair of shorts, a pair of sandals, a swimsuit, a beach towel, a beach bag, suntan lotion, a magazine, a can of soda pop, and a pair of sunglasses in a display case. When Kane finished the job, the suntan lotion was behind the sunglasses and to the left of the beach bag. The beach towel was between the shirt and the suntan lotion. The sunglasses were to the right of the shorts and swimsuit. The shorts were not in front of the beach towel. The can of soda pop was to the right of the beach bag, and the sandals were to the left of the magazine. Where was each product in the display case?

- **FIND OUT** What is the question you have to answer?
  - How many products were there?
  - What clues do you have about the location of each product?

### CHOOSE A STRATEGY

 Would it help to have pieces of paper, or something to represent each product, and be able to move them around?

- **SOLVE IT** If you use pieces of paper, how would you label them?
  - Where will you put the suntan lotion? If you don't know where to begin, make a quess.
  - · Where was the beach towel?
  - Where were the sunglasses?
  - Continue to move the "products" around until you find a place for each one.

suntan lotion

sunglasses

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your set-up of the display case. Is your answer reasonable?



### **ACT OUT OR USE OBJECTS**

### Teaching Plan

Kane works as a salesclerk at the Threads Swimshop. His manager asked him to arrange a shirt, a pair of shorts, a pair of sandals, a swimsuit, a beach towel, a beach bag, suntan lotion, a magazine, a can of soda pop, and a pair of sunglasses in a display case. When Kane finished the job, the suntan lotion was behind the sunglasses and to the left of the beach bag. The beach towel was between the shirt and the suntan lotion. The sunglasses were to the right of the shorts and swimsuit. The shorts were not in front of the beach towel. The can of soda pop was to the right of the beach bag, and the sandals were to the left of the magazine. Where was each product in the display case?

- FIND OUT What is the question you have to answer? Where was each product in the display case?
  - How many products were there? 10
  - What clues do you have about the location of each product? The suntan lotion was behind the sunglasses and to the left of the beach bag. The beach towel was between the shirt and the suntan lotion. The sunglasses were to the right of the shorts and swimsuit. The shorts were not in front of the beach towel. The can of soda pop was to the right of the beach bag. The sandals were to the left of the magazine.

### CHOOSE A STRATEGY

 Would it help to have pieces of paper, or something to represent each product, and be able to move them around? Yes, using objects makes it easy to change positions when the arrangement doesn't match a clue.

### SOLVE IT

- If you use pieces of paper, how would you label them? With the names of the 10 products
  - Where will you put the suntan lotion? Behind the sunglasses and to the left of the beach bag. If you don't know where to begin, make a guess.
- Where was the beach towel? Between the shirt and the suntan lotion
- · Where were the sunglasses? In front of the suntan lotion, and to the right of the shorts and swimsuit
- (Have the students continue to move the "products" around until they find a place for each one.) Where was each product in the display case?

### Solution:

shirt	beach	suntan	beach	can of
	towel	lotion	bag	soda
shorts	swimsuit	sunglasses	sandals	magazine

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your set-up of the display case. Is your answer reasonable?

**EXTEND IT** • Make up a similar problem, using the shop's winter products: gloves, backpacks, wool socks, sweaters, jackets, goggles, wool hats, wool scarves, earmuffs, boots.

As a marine biologist on the submarine Neptune, Petra is charting the number of fish in specific areas of the oceans. She is currently surveying an area in the Atlantic. Petra observes the fish from a 10-room viewing area in the front of the sub. The rooms are arranged on three sides, as shown below. One day as Petra was counting the fish, she saw 20 fish through the windows of Rooms A, B, C, and D; 20 fish through the windows of Rooms D, E, F, and G; and 20 fish through the windows of Rooms G, H, I, and J. She saw a different number of fish through the windows in each room. She counted at least 1 but not more than 12 through the window of each room. How many fish did Petra see through the window of each room?

D	Е	F	G
С			Н
В			ı
А			J

- **FIND OUT** What is the question you have to answer?
  - How many rooms were there on each side of the viewing area? How many rooms were there in all? Which room was shared by the left side and the front side of the sub? Which room was shared by the front side and the right side of the sub?
  - What do you know about the number of fish that Petra could see through the windows of each room?
  - How many fish did Petra see outside the left side of the sub? Outside the front side of the sub? Outside the right side of the sub?

### CHOOSE A STRATEGY

• Would it help to have pieces of paper to represent different numbers of fish, so that you could move them around?

- SOLVE IT What is the total number of fish Petra saw from each side of the sub? How many rooms were there on each side of the sub?
  - What are some combinations of four numbers that sum to 20?
  - If Petra saw at least 1 fish and not more than 12 fish from the window of each room, what numbers could you write on your pieces of paper?
  - Will you want to use the same number more than once?
  - Try using the numbers 2, 8, 6, and 4 in Rooms A, B, C, and D. Now try filling in Rooms E, F, and G. What must the numbers in those rooms sum to? Do vour numbers make the correct sum?
  - Keep moving your numbers around until you find out how many fish Petra saw from the window of each room.

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?



### ACT OUT OR USE OBJECTS

### Teaching Plan

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As a marine biologist on the submarine Neptune, Petra is charting the number of fish in specific areas of the oceans. She is currently surveying an area in the Atlantic. Petra observes the fish from a 10-room viewing area in the front of the sub. The rooms are arranged on three sides, as shown below. One day as Petra was counting the fish, she saw 20 fish through the windows of Rooms A, B, C, and D; 20 fish through the windows of Rooms D, E, F, and G; and 20 fish through the windows of Rooms G, H, I, and J. She saw a different number of fish through the windows in each room. She counted at least 1 but not more than 12 through the window of each room. How many fish did Petra see through the window of each room? DEFG

FIND OUT • What is the question you have to answer? How many fish did Petra see through the window of each room?

В

- How many rooms were there on each side of the viewing area? 4 How many rooms were there in all? 10 Which room was shared by the left side and the front side of the sub? D Which room was shared by the front side and the right side of the sub? G
- . What do you know about the number of fish that Petra could see through the windows of each room? She saw a different number of fish through the windows of each room. She saw at least 1, but not more than 12, through the windows of each room.
- How many fish did Petra see outside the left side of the sub? 20 Outside the front side of the sub? 20 Outside the right side of the sub? 20

CHOOSE A STRATEGY · Would it help to have pieces of paper to represent different numbers of fish, so that you could move them around? Yes, then we can easily change the location of a number when a clue doesn't match our arrangement.

SOLVE IT

- (This is just an example of one way to begin this problem. You will get many different kinds of responses. Encourage experimentation!)
- What is the total number of fish that Petra saw from each side of the sub? 20 How many rooms were there on each side of the sub? 4
- What are some combinations of four numbers that sum to 20? 2 + 8 + 6 + 4, 1 + 9 + 3 + 7, 1 + 2 + 5 + 12, 3 + 4 + 5 + 8
- · If Petra saw at least 1 fish and not more than 12 fish from the window of each room, what numbers could you write on your pieces of paper? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- Will you want to use the same number more than once? No, because the problem says that Petra saw a different number of fish from each window.
- Try using the numbers 2, 8, 6, and 4 in Rooms A, B, C, and D. Now try filling in Rooms E, F, and G. What must the numbers in those rooms sum to? 20 Do your numbers make the
- (Suggest that students keep experimenting by moving their numbers around. Students may find different solutions from the one shown below.) How many fish did Petra see from the window of each room?

Solution:

2	5	12	1
8	П		9
6			3
4			7

LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • If Petra had seen 24 fish from the windows on each side of the sub, how many would she have seen from the windows of each room?

PRACTICE • Similar Practice Problems: 52, 62, 78



It's the 5th Annual Fernwick Frog-Jumping Contest! Claudette and her frog Clementine have been practicing many weeks for this big event. Each frog gets a certain number of tries. A try is made up of three jumps, and the score for the try is the total of the scores for those three jumps. In the Fernwick Contest today, Clementine jumped a total of 12 feet on each of her tries. Her shortest jump was 1 foot, her longest jump was 8 feet, and she jumped a whole number of feet on each jump. How many different ways could Clementine have scored 12 feet in three jumps? (Remember that 2 + 3 + 7 is different from 3 + 7 + 2.)

- **FIND OUT** What is the question you have to answer?
  - How many jumps did Clementine make on each try?
  - · What was Clementine's total for each try?
  - What was Clementine's shortest jump? What was her longest jump? Then what are the possible scores for one of Clementine's jumps?

# STRATEGY

- CHOOSE A Let's say that Clementine scored 3 feet on the first jump, 4 feet on the second jump, and 5 feet on the third jump. Would that be the same as scoring 3 feet on the first jump, 5 feet on the second jump, and 4 feet on the third jump? Is there another way for Clementine to score a total of 12 feet with the same three scores?
  - Is there a systematic way to record all the possible ways Clementine could have scored 12 feet?

- **SOLVE IT** What are the possible scores for one jump?
  - What are the possible combinations of three scores that total 12 feet?
  - One way to set up an organized list is to have three columns labeled Jump 1, Jump 2, Jump 3. Begin with a score of 1 for Jump 1, 3 for Jump 2, and 8 for Jump 3. Is there another way to arrange these same scores, using 1 for Jump 1 again?
  - How many ways can you arrange these same scores, using the score of 3 for Jump 1?
  - How many ways can you arrange these same scores, using the score of 8 for
  - Finish your list to find all the possible ways Clementine could have scored 12 feet in three jumps.

Combinations of 12	Jump 1	Jump 2	Jump 3
1 + 3 + 8	1	3	8
1 + 4 + 7	1	8	3
	3	1	8
	3	8	1 1

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

It's the 5th Annual Fernwick Frog-Jumping Contest! Claudette and her frog Clementine have been practicing many weeks for this big event. Each frog gets a certain number of tries. A try is made up of three jumps, and the score for the try is the total of the scores for those three jumps. In the Fernwick Contest today, Clementine jumped a total of 12 feet on each of her tries. Her shortest jump was 1 foot, her longest jump was 8 feet, and she jumped a whole number of feet on each jump. How many different ways could Clementine have scored 12 feet in three jumps? (Remember that 2 + 3 + 7 is different from 3 + 7 + 2.)

- FIND OUT What is the question you have to answer? How many different ways could Clementine have scored 12 feet in three jumps?
  - How many jumps did Clementine make on each try? 3
  - What was Clementine's total for each try? 12 feet
  - What was Clementine's shortest jump? 1 foot What was her longest jump? 8 feet Then what are the possible scores for each of Clementine's jumps? 1, 2, 3, 4, 5, 6, 7, 8

### CHOOSE A STRATEGY

- Let's say that Clementine scored 3 feet on the first jump, 4 feet on the second jump, and 5 feet on the third jump. Would that be the same as scoring 3 feet on the first jump, 5 feet on the second jump, and 4 feet on the third jump? No Is there another way for Clementine to score a total of 12 feet with the same three scores? Yes: 4-3-5.
- Is there a systematic way to record all the possible ways Clementine could have scored 12 feet? Yes, we can make an organized list.

### SOLVE IT

- What are the possible scores for one jump? 1, 2, 3, 4, 5, 6, 7, 8
- What are the possible combinations of three scores that total 12 feet? 1+3+8, 1+4+7. 1+5+6, 2+2+8, 2+3+7, 2+4+6, 2+5+5, 3+3+6, 3+4+5, 4+4+4.
- One way to set up an organized list is to have three columns labeled Jump 1, Jump 2. Jump 3. Begin with a score of 1 for Jump 1, 3 for Jump 2, and 8 for Jump 3. Is there another way to arrange these same scores, using 1 for Jump 1 again? Yes, 1-8-3.
- How many ways can you arrange these same scores, using the score of 3 for Jump 1? Two ways: 3-1-8, 3-8-1.
- How many ways can you arrange these same scores, using the score of 8 for Jump 1? Two ways: 8-1-3, 8-3-1.
- (Have students finish their lists.) How many different ways could Clementine have scored 12 feet in three jumps?

Solution: 46 different ways

Combinations of 12	Jump 1	Jump 2	Jump 3	J-1	J-2	J-3	J-1	J-2	J-3
1 + 3 + 8	1	3	8	1	5	6	3	7	2
1 + 4 + 7	B / 1	8	3	1	6	5	7	2	3
1 + 5 + 6	3	1	8	5	1	6	7	3	2
2 + 2 + 8	3	8	1	5	6	1	2	4	6
2 + 3 + 7	8	3	1 1	6	1	5	2	6	4
2 + 4 + 6	8	1	3	6	5	1	4	2	6
2 + 5 + 5	1	4	7	2	2	8	4	6	2
3 + 3 + 6	1	7	4	2	8	2	6	2	4
3 + 4 + 5	4	1	7	8	2	2	6	4	2
4 + 4 + 4	4	7	1 1	2	3	7	2	5	5
	7	1	4	2	7	3	5	2	5
	7	4	1 1	3	2	7	5	5	2
		(continued)		(c	ontinue	ed)		Solution plete	ons for list.)

- LOOK BACK Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?
- **EXTEND IT** How many different ways could Clementine have scored a total of 15 feet in three jumps?

It happened one day in the towns around Fugit Sound! Television sets started broadcasting the shows they wanted to watch. Radios, stereos, and tape players began playing what they wanted to hear. It was the kind of day science fiction is made of. In one hour, Fred's Repair Shop received 17 calls from owners of sound equipment. Each call was about a radio, a television, a stereo, or a tape player. At least 2 calls came in about each kind of equipment, but the number of calls was different for each kind of equipment. How many different ways could the calls have been divided among the kinds of equipment?

- **FIND OUT** What is the question you have to answer?
  - What was happening in Fred's Repair Shop?
  - · How many calls did Fred receive in one hour?
  - · What do you know about the calls that came in?

## STRATEGY

- **CHOOSE A** Let's say that there were 2 calls about televisions, 4 calls about radios, 3 calls about stereos, and 8 calls about tape players. If you use those same four numbers, is there another way you could arrange them?
  - Is there a systematic way to record all the possible ways the calls could have been divided?

- **SOLVE IT** What is the least number of calls that came in about each kind of sound equipment? The greatest number? What are the numbers of calls possible for each kind of equipment?
  - Using those numbers, what combinations of four numbers equal 17?
  - One way to set up an organized list is to have four columns labeled TVs. Radios, Stereos, Tape Players. Begin by putting 2 under TVs, 3 under Radios, 4 under Stereos, and 8 under Tape Players. Are there other ways to arrange these numbers, putting 2 under TVs?
  - How many ways can you arrange these same numbers, putting 3 under TVs?
  - How many ways can you arrange these same numbers, putting 4 under TVs? Putting 8 under TVs?
  - Finish your list to find out how many different ways the calls could have been divided among the kinds of equipment.

Combinations = 17	TVs	Radios	Stereos	Tape Players
2 + 3 + 4 + 8	2	3	4	8
2 + 3 + 5 + 7	2	3	8	4
	2	4	3	8
	2	4		
	2	8		

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

It happened one day in the towns around Fugit Sound! Television sets started broadcasting the shows they wanted to watch. Radios, stereos, and tape players began playing what they wanted to hear. It was the kind of day science fiction is made of. In one hour, Fred's Repair Shop received 17 calls from owners of sound equipment. Each call was about a radio, a television, a stereo, or a tape player. At least 2 calls came in about each kind of equipment, but the number of calls was different for each kind of equipment. How many different ways could the calls have been divided among the kinds of equipment?

- FIND OUT What is the question you have to answer? How many different ways could the calls have been divided among the kinds of equipment?
  - · What was happening in Fred's Repair Shop? Calls were coming in from owners of television sets, radios, stereos, and tape players because the pieces of equipment were making their own decisions about what to play.
  - · How many calls did Fred receive in one hour? 17
  - What do you know about the calls that came in? At least 2 calls came in about each kind of equipment, and the number of calls was different for each kind of equipment.

### CHOOSE A STRATEGY

- Let's say that there were 2 calls about televisions, 4 calls about radios, 3 calls about stereos, and 8 calls about tape players. If you use those same four numbers, is there another way you could arrange them? Yes: 4 calls about televisions, 2 calls about radios, 3 calls about stereos, and 8 calls about tape players
- · Is there a systematic way to record all the possible ways the calls could have been divided? Yes, we can make an organized list.

- SOLVE IT What is the least number of calls that came in about each kind of sound equipment? 2 The greatest number? If the four numbers were all different, then 17 - (2 + 3 + 4) = 8, the greatest number possible. What are the numbers of calls possible for each kind of equipment? 2, 3, 4, 5, 6, 7, 8
  - Using those numbers, what combinations of four numbers equal 17? 2+3+4+8, 2+3+5+7, 2+4+5+6
  - One way to set up an organized list is to have four columns labeled TVs, Radios, Stereos, Tape Players. Begin by putting 2 under TVs, 3 under Radios, 4 under Stereos, and 8 under Tape Players. Are there other ways to arrange these numbers, putting 2 under TVs? 2-3-8-4. 2-8-3-4. 2-8-4-3. 2-4-3-8. 2-4-8-3
  - How many ways can you arrange these same numbers, putting 3 under TVs? 3-2-4-8, 3-2-8-4, 3-8-4-2, 3-8-2-4, 3-4-8-2, 3-4-2-8
  - How many ways can you arrange these same numbers, putting 4 under TVs? (See list below.) Putting 8 under TVs? (See list below.)
  - (Have students finish their lists.) How many different ways could the calls have been divided among the kinds of equipment?

Con					3 =	: 1	-
2	+	3	+	4	+	8	
2	+	3	+	5	+	7	
2	+	4	+	5	+	6	

		Players		
3	4	8		
3	8	4		
4	3	8		
4	8	3		
	3 4 4	3 8 3		

Tano

TV	R	S	TP
2	8	4	3
2	8	3	4
3	4	8	2
3	4	2	8
(See	Sol	utior	ns for
COI	mple	ete li	st.)

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

- **EXTEND IT** If the same number of calls came in for three kinds of equipment, and a different number of calls came in for the fourth kind, how many different ways could the calls have been divided among the kinds of equipment?
- PRACTICE Similar Practice Problems: 51, 60, 81

On Gildan Island there are some amazing amber trees that produce liquid gold instead of sap. Glenda, an islander, has 12 of these trees. The first year, each tree produced 3 gallons of gold. The second year, each tree tripled its production to 9 gallons. Glenda began to keep a record of the liquid gold produced by her trees. The third year, each tree doubled its production to 18 gallons. The fourth year, each tree tripled its output to 54 gallons. The fifth year, each tree doubled its output again. Glenda's record showed a pattern in the way her trees were producing liquid gold: triple output one year, double the next, triple the next, double the next, and so on. If Glenda's trees continue to produce gold at this rate, in what year will her trees produce 125,000 gallons of liquid gold?

- **FIND OUT** What is the question you have to answer?
  - What were Glenda's amber trees producing?
  - · How many gallons of liquid gold did each tree produce the first year? The second year? The third year? The fourth year? The fifth year?
  - Is the rate of change in production the same every year?

### CHOOSE A STRATEGY

- How did the number of gallons of liquid gold change from the first year to the second year? How did the number of gallons change from the second year to the third year?
- Can you use this pattern of change to help you solve the problem?
- Is there a systematic way to record the information?

- **SOLVE IT** If you set up a table, what will you keep track of in the top row?
  - What will you keep track of in the second row?
  - What will you keep track of in the third row?
  - If the number of gallons tripled the second year, doubled the third year, tripled the fourth year, and doubled the fifth year, how many gallons did each tree produce the fifth year? How many gallons will 12 trees produce?
  - Will the number of gallons double or triple in the sixth year? How many gallons will each tree produce the sixth year? How many gallons will 12 trees produce?
  - Continue filling in your table until you find the year in which the 12 trees will produce 125,000 gallons altogether.

Year	1	2	3	4	5	
Gallons per tree	3	9	18	54		
Gallons per 12 trees	36	108	216			

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of change. Is your answer reasonable?

On Gildan Island there are some amazing amber trees that produce liquid gold instead of sap. Glenda, an islander, has 12 of these trees. The first year, each tree produced 3 gallons of gold. The second year, each tree tripled its production to 9 gallons. Glenda began to keep a record of the liquid gold produced by her trees. The third year, each tree doubled its production to 18 gallons. The fourth year, each tree tripled its output to 54 gallons. The fifth year, each tree doubled its output again. Glenda's record showed a pattern in the way her trees were producing liquid gold: triple output one year, double the next, triple the next, double the next, and so on. If Glenda's trees continue to produce gold at this rate, in what year will her trees produce 125,000 gallons of liquid gold?

- FIND OUT What is the question you have to answer? In what year will Glenda's trees produce 125,000 gallons of liquid gold?
  - What were Glenda's amber trees producing? Liquid gold
  - · How many gallons of liquid gold did each tree produce the first year? 3 The second year? 9 The third year? 18 The fourth year? 54 The fifth year? Two times 54
  - · Is the rate of change in production the same every year? No, it triples one year, doubles the next, triples the next, and doubles the next year.

### CHOOSE A STRATEGY

- · How did the number of gallons of liquid gold change from the first year to the second year? It increased to 3 times the number of gallons in the first year. How did the number of gallons change from the second year to the third year? It increased to 2 times the number of gallons in the second year.
- · Can you use this pattern of change to help you solve the problem? Yes, we can use the pattern to find out when the trees will produce 125,000 gallons of liquid gold altogether.
- Is there a systematic way to record the information? We can make a table.

### SOLVE IT

- If you set up a table, what will you keep track of in the top row? The number of years
- What will you keep track of in the second row? The number of gallons produced by each
- What will you keep track of in the third row? The total number of gallons produced by the 12 trees
- If the number of gallons tripled the second year, doubled the third year, tripled the fourth year, and doubled the fifth year, how many gallons did each tree produce the fifth year?  $54 \times 2 = 108$  How many gallons will 12 trees produce? 1,296
- Will the output double or triple in the sixth year? Triple How many gallons will each tree produce the sixth year? 324 How many gallons will 12 trees produce? 3,888
- (Have the students finish their tables.) In what year will Glenda's trees produce 125,000 gallons of liquid gold?

Solution: 10th

Year	1	2	3	4	5	6	7	8	9	10
Gallons per tree	3	9	18	54	108	324	648	1,944	3,888	11,664
Gallons per 12 trees	36	108	216	648	1,296	3,888	7,776	23,328	46,656	139,968

### LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of change. Is your answer reasonable?

**EXTEND IT** • How many trees would Glenda need if she wanted the total production to be 1 million gallons in the twelfth year?



### USE OR LOOK FOR A PATTERN

Name

Natasha put \$10.00 into a savings account at the Itzakick Bank. Itzakick increased Natasha's savings by 100% the second month, then decreased it by 10% the third month. The bank continued to increase Natasha's savings by 100% one month, decrease it by 10% the next, and increase it by 100% the next. In what month will Natasha's savings account grow to \$600?

- **FIND OUT** What is the question you have to answer?
  - How much money did Natasha put into a savings account at the Itzakick
  - By what percent did the bank increase Natasha's savings the second month? By what percent did the bank decrease Natasha's savings the third month? Did the bank continue to increase and decrease Natasha's savings in that way?

- **CHOOSE A** What is the pattern of change in Natasha's savings account?
- STRATEGY Can you use this pattern of increase and decrease to help you solve the problem?
  - How can you organize and record the information?

- SOLVE IT When you set up your table, what will you keep track of in the first row? In the second row? In the third row? In the fourth row?
  - What amount did Natasha put into her savings?
  - What was the percent of increase in the second month? What was the amount of increase? Then what was the total amount in Natasha's savings?
  - What was the percent of decrease in the third month? What was the amount of decrease? Then what was the total amount in Natasha's savings?
  - Continue using the pattern to fill in your table to find the month in which Natasha's savings will grow to \$600.

Month	1	2	3	4	5	
% +/-		+100%				
\$ +/-		+\$10.00				
Total Savings	\$10.00					

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your table and the pattern of change. Is your answer reasonable?



## USE OR LOOK FOR A PATTERN

Teaching Plan

Natasha put \$10.00 into a savings account at the Itzakick Bank. Itzakick increased Natasha's savings by 100% the second month, then decreased it by 10% the third month. The bank continued to increase Natasha's savings by 100% one month, decrease it by 10% the next, and increase it by 100% the next. In what month will Natasha's savings account grow to \$600?

- FIND OUT What is the question you have to answer? In what month will Natasha's savings account
  - How much money did Natasha put into a savings account at the Itzakick Bank? \$10.00
  - By what percent did the bank increase Natasha's savings the second month? 100% By what percent did the bank decrease Natasha's savings the third month? 10% Did the bank continue to increase and decrease Natasha's savings in that way? Yes

### CHOOSE A STRATEGY

- What is the pattern of change in Natasha's savings account? +100%, -10%, +100%, -10%
- Can you use this pattern of increase and decrease to help you solve the problem? Yes.
- · How can you organize and record the information? We can make a table.

### SOLVE IT

- . When you set up your table, what will you keep track of in the first row? The number of months In the second row? The percent of increase or decrease In the third row? The amount of increase or decrease in the fourth row? The total amount in Natasha's savings
- What amount did Natasha put into her savings? \$10
- What was the percent of increase in the second month? 100% What was the amount of increase? \$10 Then what was the total amount in Natasha's savings? \$20
- What was the percent of decrease in the third month? 10% What was the amount of decrease? \$2.00 Then what was the total amount in Natasha's savings? \$18.00
- (Have students continue to fill in their tables.) In what month will Natasha's savings account grow to \$600?

Solution: 14th

Month	1	2	3	4	5	6	7
% +/-		+100%	-10%	+100%	-10%	+100%	-10%
\$ +/-		+\$10.00	-\$2.00	+\$18.00	-\$3.60	+\$32.40	-\$6.48
Total Savings	\$10.00	\$20.00	\$18.00	\$36.00	\$32.40	\$64.80	\$58.32

Month	8	9	10	11	12	13	14
% +/-			+100%	-10%	+100%	-10%	+100%
\$ +/-	+\$58.32	-\$11.66	+\$104.98	-\$21.00	+\$188.96	-\$37.79	+\$340.13
Total Savings	\$116.64	\$104.98	\$209.96	\$188.96	\$377.92	\$340.13	\$680.26

# LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of change. Is your answer reasonable?

### EXTEND IT

In what month will the decrease in Natasha's savings be between \$100 and \$200?

### **PRACTICE**

• Similar Practice Problems: 53, 64, 77

Professor Orion and Professor Polaris are working every night this month at the Mt. Olympus Observatory. On Monday night, during his shift from 8-10 P.M., Professor Orion notices 12 new stars. He finds 18 new stars on Tuesday, 30 on Wednesday, 36 on Thursday, and 48 on Friday. Professor Polaris, working the 10-12 P.M. shift, notices 18 new stars on Monday, 26 on Tuesday, 44 on Wednesday, 52 on Thursday, and 70 on Friday. If the professors continue to spot new stars at the same rate, during what day of the week will their combined total equal the number of days in a year?

- **FIND OUT** What is the question you have to answer?
  - When are the professors working?
  - How many stars does Professor Orion notice on Monday? On Tuesday? On Wednesday? On Thursday? On Friday?
  - How many stars does Professor Polaris notice on Monday? On Tuesday? On Wednesday? On Thursday? On Friday?

### CHOOSE A STRATEGY

- Would it help to keep track of each day, and how many new stars each professor finds?
- What strategy can help you figure out how the number of new stars changes from day to day?

- **SOLVE IT** When you set up a table, what do you want to use as labels for the columns? For the rows?
  - What is the difference between the number of stars Professor Orion notices on Monday and Tuesday? Between Tuesday and Wednesday? Between Wednesday and Thursday? Between Thursday and Friday?
  - Do you see a pattern in the way the numbers change?
  - Continue to fill in your table to find out what night the professors will notice a number of stars equal to the days of the year.

Day	Mon.	Tues.	Wed.	Thurs.	
Orion	12	18	30		
Polaris	18	26			
Total	30				



### **USE OR MAKE A TABLE**

## Teaching Plan

Professor Orion and Professor Polaris are working every night this month at the Mt. Olympus Observatory. On Monday night, during his shift from 8-10 P.M., Professor Orion notices 12 new stars. He finds 18 new stars on Tuesday, 30 on Wednesday, 36 on Thursday, and 48 on Friday. Professor Polaris, working the 10-12 P.M. shift, notices 18 new stars on Monday, 26 on Tuesday, 44 on Wednesday, 52 on Thursday, and 70 on Friday. If the professors continue to spot new stars at the same rate, during what day of the week will their combined total equal the number of days in a year?

- FIND OUT What is the question you have to answer? If the professors continue to spot new stars at the same rate, during what day of the week will their combined total equal the number of days in a year?
  - When are the professors working? Every night from 8 to 12 P.M.
  - How many stars does Professor Orion notice on Monday? 12 On Tuesday? 18 On Wednesday? 30 On Thursday? 36 On Friday? 48
  - How many stars does Professor Polaris notice on Monday? 18 On Tuesday? 26 On Wednesday? 44 On Thursday? 52 On Friday? 70

### CHOOSE A STRATEGY

- · Would it help to keep track of each day, and how many new stars each professor finds? Yes, we can make a table.
- What strategy can help you figure out how the number of new stars changes from day to day? Look for a pattern in the way the numbers change.

### SOLVE IT

- When you set up a table, what do you want to use as labels for the columns? Days of the week For the rows? The professors, and Total
- What is the difference between the number of stars Professor Orion notices on Monday and Tuesday? +6 Between Tuesday and Wednesday? +12 Between Wednesday and Thursday? +6 Between Thursday and Friday? +12
- Do you see a pattern in the way the numbers change? Yes, the difference increases by 6 and then by 12 and this repeats.
- (Have the students continue to fill in their tables.) During what day of the week will the professors notice a combined number of new stars equal to the number of days in the year?

Solution: Wednesday

Days	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.
Orion	12	18	30	36	48	54	66	72	84	90
Polaris	18	26	44	52	70	78	96	104	122	130
Total	30	44	74	88	118	132	162	176	206	220

(continued)

Days	Thurs.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.*
Orion	102	108	120	126	138	144	156
Polaris	148	156	174	182	200	208	226
Total	250	264	294	308	338	352	382

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

**EXTEND IT** • A year later the professors are at Mt. Olympus again. On Monday Professor Polaris finds 8 new stars, on Tuesday 16, on Wednesday 11, on Thursday 22, and 17 on Friday, Professor Orion notices 7 on Monday, 21 on Tuesday, 22 on Wednesday, 33 on Thursday, and 23 on Friday. When will Professor Orion first notice more than three times the number of new stars Professor Polaris does?

Water is scarce in the Kalahari desert, and biologists are counting the animals that come to one of the watering holes. They notice that the animals that are fierce enemies vary their schedules during the day, to avoid confrontations. The first day the biologists count 7 wildebeests, 4 elephants, 6 zebras, and 5 lions. On the second day they count 10 wildebeests, 2 elephants, 8 zebras, and 4 lions; on the third day 9 wildebeests, 6 elephants, 5 zebras, and 7 lions; on the fourth day 12 wildebeests, 4 elephants, 7 zebras, and 6 lions; and 11 wildebeests, 8 elephants, 4 zebras, and 9 lions on the fifth day. If this pattern continues, how many animals will they have counted altogether on the first day that they don't see any zebras?

- FIND OUT What is the question you have to answer?
  - What animals are the biologists counting?
  - How many of each kind of animal do they count on the first day? On the second day? On the third day? On the fourth day? On the fifth day?

# CHOOSE A

- · Would it help to keep track of each day and each animal?
- STRATEGY What other strategy could help you find out how the number of animals is changing from day to day?

- SOLVE IT When you set up a table, what will you use as labels for the columns? For the
  - What is the difference between the number of elephants they count the first day and the second day? The second day and the third day? The third day and the fourth day? Between the fourth day and the fifth day?
  - Do you see a pattern in the way the numbers change?
  - Continue to fill in the table, looking at the pattern of changes for each animal, until you find a day when they don't see any zebras.

Day	1	2	3	
Wildebeest	7	10	9	
Elephant	4	2		
Zebra	6			
Lion	5			
Total				



### USE OR MAKE A TABLE

## Teaching Plan

Water is scarce in the Kalahari desert, and biologists are counting the animals that come to one of the watering holes. They notice that the animals that are fierce enemies vary their schedules during the day, to avoid confrontations. The first day the biologists count 7 wildebeests, 4 elephants, 6 zebras, and 5 lions. On the second day they count 10 wildebeests, 2 elephants, 8 zebras, and 4 lions; on the third day 9 wildebeests, 6 elephants, 5 zebras, and 7 lions; on the fourth day 12 wildebeests, 4 elephants, 7 zebras, and 6 lions; and 11 wildebeests, 8 elephants, 4 zebras, and 9 lions on the fifth day. If this pattern continues, how many animals will they have counted altogether on the first day that they don't see any zebras?

- FIND OUT What is the question you have to answer? If this pattern continues, how many animals will they have counted altogether on the first day that they don't see any zebras?
  - What animals are the biologists counting? Wildebeests, elephants, zebras, and lions
  - How many of each kind of animal do they count on the first day? 7 wildebeests, 4 elephants, 6 zebras, 5 lions On the second day? 10 wildebeests, 2 elephants, 8 zebras, 4 lions On the third day? 9 wildebeests, 6 elephants, 5 zebras, 7 lions On the fourth day? 12 wildebeests, 4 elephants, 7 zebras, 6 lions On the fifth day? 11 wildebeests, 8 elephants, 4 zebras, 9 lions

## CHOOSE A STRATEGY

- · Would it help to keep track of each day and each animal? Yes, we can make a table.
- What other strategy could help you find out how the number of animals is changing from day to day? We can look for a pattern in the numbers.

### SOLVE IT

- When you set up a table, what will you use as labels for the columns? Days For the rows? Names of animals and Total
- · What is the difference between the number of elephants they count the first day and the second day? -2 The second day and the third day? +4 The third day and the fourth day? -2 Between the fourth day and the fifth day? +4
- Do you see a pattern in the way the numbers change? Yes, -2, +4
- (Have the students continue to fill in their tables.) How many animals will they have counted altogether on the first day they don't see any zebras?

Solution: 478 animals

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
Wildebeest	7	10	9	12	11	14	13	16	15	18	17	20	19
Elephant	4	2	6	4	8	6	10	8	12	10	14	12	16
Zebra	6	8	5	7	4	6	3	5	2	4	1	3	0 *
Lion	5	4	7	6	9	8	11	10	13	12	15	14	17
Total	22	46	73	102	134	168	205	244	286	330	377	426	478 *

### LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

**EXTEND IT** • The year before they counted 3 wildebeests, 4 elephants, 6 zebras, and 5 lions on the first day; 6 wildebeests, 16 elephants, 18 zebras, and 18 lions on the second; 4 wildebeests, 11 elephants, 7 zebras, 19 lions on the third; 8 wildebeests, 23 elephants, 21 zebras, 32 lions on the fourth; 6 wildebeests, 18 elephants, 10 zebras, and 33 lions on the fifth day. How many animals would they have counted altogether on the day when they see 88 lions?

**PRACTICE** • Similar Practice Problems: 56, 70, 88

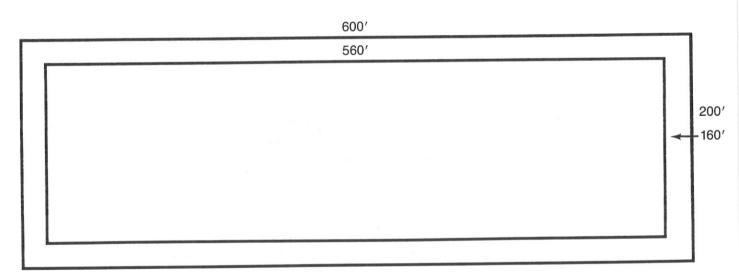
C 623 and C 279 are Cyborgs, who work together on interstellar developments for the Cosmos Construction Company. Their last job was to clear a small rectangular area 200 feet wide and 600 feet long for a new dance club. They used a special planet planer, a bulldozer-type machine, to clear loose rocks and smooth the surface. The planet planer covers an area 20 feet across. If C 623 begins at an outside corner, and works his way into the center, on which trip around will he clear his half of the area?

- FIND OUT What is the question you have to answer?
  - What are C 623 and C 279 doing?
  - What is the shape and dimensions of the area for the dance club?
  - How wide is the area that the planer clears?
  - Where does C 623 begin clearing and how does he move?

# CHOOSE A STRATEGY

Would it help to draw a diagram so that you can see how the planer moves?

- SOLVE IT Begin by drawing the area that is being cleared for the dance club. Write the dimensions on the sides.
  - What is the area of the space being cleared? What is half of the area?
  - How wide is the strip being cleared by the planer? On your diagram, show the strip that will be cleared the first time around the rectangle.
  - What are the dimensions of the part still to be cleared after one trip? What is the area of the part still to be cleared? Is it more, or less, than half?
  - · Continue to show the strips on your diagram that are cleared. Record the dimensions and figure out the area remaining after each trip around until half the area has been cleared.



C 623 and C 279 are Cyborgs, who work together on interstellar developments for the Cosmos Construction Company. Their last job was to clear a small rectangular area 200 feet wide and 600 feet long for a new dance club. They used a special planet planer, a bulldozer-type machine, to clear loose rocks and smooth the surface. The planet planer covers an area 20 feet across. If C 623 begins at an outside corner, and works his way into the center, on which trip around will he clear his half of the area?

- FIND OUT What is the question you have to answer? If C 623 begins at an outside edge and works his way into the center, on which trip around will he clear his half of the area?
  - What are C 623 and C 279 doing? Clearing a small rectangular area for a dance club
  - What is the shape and dimensions of the area for the dance club? It is 200 feet wide and 600 feet long.
  - How wide is the area that the planer clears? 20 feet across
  - Where does C 623 begin clearing and how does he move? He begins at an outside edge and works his way into the center.

### CHOOSE A STRATEGY

Would it help to draw a diagram so that you can see how the planer moves? Yes

## SOLVE IT

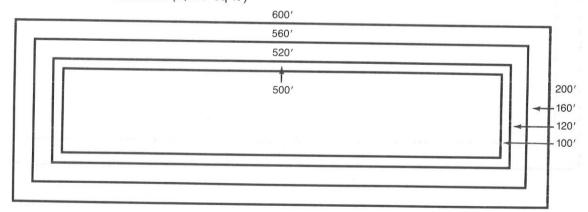
- Begin by drawing the area that is being cleared for the dance club. Write the dimensions on the sides.
- What is the area of the space being cleared? 120,000 square feet What is half of the area? 60,000 square feet
- · How wide is the strip being cleared by the planer? 20 feet On your diagram, show the strip that will be cleared the first time around the rectangle.
- What are the dimensions of the part still to be cleared after one trip? 560 feet by 160 feet What is the area of the part still to be cleared? 89,600 square feet Is it more, or less, than half? More
- (Have students continue this process of showing strips cleared on their diagram and then determining the area remaining.) On which trip around will C 623 have cleared his half of the area?

Solution: During the third trip

Area = 120,000 sq ft $\frac{1}{2}$  area = 60.000 sq ft

After 1st trip - 89,600 sq ft left (560  $\times$  160) After 2nd — 62,400 sq ft left (520  $\times$  120)

Need to clear additional (2,400 sq ft)



LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT • How many trips around would C 623 make if the planer was 15 feet across and he had to clean two thirds of the area?

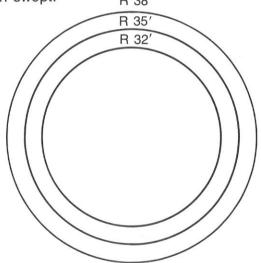
As Clyde moves his broom around the circus ring, he thinks that he has finally found a job where he can make a clean sweep of things. Clyde is sweeping the ring where the lions perform in the Filbert Family Circus. The ring is 76 feet across and Clyde's broom is 3 feet wide. He starts at the outside edge and works his way to the middle, making circles around the ring. After sweeping three fourths of the ring, Clyde sees the lions coming with their trainer and scurries out of the ring. How many trips around the ring did he make?

- FIND OUT What is the question you have to answer?
  - What are the dimensions of the circus ring that Clyde is sweeping?
  - · What is the size of the broom Clyde is using?
  - · Where does Clyde begin sweeping?
  - · How much of the ring did Clyde clean before he left?

## CHOOSE A STRATEGY

 Would it help to draw a diagram so that you can see where Clyde is after each trip around the ring?

- **SOLVE IT** Begin by drawing the circus ring. Write the diameter on the circle.
  - How do you go about finding the area of the ring? What is the radius of the ring? What do you multiply with for  $\pi$ ? What is the area of the ring? What is three fourths of the area? When he has swept three fourths of the ring, what will be the area of the one fourth left?
  - · How wide is the strip being swept by Clyde's broom? On your diagram, show the strip that will be cleaned the first time around the circle.
  - · What is the radius of the remaining part of the circle? What is the area of the remaining part of the circle? Is it more, or less, than one fourth of the ring?
  - Continue to show each strip on your diagram that Clyde sweeps. Record the dimensions and figure out the area remaining after each trip around until three fourths of the area has been swept. R 38'





## MAKE A PICTURE OR DIAGRAM

Teaching Plan

As Clyde moves his broom around the circus ring, he thinks that he has finally found a job where he can make a clean sweep of things. Clyde is sweeping the ring where the lions perform in the Filbert Family Circus. The ring is 76 feet across and Clyde's broom is 3 feet wide. He starts at the outside edge and works his way to the middle, making circles around the ring. After sweeping three fourths of the ring, Clyde sees the lions coming with their trainer and scurries out of the ring. How many trips around the ring did he make?

# FIND OUT

- What is the question you have to answer? How many trips around the ring did he make?
- What are the dimensions of the circus ring that Clyde is sweeping? The ring is 76 feet straight across, or the diameter is 76 feet.
- What is the size of the broom Clyde is using? It is 3 feet wide.
- Where does Clyde begin sweeping? At the outside of the ring
- · How much of the ring did Clyde clean before he left? He swept three fourths of the ring.

# CHOOSE A STRATEGY

· Would it help to draw a diagram so that you can see where Clyde is after each trip around the ring? Yes

SOLVE IT

- Begin by drawing the circus ring. Write the diameter on the circle.
- How do you go about finding the area of the ring? We use the formula  $\pi r^2$ . What is the radius of the ring? 38 feet What do you multiply with for  $\pi$ ? 3.14 What is the area of the ring? 4,534.16 square feet What is three fourths of the area? 3400.62 square feet When he has swept three fourths of the ring, what will be the area of the one fourth left? 1133.54 square feet
- How wide is the strip being swept by Clyde's broom? 3 feet On your diagram, show the strip that will be cleaned the first time around the circle.
- What is the radius of the remaining part of the circle? The radius is 35 feet What is the area of the remaining part of the circle? 3,846.50 square feet Is it more, or less, than one fourth of the ring? More
- (Have the students continue to show each strip cleaned on their diagrams.) How many trips did Clyde make around the ring? R 38'

Solution: 6 plus part of a 7th trip

radius =  $38 - \text{area} = 4534.16 \div 4 = 1133.54 \text{ sq ft}$ 

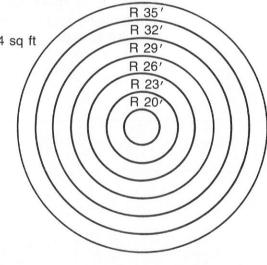
1st trip - r 35, 3846.5 sq ft left

2nd - r 32, 3215.36 sq ft left

3rd - r 29, 2640.74 sq ft left 4th - r 26, 2122.64 sq ft left

5th - r 23, 1661.06 sq ft left

6th - r 20, 1256 sq ft left



LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

**EXTEND IT** • If Clyde's broom is 2½ feet wide, how many trips around the ring will it take him to sweep one half of the ring?

**PRACTICE** • Similar Practice Problems: 57, 75, 91



Business was booming for the Sensational Events Company. During the past year they supplied 1295 entertainers for parties. They sent three times as many magicians as clowns, 25% as many fire-eaters as clowns, one-third as many people in gorilla suits as magicians, and a number of jugglers equal to the combined number of clowns and magicians. How many magicians, clowns, fire-eaters, people in gorilla suits, and jugglers had Sensational Events supplied for parties during the past year?

- **FIND OUT** What is the question you have to answer?
  - · How many entertainers had they supplied?
  - What do you know about the requests for magicians? For fire-eaters? For people in gorilla suits? For jugglers? For clowns?

# STRATEGY

- **CHOOSE A** Can guessing an answer help you solve this problem?
  - How can you use the information from an incorrect guess?

- **SOLVE IT** How many entertainers did they send out?
  - If you make a guess for one of the types of entertainers, which one would you begin with? Why?
  - What is your guess?
  - If you guess a number for one of the entertainers, then can you figure out what the other numbers are?
  - How can you check your guess?
  - How was your guess? If your guess was wrong, how can you improve your next guess?
  - Continue to make guesses and check them until you find the answer.

Business was booming for the Sensational Events Company. During the past year they supplied 1295 entertainers for parties. They sent three times as many magicians as clowns, 25% as many fire-eaters as clowns, one-third as many people in gorilla suits as magicians, and a number of jugglers equal to the combined number of clowns and magicians. How many magicians, clowns, fire-eaters, people in gorilla suits, and jugglers had Sensational Events supplied for parties during the past year?

FIND OUT • What is the question you have to answer? How many of each kind of entertainer had Sensational Events supplied for parties during the past year?

How many entertainers had they supplied? 1295

What do you know about the requests for magicians? There were three times as many magicians as clowns. For fire-eaters? 25% of the number of clowns For people in gorilla suits? One-third the number of magicians For jugglers? Equal to the combined total of clowns and magicians For clowns? We don't know very much.

# CHOOSE A STRATEGY

- · Can guessing an answer help you solve this problem? Yes, because we don't have a number for any kind, just clues.
- How can you use the information from an incorrect guess? We can get a total and check it against 1295 and see if our guess was too high or too low. Then our next guess can be based on that information.

### SOLVE IT

- How many entertainers did they send out? 1295
- If you make a guess for one of the types of entertainers, which one would you begin with? The clowns Why? Because we know the least about that group

What is your guess? 100 (This is an example of one possible guess.)

• If you guess a number for one of the entertainers, then can you figure out what the other numbers are? Yes, if the clowns are 100 then the magicians are 3 times 100, or 300; the gorillas are one-third the magicians, or 100; the fire-eaters are 25% of the clowns, or 25; the jugglers are the total of clowns and magicians, or 400.

How can you check your guess? Add up the numbers we have: 100 + 300 + 25 + 100 + 400 = 925, and compare this with 1295

How was your guess? Too low If your guess was wrong, how can you improve your next

guess? We need to make our next guess higher. (Have students continue to make guesses until they solve the problem.) How many of each kind of entertainer had they supplied for parties?

Solution: Clowns—140, magicians—420, fire-eaters—35, people in gorilla suits—140, jugglers-560

# LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • This year they added sword swallowers and supplied a total of 1458 entertainers. They sent 21/2 times as many clowns as sword swallowers; 6 times as many magicians as clowns; 1/4 as many fire-eaters as clowns; as many people in gorilla suits as the combined total of clowns, fire-eaters, and sword swallowers; and 21/2 times as many jugglers as magicians. How many did they send of each kind?

16

The First International Vegetable Extravaganza (FIVE) Program was almost finished. Zachary Zucchini was reporting on the attendance figures to Ruby Radish. He said that there were three times as many zucchini as radishes present,  $\frac{5}{9}$  as many heads of lettuce as zucchini, five more potatoes than the combined number of heads of lettuce and radishes, one half as many artichokes as zucchinis and radishes combined, and that the number of carrots was 20% of the heads of lettuce. He said they had a marvelous turnout of 1381 vegetables in all. How many of each kind of vegetable attended the conference?

- **FIND OUT** What is the question you have to answer?
  - · How many vegetables attended the conference?
  - What do you know about the number of zucchinis that attended? The radishes? The heads of lettuce? The potatoes? The artichokes? The carrots?

- CHOOSE A Can guessing an answer help you solve this problem?
- **STRATEGY** How can you use the information from an incorrect guess?

- **SOLVE IT** How many vegetables were there altogether?
  - If you make a guess for one of the vegetables, which one would you begin with? Why?
  - What is your guess?
  - · If you guess a number for one of the vegetables, then can you figure out what the other numbers are?
  - How can you check your guess?
  - How was your guess? If your guess was wrong, how can you improve your next quess?
  - Continue to make guesses and check them until you find the answer.

The First International Vegetable Extravaganza (FIVE) Program was almost finished. Zachary Zucchini was reporting on the attendance figures to Ruby Radish. He said that there were three times as many zucchini as radishes present, 5/9 as many heads of lettuce as zucchini, five more potatoes than the combined number of heads of lettuce and radishes, one half as many artichokes as zucchinis and radishes combined, and that the number of carrots was 20% of the heads of lettuce. He said they had a marvelous turnout of 1381 vegetables in all. How many of each kind of vegetable attended the conference?

- FIND OUT What is the question you have to answer? How many of each kind of vegetable attended the conference?
  - How many vegetables attended the conference? 1381
  - What do you know about the number of zucchinis that attended? There were 3 times as many zucchini as radishes. The radishes? We don't know very much. The heads of lettuce? 5/9 number of zucchini The potatoes? 5 more than combined number of heads of lettuce and radishes The artichokes? 1/2 number of combined total of zucchini and radishes. The carrots? 20% of the heads of lettuce

### CHOOSE A STRATEGY

- Can guessing an answer help you solve this problem? Yes, because we don't have a definite number for any of the vegetables.
- How can you use the information from an incorrect guess? We can find out if our guess was too high or too low and then make our next guess based on that information.

## SOLVE IT

- How many vegetables were there altogether? 1381
- If you make a guess for one of the vegetables, which one would you begin with? Radishes Why? Because we have the least amount of information about the radishes
- What is your guess? 81 (This is an example of one possible guess.)
- If you guess a number for one of the vegetables, then can you figure out what the other numbers are? Yes, if the number for the radishes is 81, then the zucchini is 3 times that or 243; the heads of lettuce are 5/9 of the zucchini, or 135; the potatoes are 5 more than the heads of lettuce and radishes, 221; the artichokes are 1/2 zucchini + radishes, or 162; the carrots are 20% of the heads of lettuce, 27.
- How can you check your guess? We can add all our numbers together: 81 + 243 + 135+ 221 + 162 + 27 = 869.
- How was your guess? Too low If your guess was wrong, how can you improve your next guess? We need to make a higher guess next time.
- (Have students continue to make guesses and check them, until they solve the problem.) How many of each kind of vegetable were at the conference?

Solution: Radishes—129, zucchini—387, heads of lettuce—215, potatoes—349, artichokes-258, carrots-43

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • The next year there were 1665 vegetables, with the addition of beets and onions. There were 2½ more radishes than carrots; 4 times as many zucchini as radishes, heads of lettuce equal to the combined total of artichokes and carrots; 40% as many potatoes as zucchini; 1/5 as many artichokes as radishes; 2 times as many beets as carrots; and 6 times as many onions as heads of lettuce. How many were there of each kind of vegetable?

PRACTICE • Similar Practice Problems: 54, 69, 93

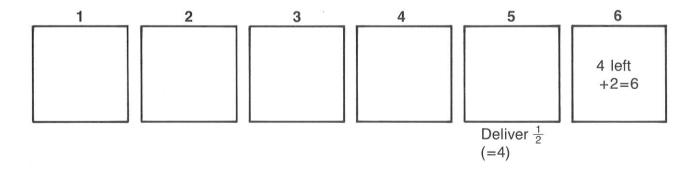
Annie has the Saturday delivery schedule for chicken chunks from Rooster-on-the-Run. One Saturday she set out in the truck and left  $\frac{3}{5}$  of her orders at the college. She went back to the restaurant and picked up 6 orders. Her second delivery was for  $\frac{3}{7}$  of the total amount that she had in the truck. She headed back to pick up 4 orders and then delivered  $\frac{3}{4}$  of her total. She picked up 6 more before heading to the local businesses to deliver  $\frac{1}{3}$  of her orders. She went back for 2 more orders, and delivered ½ of her total amount. She returned for 2 orders and made her last delivery of 6 orders at the hospital. How many orders did she deliver altogether?

- **FIND OUT** What is the question you have to answer?
  - What is Annie doing?
  - What do you know about her first delivery? Then what did she pick up? What about her second delivery? Then how many did she get? What about her third delivery? How many did she pick up next? What about her fourth delivery? What did she pick up next? Her fifth delivery? How many did she get then? Her sixth delivery?

# STRATEGY

- **CHOOSE A** Think about the best way to begin solving this problem. The only total given for the number of orders delivered is at the end of the problem. Therefore you need to work backwards from this given amount.
  - Is there another strategy that would be helpful?

- **SOLVE IT** If you begin with the last delivery, how many orders was that?
  - · How many did she pick up before delivering the 6? If you subtract that amount from the 6, this is the  $\frac{1}{2}$  left after the fifth delivery. How many did she deliver? Then how many orders did she have in the truck before she made her fifth delivery?
  - Now you can take this number and subtract the number picked up before the fifth delivery. This number is the  $\frac{2}{3}$  left after the fourth delivery. How many did she deliver? How many did she have before the fourth delivery?
  - Continue to work backwards, until you find the number of orders that she started with.





### WORK BACKWARDS

## Teaching Plan

17

Annie has the Saturday delivery schedule for chicken chunks from Rooster-on-the-Run. One Saturday she set out in the truck and left ½ of her orders at the college. She went back to the restaurant and picked up 6 orders. Her second delivery was for ¾ of the total amount that she had in the truck. She headed back to pick up 4 orders and then delivered ¾ of her total. She picked up 6 more before heading to the local businesses to deliver ⅓ of her orders. She went back for 2 more orders, and delivered ½ of her total amount. She returned for 2 orders and made her last delivery of 6 orders at the hospital. How many orders did she deliver altogether?

### FIND OUT

• What is the question you have to answer? How many orders did she deliver altogether?

• What is Annie doing? She is delivering orders of chicken chunks.

• What do you know about her first delivery? She left 3/6 of the orders she had. Then what did she pick up? 6 orders What about her second delivery? She left 3/7 of her total Then how many did she get? 4 What about her third delivery? She left 3/4 of her orders. How many did she pick up next? 6 What about her fourth delivery? 1/3 of the total What did she pick up next? 2 Her fifth delivery? She left 1/2 of her total orders. How many did she get then? 2 Her sixth delivery? She left 6.

# CHOOSE A STRATEGY

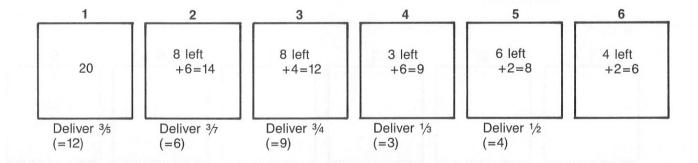
- Think about the best way to begin solving this problem. The only total given for the number of orders delivered is at the end of the problem. Therefore you need to work backwards from this given amount.
- Is there another strategy that would be helpful? Yes, we can make a diagram.

## SOLVE IT

• If you begin with the last delivery, how many orders was that? 6

- How many did she pick up before delivering the 6? 2 If you subtract that amount from the 6, this is the ½ left after the fifth delivery. How many did she deliver? 4 Then how many orders did she have in the truck before she made her fifth delivery? 8
- Now you can take this number and subtract the number picked up before the fifth delivery. This number is the ½ left after the fourth delivery. How many did she deliver? 3 How many did she have before the fourth delivery? 9
- (Have students continue to work backwards until they find out how many orders she started with.) How many orders did she deliver altogether?

Solution: 40



## LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

### EXTEND IT

• If she delivered 8 on her last delivery, ½ of the total for her first delivery, and ½ of the total for her second delivery, and otherwise made the same pick-ups and deliveries, how many orders would she have delivered altogether?

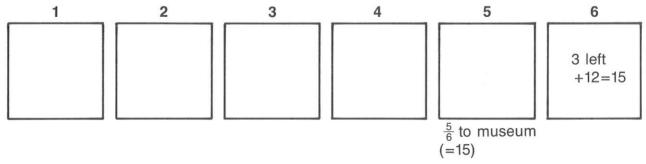
Maresa and her brother Andrew are on a dinosaur dig in New Mexico. Each day they dig for fossils, and then check in with the resident paleontologist at the end of the day. He helps them to separate the fossils, those for the museum and those they can keep. At the end of the first day  $\frac{2}{5}$  of the total goes to the museum. On the second day they find 6 new ones and then  $\frac{3}{8}$  of the total goes to the museum. On the third day they find 13 new fossils and that night the museum keeps  $\frac{2}{7}$  of the total. On the fourth day they add 7 and  $\frac{4}{9}$  goes to the museum. On the fifth day they find 3 and the museum keeps  $\frac{5}{6}$  of their total. They add 12 on the sixth day, and the museum keeps  $\frac{1}{3}$  of their collection. At the end of their dig Andrew and Maresa have 10 fossils to take home. How many new fossils did they collect for the museum?

- **FIND OUT** What is the question you have to answer?
  - What are Maresa and Andrew doing?
  - What do you know about their first day? What do you know about the second day? What do you know about the third day? What do you know about the fourth day? What do you know about the fifth day? What do you know about the sixth day?
  - How many fossils do Andrew and Maresa take home?

# STRATEGY

- **CHOOSE A** Think about the best way to begin solving this problem. The only total given for the number of fossils that they kept is at the end of the problem. Therefore you need to work backwards from this given amount.
  - Is there another strategy that would be helpful?

- **SOLVE IT** If you begin with the total number that they take home, you know that this is two-thirds of the total number that they have at the end of this day. How many do they have altogether at the end of the sixth day? How many go to the museum?
  - If you work backwards to the fifth day, you know that  $\frac{5}{6}$  goes to the museum, so the number at the beginning of the sixth day is  $\frac{1}{6}$ . How many is this? Then how much goes to the museum at the end of the fifth day? Then what is the total number of fossils at the end of the fifth day?
  - · Continue to work backwards until you know how many fossils they find on the first day. Then you can figure out how many the museum has at the end.



Maresa and her brother Andrew are on a dinosaur dig in New Mexico. Each day they dig for fossils, and then check in with the resident paleontologist at the end of the day. He helps them to separate the fossils, those for the museum and those they can keep. At the end of the first day 2/5 of the total goes to the museum. On the second day they find 6 new ones and then 3% of the total goes to the museum. On the third day they find 13 new fossils and that night the museum keeps 2/7 of the total. On the fourth day they add 7 and 1/9 goes to the museum. On the fifth day they find 3 and the museum keeps 5/6 of their total. They add 12 on the sixth day, and the museum keeps 1/3 of their collection. At the end of their dig they have 10 fossils to take home. How many new fossils did they collect for the museum?

# FIND OUT

· What is the question you have to answer? How many new fossils did they collect for the museum?

What are Maresa and Andrew doing? They are digging for dinosaur fossils.

• What do you know about their first day? The museum keeps 2/5 of the fossils collected. What do you know about the second day? They find 6 new ones and the museum keeps 3/8 of total amount. What do you know about the third day? They find 13 and the museum keeps 2/7 of the total. What do you know about the fourth day? They find 7 and the museum keeps 46. What do you know about the fifth day? They find 3 and the museum keeps 56. What do you know about the sixth day? They find 12 and the museum keeps 1/3.

How many fossils do Andrew and Maresa take home? 10

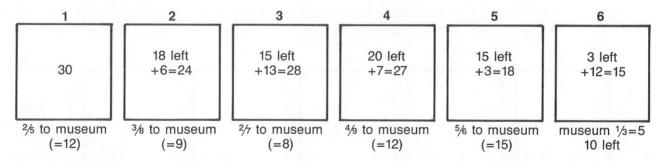
# CHOOSE A STRATEGY

- Think about the best way to begin solving this problem. The only total given for the number of fossils that they kept is at the end of the problem. Therefore you need to work backwards from this given amount.
- Is there another strategy that would be helpful? We can make a diagram.

### SOLVE IT

- If you begin with the total number that they take home, you know that this is two-thirds of the total number that they have at the end of this day. How many do they have altogether at the end of the sixth day? 15 How many go to the museum? 5
- If you work backwards to the fifth day, you know that 5/6 goes to the museum, so the number at the beginning of the sixth day is 1/6. How many is this? 3 Then how much goes to the museum at the end of the fifth day? 15 Then what is the total number of fossils at the end of the fifth day? 18
- (Have the students continue to work backwards until they find the number of fossils they collected the first day.) How many fossils did they collect for the museum?

Solution: 61



LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • Make up a similar problem, and begin with Andrew and Maresa finding 45 fossils on the

PRACTICE • Similar Practice Problems: 59, 65, 79

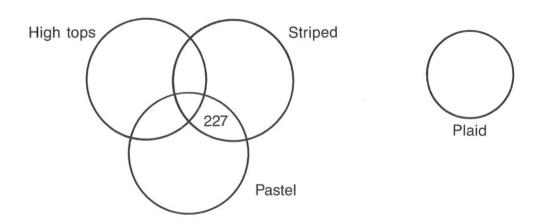
Tyrone's Fantastic Footwear uses tennis-playing chimps to advertise their high tops. The store carries shoes for a variety of sports. When Serena entered the store, a salesman ran up to announce that the store had 555 pairs of shoes with high tops, 730 pairs with striped laces, and 654 pairs of pastel-colored shoes. He said that 227 pairs were pastel with striped laces; 265 pairs were high tops with striped laces; 143 pairs were high tops and pastel; and 65 pairs were high tops, pastel, and had striped laces. The salesman added that they also had 277 special pairs that were plaid. How many pairs of shoes did Serena have to choose from?

- **FIND OUT** What is the question you have to answer?
  - How many pairs of shoes were high tops? Had striped laces? Were pastel?
  - How many pairs of shoes were high tops with striped laces? Were pastel with striped laces? Were high tops and pastel? Were high tops, pastel, and had striped laces?

### CHOOSE A STRATEGY

• Is there a special kind of diagram that can help you organize all the information in the problem?

- SOLVE IT If you make a Venn diagram, how many circles will you have to draw? What will you label each of the circles? Do the circles intersect? Why or why not?
  - How many pairs of shoes are high tops? Where will you write this number? How many are high tops and have striped laces? Where will you write this number? How many are high tops and pastel? Where will you write this number? How many are high tops with striped laces and are pastel? Where will you write this number? Then how many are high tops and are not pastel and do not have striped laces? Where does this number go?
  - Keep filling in your diagram to find out the total number of pairs of shoes.





# MAKE A PICTURE OR DIAGRAM

Teaching Plan

Tyrone's Fantastic Footwear uses tennis-playing chimps to advertise their high tops. The store carries shoes for a variety of sports. When Serena entered the store, a salesman ran up to announce that the store had 555 pairs of shoes with high tops, 730 pairs with striped laces, and 654 pairs of pastel-colored shoes. He said that 227 pairs were pastel with striped laces; 265 pairs were high tops with striped laces; 143 pairs were high tops and pastel; and 65 pairs were high tops, pastel, and had striped laces. The salesman added that they also had 277 special pairs that were plaid. How many pairs of shoes did Serena have to choose from?

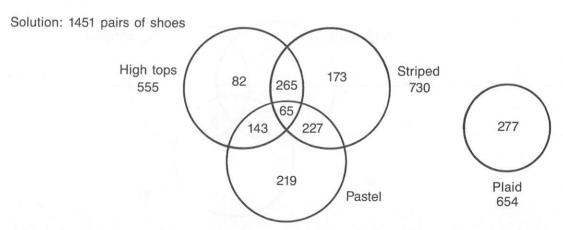
- FIND OUT What is the question you have to answer? How many pairs of shoes did Serena have to choose from?
  - How many pairs of shoes were high tops? 555 Had striped laces? 730 Were pastel? 654
  - How many pairs of shoes were high tops with striped laces? 265 Were pastel with striped laces? 227 Were high tops and pastel? 143 Were high tops, pastel, and had striped laces?

### CHOOSE A STRATEGY

 Is there a special kind of diagram that can help you organize all the information in the problem? Yes, a Venn diagram.

## SOLVE IT

- If you make a Venn diagram, how many circles will you have to draw? 4 What will you label each of the circles? High tops, striped laces, pastel, plaid Do the circles intersect? Yes Why or why not? Three circles intersect to show the shoes that are high tops with striped laces; the ones that are high tops and pastel; the ones that are pastel with striped laces; and the high tops with striped laces that are pastel.
- How many pairs of shoes are high tops? 555 Where will you write this number? Under the label outside the circle How many are high tops and have striped laces? 265 Where will you write this number? In the intersection of high tops and striped laces How many are high tops and pastel? 143 Where will you write this number? In the intersection of high tops and pastel How many are high tops with striped laces and are pastel? 65 Where will you write this number? In the center intersection between all 3 circles Then how many are high tops and are not pastel and do not have striped laces? 82 Where does this number go? In the circle labeled high tops, outside the intersections
- (Have students complete their diagrams.) How many pairs of shoes does Serena have to choose from?



LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

**EXTEND IT** • Tyrone's expanded! They have 1400 pairs of high tops, 1100 with striped laces, 863 pastel, 571 plaid, and 450 polka-dotted. There are 349 high tops with striped laces; 458 striped and pastel; 640 pastel and high tops; 355 striped, high, and pastel. How many pairs do they have now?

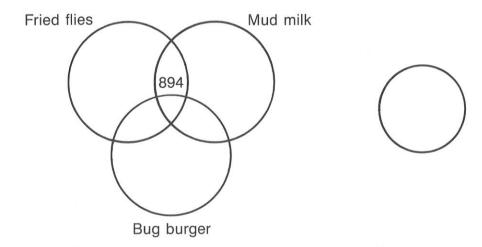
The demand for earthworm specialties at Elliot's Underground Gourmet Shop is growing. During the past month Elliot sold 7,584 glasses of mud milk, 7,872 orders of fried flies, and 8,712 bug burgers. He had 2,866 customers buy all three on the combination plate; 2,108 bought just the sawdust salad platter; 894 bought fried flies and a glass of mud milk; 2,769 bought a bug burger and some fried flies; and 2,311 bought mud milk and a bug burger. How many customers visited Elliot's Gourmet Shop in the past month?

- **FIND OUT** What is the question you have to answer?
  - How many glasses of mud milk were sold? Orders of fried flies? Bug burgers?
  - How many customers bought the combination plate? The sawdust platter? Fried flies and mud milk? A bug burger and fried flies? A bug burger and mud milk?

## CHOOSE A STRATEGY

· Is there a special kind of diagram that can help you organize all the information in the problem?

- SOLVE IT If you make a Venn diagram, how many circles will you have to draw? What will you label each of the circles? Do the circles intersect? Why or why not?
  - How many fried flies were sold? Where will you write this number? How many fried flies with mud milks were sold? Where will you write this number? How many fried flies with bug burgers were sold? Where will you write this number? How many fried flies with mud milk and bug burgers were sold? Where will you write this number? Then how many fried flies were sold without mud milk or without bug burgers? Where does this number go?
  - Keep filling in your diagram to find out the total number of customers at Elliot's during the past month.





## MAKE A PICTURE OR DIAGRAM

Teaching Plan

The demand for earthworm specialties at Elliot's Underground Gourmet Shop is growing. During the past month Elliot sold 7,584 glasses of mud milk, 7,872 orders of fried flies, and 8,712 bug burgers. He had 2,866 customers buy all three on the combination plate; 2,108 bought just the sawdust salad platter; 894 bought fried flies and a glass of mud milk; 2,769 bought a bug burger and some fried flies; and 2,311 bought mud milk and a bug burger. How many customers visited Elliot's Gourmet Shop in the past month?

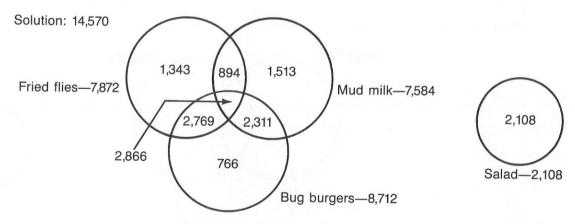
- FIND OUT What is the question you have to answer? How many customers visited Elliot's Gourmet Shop in the past month?
  - How many glasses of mud milk were sold? 7,584 Orders of fried flies? 7,872 Bug burgers?
  - How many customers bought the combination plate? 2,866 The sawdust platter? 2,108 Fried flies and mud milk? 894 A bug burger and fried flies? 2,769 A bug burger and mud milk? 2.311

# CHOOSE A STRATEGY

 Is there a special kind of diagram that can help you organize all the information in the problem? Yes, we can make a Venn diagram.

SOLVE IT

- If you make a Venn diagram, how many circles will you have to draw? 4 What will you label each of the circles? Fried flies, mud milk, bug burgers, and salad Do the circles intersect? Three of the circles intersect. Why or why not? Three intersect to show that more than one kind of food was bought by a customer; one circle is for the customers who bought only salad.
- How many fried flies were sold? 7,872 Where will you write this number? Under the label outside the circle How many fried flies with mud milks were sold? 894 Where will you write this number? In the intersection between fried flies and mud milk How many fried flies with bug burgers were sold? 2,769 Where will you write this number? In the intersection between fried flies and bug burgers How many fried flies with mud milk and bug burgers were sold? 2,866 Where will you write this number? In the center intersection between all 3 circles Then how many fried flies were sold without mud milk or without bug burgers? 1,343 Where does this number go? Outside the intersections in the fried flies circle
- (Have the students finish their diagrams.) What was the total number of customers at Elliot's during the past month?



# LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

# EXTEND IT

• Elliot's Outlet sold 582 fried flies, 669 burgers, 934 mud milks, 323 salads, and 295 spider soups. They sold 320 flies with burgers; 508 burgers with milk; 411 milk with flies; 416 flies with burgers and milk; 301 salads with milk. How many customers did the outlet have?

**PRACTICE** • Similar Practice Problems: 55, 68, 95

The planets in the Andromedia galaxy are working out a system of trade. The main resource of cold, dark Dox is ice. Hot, dry Sterp has a large quantity of warm sand, which the Doxians need. Aquam, a watery planet with little land, could also use the sand. Torf, which has acres of ancient forests, is a source of fuel for the other planets. So far the planets have worked out this exchange rate: 6 pounds of Doxian ice for 4 pounds of sand from Sterp; 9 pounds of Aquam water for 11 pounds of fuel from Torf; and 6 pounds of Aquam water for 8 pounds of Doxian ice. The residents of Sterp would also like to trade with Torf. Based on the current rates, what would be the exchange rate for fuel and sand?

- FIND OUT What is the question you have to answer?
  - What does each planet have to trade?
  - Six pounds of Doxian ice are worth how many pounds of sand from Sterp? pounds of Aquam water are worth how many pounds of fuel from Torf? 6 pounds of Aquam water are worth how many pounds of Doxian ice?

# CHOOSE A STRATEGY

- You can use a series of "If this is true, then this must be true" statements to help you solve this problem. What kind of thinking is that called?
  - How can you record your "If...then" statements?

- SOLVE IT You can begin by writing the information given in the problem as equations.
  - If 6 pounds of ice are exchanged for 4 pounds of sand, then how many pounds of sand would be exchanged for 24 pounds of ice?
  - If 9 pounds of water are exchanged for 11 pounds of fuel, then how many pounds of fuel would be exchanged for 18 pounds of water?
  - If 6 pounds of water are exchanged for 8 pounds of ice, then how many pounds of ice would be exchanged for 18 pounds of water?
  - If you have the same amount of water equal to the fuel and the ice, then what would be the rate of exchange of fuel for ice?
  - Now can you make a statement about the exchange of fuel and sand?

6 pounds ice = 4 pounds sand If

Then



# USE LOGICAL REASONING

## Teaching Plan

The planets in the Andromedia galaxy are working out a system of trade. The main resource of cold, dark Dox is ice. Hot, dry Sterp has a large quantity of warm sand, which the Doxians need. Aquam, a watery planet with little land, could also use the sand. Torf, which has acres of ancient forests, is a source of fuel for the other planets. So far the planets have worked out this exchange rate: 6 pounds of Doxian ice for 4 pounds of sand from Sterp; 9 pounds of Aquam water for 11 pounds of fuel from Torf; and 6 pounds of Aquam water for 8 pounds of Doxian ice. The residents of Sterp would also like to trade with Torf. Based on the current rates, what would be the exchange rate for fuel and sand?

- FIND OUT What is the question you have to answer? Based on the current rates, what would be the exchange rate for fuel and sand?
  - What does each planet have to trade? Dox-ice, Sterp-sand, Torf-fuel, Aquam-water
  - Six pounds of Doxian ice are worth how many pounds of sand from Sterp? 4 9 pounds of Aquam water are worth how many pounds of fuel from Torf? 11 6 pounds of Aquam water are worth how many pounds of Doxian ice? 8

### CHOOSE A STRATEGY

- You can use a series of "If this is true, then this must be true" statements to help you solve this problem. What kind of thinking is that called? Logical reasoning
- How can you record your "If... then" statements? As a series of equations

### SOLVE IT

- You can begin by writing the information given in the problem as equations.
- If 6 pounds of ice are exchanged for 4 pounds of sand, then how many pounds of sand would be exchanged for 24 pounds of ice? 16
- If 9 pounds of water are exchanged for 11 pounds of fuel, then how many pounds of fuel would be exchanged for 18 pounds of water? 22
- If 6 pounds of water are exchanged for 8 pounds of ice, then how many pounds of ice would be exchanged for 18 pounds of water? 24
- If you have the same amount of water equal to the fuel and the ice, then what would be the rate of exchange of fuel for ice? 24 pounds of ice would be equal to 22 pounds of fuel
- Now can you make a statement about the exchange of fuel and sand? Yes, now we can go back to the beginning where we have 24 pounds of ice equal to 16 pounds of sand, so then 16 pounds of sand must be equal to 22 pounds of fuel.

Solution: 16 pounds of sand for 22 pounds of fuel

If 6 pounds ice = 4 pounds sand

9 pounds water = 11 pounds fuel

6 pounds water = 8 pounds ice

24 pounds ice = 16 pounds sand (4  $\times$  6 and 4  $\times$  4) Then

18 pounds water = 22 pounds fuel (2  $\times$  9 and 2  $\times$  11)

18 pounds water = 24 pounds ice (3  $\times$  6 and 3  $\times$  8)

Then 24 pounds ice = 22 pounds fuel

22 pounds fuel = 16 pounds sand and

### LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

### EXTEND IT

• What if the rate of exchange was 4 pounds of ice for 10 pounds of sand, 3 pounds of water for 13 pounds of fuel, and 8 pounds of water for 5 pounds of ice. What would be the exchange of fuel and sand?



### USE LOGICAL REASONING

Name

A hotly-debated issue in the enchanted forest is whether to seed the clouds in order to produce more rain. There is a definite difference of opinion among the various types of trees. The forest tree leaders want any vote to be equitable and realize that some types of trees are more numerous than others. The leaders have set up this system: 6 pine tree votes are equal to 9 oak tree votes, 4 beech tree votes are equal to 13 maple tree votes, and 6 beech tree votes are equal to 8 pine tree votes. What would be the balance of maple tree votes to oak tree votes?

- FIND OUT What is the question you have to answer?
  - What did the forest tree leaders work out?
  - Six pine tree votes are equal to how many oak tree votes? are equal to how many maple tree votes? 6 beech tree votes are equal to how many pine tree votes?

# STRATEGY

- CHOOSE A You can use a series of "If this is true, then this must be true" statements to help you solve this problem. What kind of thinking is that called?
  - How can you record your "If...then" statements?

- **SOLVE IT** You can begin by writing the information given in the problem as equations.
  - If 4 beech votes are equal to 13 maple votes, how many maples would be equal to 12 beech votes?
  - If 6 beech votes are equal to 8 pine votes, how many pine votes would be equal to 12 beech votes?
  - Then how many pine votes would be equal to how many maple votes?
  - Keep making equations until you can equate a number of oak votes with maple votes.

If

6 pine tree votes = 9 oak tree votes

Then



# **USE LOGICAL REASONING**

# Teaching Plan

A hotly-debated issue in the enchanted forest is whether to seed the clouds in order to produce more rain. There is a definite difference of opinion among the various types of trees. The forest tree leaders want any vote to be equitable and realize that some types of trees are more numerous than others. The leaders have set up this system: 6 pine tree votes are equal to 9 oak tree votes, 4 beech tree votes are equal to 13 maple tree votes, and 6 beech tree votes are equal to 8 pine tree votes. What would be the balance of maple tree votes to oak tree votes?

FIND OUT • What is the question you have to answer? What would be the balance of maple tree votes to oak tree votes?

What did the forest tree leaders work out? A system to make the votes equal

 Six pine tree votes are equal to how many oak tree votes? 9 4 beech tree votes are equal to how many maple tree votes? 13 6 beech tree votes are equal to how many pine tree

### CHOOSE A STRATEGY

- You can use a series of "If this is true, then this must be true" statements to help you solve this problem. What kind of thinking is that called? Logical thinking
- How can you record your "If...then" statements? In a series of equations

### SOLVE IT

- You can begin by writing the information given in the problem as equations.
- If 4 beech votes are equal to 13 maple votes, how many maples would be equal to 12 beech votes? 39
- If 6 beech votes are equal to 8 pine votes, how many pine votes would be equal to 12 beech votes? 16
- Then how many pine votes would be equal to how many maple votes? 16 pine = 39
- (Have the students continue solving the problem using equations.) What is the balance of maple tree votes to oak tree votes?

Solution: 8 oak tree votes equal 13 maple tree votes

If 6 pine tree votes = 9 oak tree votes 4 beech votes = 13 maple votes

6 beech votes = 8 pine votes

Then 12 beech = 39 maple 12 beech = 16 pine

Then

16 pine = 39 maple

6 pine = 9 oak 16 pine = 39 maple

Then 48 pine = 72 oak 48 pine = 117 maple

and 72 oak = 117 maple  $\div$  9 = 8 oaks = 13 maples

# LOOK BACK

- **EXTEND IT** Make up a similar problem and use different types of trees.
- PRACTICE Similar Practice Problems: 58, 92, 101

23

Marta made out the order for Jaguar Jeans. She ordered 7 pairs of jeans for every jacket, 1 shirt for every 2 pairs of jeans, 3 sweaters for every jacket, 2 pairs of shoes for each 3 sweaters, and 1 belt for every 3 jackets. If Marta ordered 36 jackets, how many items did she order altogether?

- FIND OUT What is the question you have to answer?
  - How many pairs of jeans did Marta order for every jacket? How many shirts for every 2 pairs of jeans? How many sweaters for every jacket? How many pairs of shoes for each 3 sweaters? How many belts for every 3 jackets?
  - · How many jackets did Marta order?

# STRATEGY

CHOOSE A • What strategy can help you keep track of all the items Marta is ordering?

- SOLVE IT When you set up a table, what will you use as labels for the columns? For the rows?
  - If you put jackets in the first row, what numbers go in the columns?
  - · How many jeans will go in the first column? Which other rows will have a number in the first column?
  - Which column will have the first number for shirts? For belts?
  - · Continue to fill in your table until you can find out how many items she ordered, if she ordered 36 jackets.

Jacket	1	2	3	4	5	6	7	8	
Jeans									
Shirt									
Sweater									
Shoes									
Belt									
Total									



### **USE OR MAKE A TABLE**

# Teaching Plan

23

Marta made out the order for Jaguar Jeans. She ordered 7 pairs of jeans for every jacket, 1 shirt for every 2 pairs of jeans, 3 sweaters for every jacket, 2 pairs of shoes for each 3 sweaters, and 1 belt for every 3 jackets. If Marta ordered 36 jackets, how many items did she order altogether?

## FIND OUT

- · What is the question you have to answer? If Marta ordered 36 jackets, how many items did she order altogether?
- · How many pairs of jeans did Marta order for every jacket? 7 How many shirts for every 2 pairs of jeans? 7 How many sweaters for every jacket? 3 How many pairs of shoes for each 3 sweaters? 2 How many belts for every 3 jackets? 1

How many jackets did Marta order? 36

# CHOOSE A STRATEGY

What strategy can help you keep track of all the items Marta is ordering? A table

# SOLVE IT

• When you set up a table, what will you use as labels for the columns? The number of jackets For the rows? The names of the different items

• If you put jackets in the first row, what numbers go in the columns? 1 to 18

· How many jeans will go in the first column? 7 Which other rows will have a number in the first column? Sweaters and shoes

Which column will have the first number for shirts? Second For belts? Third

(Have students finish filling in their tables.) Do you need to fill in the number for jackets from 1 to 36? No, we can figure out 18 and then double the amount. How many items did she order?

Solution: 606

Jacket	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18		
Jeans	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126		
Shirt		7		14		21		28		35		42	4	49		56		63		
Sweater	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54		
Shoes	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36		
Belt			1			2			3			4			5		-	6		
Total									3							TUE		000	× 2 =	606

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT • At another Jaguar Jeans store they ordered 8 pairs of jeans for every jacket, 5 shirts for each 16 pairs of jeans, 2 sweaters for each jacket, 1/4 as many shoes as jeans, and 3 belts for each 2 jackets. If they ordered 320 pairs of jeans, how many items did they order altogether?

24

Q4T4 is the number one Martian vehicle manufacturer. They make six different forms of transportation for sweeping around the Martian countryside. For every 12 Crater Crawlers they make 4 Canal Cruisers;  $\frac{3}{4}$  the number of Dusty Dippers as Crawlers; 4½ times as many Ice Cap Crunchers as Cruisers; one-half as many Rocket Rollers as Crunchers; and 2 Rotating Ribbits for every 4 Crawlers. If in one year Q4T4 produced 252 Ice Cap Crunchers, how many vehicles did they make altogether?

- **FIND OUT** What is the question you have to answer?
  - How many Crater Crawlers did Q4T4 make for every 4 Canal Cruisers? How many Dusty Dippers? How many Ice Cap Crunchers? How many Rocket Rollers? How many Rotating Ribbits?
  - How many Ice Cap Crunchers did they make altogether this year?

# STRATEGY

CHOOSE A • What strategy can help you keep track of all the vehicles made by Q4T4?

- **SOLVE IT** When you set up a table, what will you use as labels for the columns? For the rows?
  - If you put Canal Cruisers in the first row, what numbers go in the columns?
  - How many Crawlers go in the first column? Dippers? Crunchers? Rollers?
  - Continue to fill in your table until you find 252 Crunchers.

Cruisers	4	8	12	16	20	
Crawlers	12	24	36			
Dippers	9	18				
Crunchers						
Rollers						
Ribbits						
Total						

Q4T4 is the number one Martian vehicle manufacturer. They make six different forms of transportation for sweeping around the Martian countryside. For every 12 Crater Crawlers they make 4 Canal Cruisers; 3/4 the number of Dusty Dippers as Crawlers; 41/2 times as many Ice Cap Crunchers as Cruisers; one-half as many Rocket Rollers as Crunchers; and 2 Rotating Ribbits for every 4 Crawlers. If in one year Q4T4 produced 252 Ice Cap Crunchers, how many vehicles did they make altogether?

- FIND OUT What is the question you have to answer? If in one year Q4T4 produced 252 Ice Cap Crunchers, how many vehicles did they make altogether?
  - How many Crater Crawlers did Q4T4 make for every 4 Canal Cruisers? 12 How many Dusty Dippers? 3/4 the number of Crawlers How many Ice Cap Crunchers? 41/2 times the number of Cruisers How many Rocket Rollers? 1/2 the number of Crunchers How many Rotating Ribbits? 2 Ribbits for every 4 Crawlers
  - How many Ice Cap Crunchers did they make altogether this year? 252

### CHOOSE A STRATEGY

What strategy can help you keep track of all the vehicles made by Q4T4? Making a table

# SOLVE IT

- When you set up a table, what will you use as labels for the columns? The number of Cruisers For the rows? The names of the vehicles
- If you put Canal Cruisers in the first row, what numbers go in the columns? 4, 8, 12, 16, etc.
- How many Crawlers go in the first column? 12 Dippers? 9 Crunchers? 18 Rollers? 9 Ribbits? 6
- (Have students continue to fill in the table.) When Q4T4 makes 252 Ice Cap Crunchers, how many vehicles will they have completed altogether?

Solution: 812

Cruisers	4	8	12	16	20	24	28	32	36	40	44	48	52	56
Crawlers	12	24	36	48	60	72	84	96	108	120	132	144	156	168
Dippers	9	18	27	36	45	54	63	72	81	90	99	108	117	126
Crunchers	18	36	54	72	90	108	126	144	162	180	198	216	234	252*
Rollers	9	18	27	36	45	54	63	72	81	90	99	108	117	126
Ribbits	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Total				S Roman		- 60.7								812*

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

### EXTEND IT •

Another year they made 7 Crawlers for every 5 Cruisers, twice as many Dippers as Cruisers, a number of Crunchers equal to the combined number of Cruisers and Crawlers, one-half as many Rollers as Crunchers, and 11/2 times as many Ribbits as Dippers. When they finished 150 Dippers, how many vehicles were completed altogether?

PRACTICE • Similar Practice Problems: 61, 89, 103

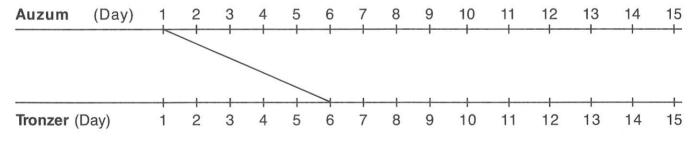
Scotty and Emma drive space shuttles between Tronzer and Auzum for ITS, the Intergalactic Transport System. That shuttle run takes 5 days one way. Scotty's spacecraft, a Tronzer Shuttle, departs from Tronzer at 10:00 A.M. and arrives on Auzum at 10:00 A.M. five days later. Emma's spacecraft, an Auzum Shuttle, starts out from Auzum at 10:00 A.M. and arrives on Tronzer five days later. There are many shuttles flying that same run. At 10:00 A.M. every day of the year, one shuttle is departing from Tronzer and one shuttle is arriving on Tronzer; and, one shuttle is departing from Auzum and one shuttle is arriving on Auzum. How many Auzum Shuttles will Scotty see on one of his five-day runs from Tronzer?

- **FIND OUT** What is the question you have to answer?
  - Between what two places do Scotty and Emma drive space shuttles?
  - How long does it take a shuttle to fly from Tronzer to Auzum? From Auzum to
  - How often, and at what time, do Tronzer Shuttles depart from Tronzer? How many days after it departs from Tronzer does a shuttle arrive on Auzum? At what time?
  - How often, and at what time, do Auzum Shuttles depart from Auzum? How many days after it departs from Auzum does a shuttle arrive on Tronzer? At what time?

# CHOOSE A STRATEGY

 Is there a good way to keep track of the shuttles coming and going, so you can "see" when their paths cross?

- **SOLVE IT** What do you want to keep track of in your diagram?
  - If a shuttle departs from Auzum on day 1, on what day will it arrive in Tronzer? Draw a line from Auzum day 1 to Tronzer day 6. If a shuttle departs from Auzum on day 2, on what day will it arrive in Tronzer? Draw lines to show the Auzum Shuttle departures and arrivals.
  - If a shuttle departs from Tronzer on day 1, on what day will it arrive in Auzum? Draw lines to show the Tronzer Shuttle departures and arrivals.
  - Let's say that Scotty's shuttle starts out from Tronzer on day 7. Does he see an Auzum Shuttle before he leaves Tronzer? Make a dot at Tronzer and at each place that his shuttle crosses paths with an Auzum Shuttle after he leaves Tronzer. Does he see a shuttle when he arrives in Auzum?
  - How many Auzum Shuttles does Scotty see on his flight?



Scotty and Emma drive space shuttles between Tronzer and Auzum for ITS, the Intergalactic Transport System. That shuttle run takes 5 days one way. Scotty's spacecraft, a Tronzer Shuttle, departs from Tronzer at 10:00 A.M. and arrives on Auzum at 10:00 A.M. five days later. Emma's spacecraft, an Auzum Shuttle, starts out from Auzum at 10:00 A.M. and arrives on Tronzer five days later. There are many shuttles flying that same run. At 10:00 A.M. every day of the year, one shuttle is departing from Tronzer and one shuttle is arriving on Tronzer; and, one shuttle is departing from Auzum and one shuttle is arriving on Auzum. How many Auzum Shuttles will Scotty see on one of his five-day runs from Tronzer?

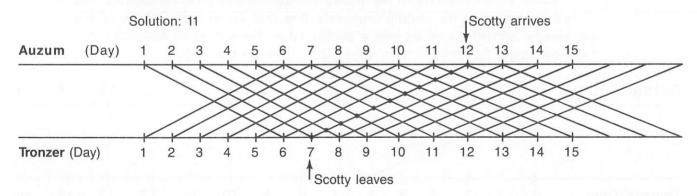
- FIND OUT What is the question you have to answer? How many Auzum Shuttles will Scotty see on one of his five-day runs from Tronzer?
  - Between what two places do Scotty and Emma drive space shuttles? Tronzer and Auzum
  - How long does it take a shuttle to fly from Tronzer to Auzum? 5 days From Auzum to Tronzer? 5 days
  - · How often, and at what time, do Tronzer Shuttles depart from Tronzer? Every day of the year, at 10:00 A.M. How many days after it departs from Tronzer does a shuttle arrive on Auzum? 5 At what time? 10:00 A.M.
  - How often, and at what time, do Auzum Shuttles depart from Auzum? Every day of the year, at 10:00 A.M. How many days after it departs from Auzum does a shuttle arrive on Tronzer? 5 At what time? 10:00 A.M.

# CHOOSE A STRATEGY

• Is there a good way to keep track of the shuttles coming and going, so you can "see" when their paths cross? Yes, we can make a diagram.

# SOLVE IT

- What do you want to keep track of in your diagram? The days on which shuttles depart from Tronzer and arrive in Auzum; and the days on which shuttles depart from Auzum and arrive in Tronzer
- If a shuttle departs from Auzum on day 1, on what day will it arrive in Tronzer? Day 6 Draw a line from Auzum day 1 to Tronzer day 6. If a shuttle departs from Auzum on day 2, on what day will it arrive in Tronzer? Day 7 Draw lines to show the Auzum Shuttle departures and arrivals.
- · If a shuttle departs from Tronzer on day 1, on what day will it arrive in Auzum? Day 6 Draw lines to show the Tronzer Shuttle departures and arrivals.
- Let's say that Scotty's shuttle starts out from Tronzer on day 7. Does he see an Auzum Shuttle before he leaves Tronzer? Yes Make a dot at Tronzer and at each place that his shuttle crosses paths with an Auzum Shuttle after he leaves Tronzer. Does he see a shuttle when he arrives in Auzum? Yes
- How many Auzum Shuttles does Scotty see on his flight?



# LOOK BACK

- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?
- **EXTEND IT** Make up another similar problem. Make the shuttle trip 6 days long.

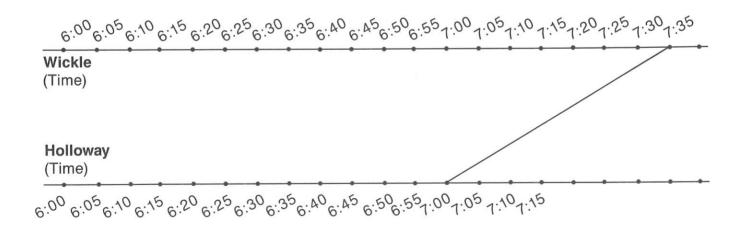
At 7:00 one summer morning, Tobie and Helene hopped into their speedy motorboat and left the Holloway dock. They arrived at Wickle Wharf 35 minutes later. During busy summer months, a car ferry departs from the Wickle Wharf every 5 minutes, beginning at 6:00 A.M. Tourists and local people crowd onto the ferry for the 55-minute ride to Holloway. If Tobie and Helene followed the route that the car ferries take between Wickle and Holloway, how many car ferries did they see on their trip?

- FIND OUT What is the question you have to answer?
  - Between what two places do the car ferries run?
  - At what time did Tobie and Helene leave Holloway in their motorboat? How long did it take them to get to Wickle? What time did they arrive at Wickle Wharf?
  - At what times of the day do car ferries depart from Wickle on summer days?
  - How long does it take a car ferry to go from Wickle to Holloway?

## CHOOSE A STRATEGY

• Is there a good way to keep track of the car ferry trips and Tobie and Helene's trip, so that you can "see" when their paths cross?

- SOLVE IT What do you want to keep track of in your diagram? Do you have to keep track of every minute of the day?
  - What time did Tobie and Helene leave Holloway? What time did they arrive at Wickle? Draw a line to show the time their trip took.
  - If a car ferry left Wickle at 6:00 A.M., at what time did it arrive at Holloway? Draw a line from Wickle (6:00) to Holloway (6:55). If a car ferry left Wickle at 6:05, at what time did it arrive in Holloway? Draw a line to show that trip. Continue to draw lines to show the car ferry trips from Wickle to Holloway.
  - Find the points at which Tobie and Helene crossed paths with car ferries. How many car ferries did Tobie and Helene see on their trip?





# MAKE A PICTURE OR DIAGRAM

Teaching Plan

At 7:00 one summer morning, Tobie and Helene hopped into their speedy motorboat and left the Holloway dock. They arrived at Wickle Wharf 35 minutes later. During busy summer months, a car ferry departs from the Wickle Wharf every 5 minutes, beginning at 6:00 A.M. Tourists and local people crowd onto the ferry for the 55-minute ride to Holloway. If Tobie and Helene followed the route that the car ferries take between Wickle and Holloway, how many car ferries did they see on their trip?

FIND OUT • What is the question you have to answer? How many car ferries did Tobie and Helene see on their trip from Holloway to Wickle?

Between what two places do the car ferries run? Wickle and Holloway

- At what time did Tobie and Helene leave Holloway in their motorboat? 7:00 A.M. How long did it take them to get to Wickle? 35 minutes What time did they arrive at Wickle Wharf? 7:35 A.M.
- At what times of the day do car ferries depart from Wickle on summer days? Every five minutes, beginning at 6:00 A.M.
- How long does it take a car ferry to go from Wickle to Holloway? 55 minutes

# CHOOSE A STRATEGY

• Is there a good way to keep track of the car ferry trips and Tobie and Helene's trip, so that you can "see" when their paths cross? Yes, we can make a diagram.

SOLVE IT

• What do you want to keep track of in your diagram? The times at which car ferries depart from Wickle and arrive at Holloway; and the times at which Tobie and Helene left Holloway and arrived at Wickle Do you have to keep track of every minute of the day? No, we can just keep track of 5-minute intervals.

What time did Tobie and Helene leave Holloway? 7:00 A.M. What time did they arrive at

Wickle? 7:35 Draw a line to show the time their trip took.

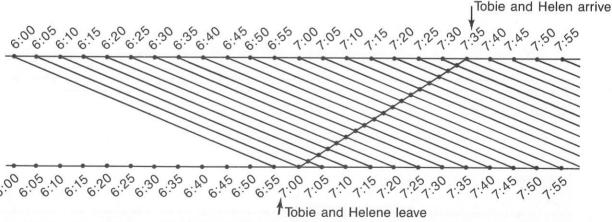
• If a car ferry left Wickle at 6:00 A.M., at what time did it arrive at Holloway? 6:55 A.M. Draw a line from Wickle (6:00) to Holloway (6:55). If a car ferry left Wickle at 6:05, at what time did it arrive in Holloway? 7:00 Draw a line to show that trip. Continue to draw lines to show the car ferry trips from Wickle to Holloway.

(Have students find the points at which Tobie and Helene crossed paths with car ferries.) How many car ferries did Tobie and Helene see on their trip?

Solution: 19

Wickle (Time)

Holloway (Time)



LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

**EXTEND IT** • Make up a similar problem about the car ferry trips during the winter months. Change the times of departure from Wickle and the length of time the trip takes. Change the length of time for Tobie and Helene's trip, also.

PRACTICE • Similar Practice Problems: 67, 94, 107

Lenny, the zookeeper's assistant, was exhausted. His new charges were eating up a storm. The tiny-toothed trilobat ate 3 ounces of food at each feeding. When Lenny fed the 3-horned hepplefoot, he had to bring 7 ounces of food, while the red-eyed rippersnout ate 29 ounces at each feeding. The fuzzy-necked fizzlenit ate 38 ounces at a feeding and the furry furkadoo ate 44 ounces at a time. After one particularly long day, Lenny made 71 separate feedings, a total of 1,773 ounces of food. How many times did each creature get fed, and how many ounces did each one eat altogether?

- FIND OUT What is the question you have to answer?
  - How much does the trilobat eat at each feeding? The hepplefoot? The rippersnout? The fizzlenit? The furkadoo?
  - · After one long day, how many separate feedings did he make? How much did they eat altogether?

## CHOOSE A STRATEGY

How can you systematically display the information for this problem?

- SOLVE IT When you make an organized list, how many columns will you need? What will you use as labels for the columns?
  - What will you record in the first row of your list? In the second row?
  - Fill in several rows of your list until you find a combination of feedings that make 71. What does the total number of ounces for these feedings need to be?
  - Continue to fill in your list and test combinations of feedings that equal 71.

	Trilobat	Hepplefoot	Rippersnout	Fizzlenit	Furkadoo
1	3	7	29	38	44
2					
3					

Lenny, the zookeeper's assistant, was exhausted. His new charges were eating up a storm. The tiny-toothed trilobat ate 3 ounces of food at each feeding. When Lenny fed the 3-horned hepplefoot, he had to bring 7 ounces of food, while the red-eyed rippersnout ate 29 ounces at each feeding. The fuzzy-necked fizzlenit ate 38 ounces a feeding and the furry furkadoo ate 44 ounces at a time. After one particularly long day, Lenny made 71 separate feedings, a total of 1,773 ounces of food. How many times did each creature get fed, and how many ounces did each one eat altogether?

- FIND OUT What is the question you have to answer? How many times did he feed each creature, and many ounces did each creature eat?
  - How much does the trilobat eat at each feeding? 3 ounces The hepplefoot? 7 ounces The rippersnout? 29 ounces The fizzlenit? 38 ounces The furkadoo? 44 ounces
  - After one long day, how many separate feedings did he make? 71 How much did they eat altogether? 1,773 ounces

## CHOOSE A STRATEGY

 How can you systematically display the information for this problem? We can make an organized list.

# SOLVE IT

- · When you make an organized list, how many columns will you need? 5 What will you use as labels for the columns? The names of the 5 creatures
- What will you record in the first row of your list? The amount that each creature eats for 1 feeding In the second row? The amount they have eaten after 2 feedings
- · Fill in several rows of your list until you find a combination of feedings that total 71. What does the total number of ounces for these feedings need to be? 1,773 ounces
- (Have the students continue to fill in their list and test combinations of feedings that add up to 71.) How many times did each creature get fed and how much did each one eat altogether?

Solution: Trilobat: 13 — 39 oz, Hepplefoot: 14 — 98 oz, Rippersnout: 14 — 406 oz, Fizzlenit: 15 — 570 oz, Furkadoo: 15 — 660 oz

	Trilobat	Hepplefoot	Rippersnout	<b>Fizzlenit</b>	Furkadoo
1	3	7	29	38	44
2	6	14	58	76	88
3	9	21	87	114	132
4	12	28	116	152	176
5	15	35	145	190	220
6	18	42	174	228	264
7	21	49	203	266	308
8	24	56	232	304	352
9	27	63	261	342	396
10	30	70	290	380	440
11	33	77	319	418	484
12	36	84	348	456	528
13	*39	91	377	494	572
14	42	*98	*406	532	616
15	45	105	435	*570	*660

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

### EXTEND IT

• The next week they had a new creature, the sillybit, which ate 63 ounces at each feeding. One day Lenny made a total of 48 feedings for a total of 1,580 ounces of food. How many feedings did each creature get and how much did each one eat altogether?

Andy and Ellen began baking early at the Muffin Mansion, which supplied muffins for local businesses. Today they baked oatmeal bran muffins, which cost \$2.31 for one dozen; banana yogurt muffins, which cost \$4.52 for one and a half dozen; and chocolate raisin muffins, which cost \$7.66 for two and a half dozen. During the breakfast rush they sold a total of 642 muffins for \$153.21. How many of each kind of muffin did they sell?

- **FIND OUT** What is the question you have to answer?
  - How much do the oatmeal bran muffins cost and how many do you get? How much do the banana yogurt muffins cost and how many do you get? How much do the chocolate raisin muffins cost and how many do you get?
  - How many muffins did they sell during the breakfast rush?

# STRATEGY

**CHOOSE A** • How can you systematically display the information for this problem?

- **SOLVE IT** When you make an organized list, how many columns will you need? What will you use as labels for the columns?
  - What will you record in the first row of your list? In the second row?
  - Fill in several rows of your list until you find a combination of muffins that makes 642. What does the total cost for these muffins have to be?
  - Continue to fill in your list and test combinations of muffins that equal 642.

Banana yogurt	Bran oatmeal	Chocolate raisin	_
1.5 doz = \$4.52	1  doz = \$2.31	2.5  doz = \$7.66	
3 doz			

Andy and Ellen began baking early at the Muffin Mansion, which supplied muffins for local businesses. Today they baked oatmeal bran muffins, which cost \$2.31 for one dozen; banana yogurt muffins, which cost \$4.52 for one and a half dozen; and chocolate raisin muffins, which cost \$7.66 for two and a half dozen. During the breakfast rush they sold a total of 642 muffins for \$153.21. How many of each kind of muffin did they sell?

### FIND OUT

- What is the question you have to answer? How many of each kind of muffin did they sell?
- How much do the oatmeal bran muffins cost and how many do you get? \$2.31 for 1 dozen How much do the banana yogurt muffins cost and how many do you get? 11/2 dozen for \$4.52 How much do the chocolate raisin muffins cost and how many do you get? \$7.66 for 21/2 dozen
- How many muffins did they sell during the breakfast rush? 642

# CHOOSE A STRATEGY

 How can you systematically display the information for this problem? We can make an organized list.

# SOLVE IT

- When you make an organized list, how many columns will you need? 3 What will you use as labels for the columns? The names of the muffins
- What will you record in the first row of your list? The quantity each muffin is sold in and the price In the second row? The price of twice the amount from the first row
- Fill in several rows of your list until you find a combination of muffins that makes 642. What does the total cost for these muffins have to be? \$153.21
- (Have students continue to fill in their lists and test combinations of numbers, until they find 642.) How many of each kind of muffin did they sell?

Solution: Banana yogurt = 216, Bran oatmeal = 156, Chocolate = 270

Banana yogurt	Bran oatmeal	Chocolate raisin
1.5  doz = \$4.52	1 doz = \$2.31	2.5 doz = \$7.66
3 = 9.04	2 = 4.62	5 = 15.32
4.5 = 13.56	3 = 6.93	7.5 = 22.98
6 = 18.08	4 = 9.24	10 = 30.64
7.5 = 22.60	5 = 11.55	12.5 = 38.30
9 = 27.12	6 = 13.86	15 = 45.96
10.5 = 31.64	7 = 16.17	17.5 = 53.62
12 = 36.16	8 = 18.48	20 = 61.28
13.5 = 40.68	9 = 20.79	*22.5 = 68.94
15 = 45.20	10 = 23.10	25 = 76.60
16.5 = 49.72	11 = 25.41	
*18 = 54.24	12 = 27.72	
	*13 = 30.03	

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

**EXTEND IT** • The following week they had the same muffins along with apple granola, which sold in quantities of 31/4 dozen for \$5.80. If they sold 687 muffins for \$140.27 one day, how many of each kind of muffin did they sell?

PRACTICE • Similar Practice Problems: 71, 96, 111



Name

"Seven eighths of the number, added to 72, doubles the number," read Soren. "Is this a joke?" Larry looked at the paper and said, "No, it's the number of my locker." What is the number of Larry's locker?

- **FIND OUT** What is the question you have to answer?
  - What do you know about the number of the locker?

- **CHOOSE A** Will guessing the answer help you to solve this problem?
- STRATEGY How can you use the information from an incorrect guess?

- **SOLVE IT** If you have to find  $\frac{7}{8}$  of the number, will you want to begin with a whole number that is a multiple of 8?
  - What is your guess? What is  $\frac{7}{8}$  of that number?
  - What number do you get if you add 72 to your guess? Is that double your quess?
  - If your guess was wrong, how can you use the information to make your next guess?
  - Continue guessing and checking until you find the number of Larry's locker.



### Teaching Plan

29

"Seven eighths of the number, added to 72, doubles the number," read Soren. "Is this a joke?" Larry looked at the paper and said, "No, it's the number of my locker." What is the number of Larry's locker?

### FIND OUT

- What is the question you have to answer? What is the number of Larry's locker?
- What do you know about the number of the locker? 7/8 of the number, added to 72, doubles the number.

# CHOOSE A STRATEGY

- Will guessing the answer help you to solve this problem? Yes, because we're not really sure what to do.
- How can you use the information from an incorrect guess? The incorrect guess can help us decide if the the next guess should be higher or lower.

### SOLVE IT

- If you have to find % of the number, will you want to begin with a whole number that is a multiple of 8? Yes
- What is your guess? 16 (This is an example of a guess.) What is 1/8 of that number? 14
- What number do you get if you add 72 to your guess? 88 Is that double your guess? No, 16 × 2 is 32.
- If your guess was wrong, how can you use the information to make your next guess? Our next guess should be higher.
- (Have students continue guessing and checking until they find a solution.) What is the number of Larry's locker?

Solution: 64

 $\frac{7}{8} \times 64 = 56$ 

56 + 72 = 128

 $64 \times 2 = 128$ 

### LOOK BACK

• Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

### EXTEND IT

• What would the locker number be if the paper said that 11/7 of the number, plus 90, triples the locker number.



Name

The Split Ends are hot! They have played 20 live shows this week, and every record store in town is sold out of their albums. Their manager told reporters that over a thousand records were sold in one hour after the last concert. He said, "If you take  $\frac{4}{6}$  of the number and double it, that's 500 more than the number of records sold." Give the reporters a break. How many records were sold?

- FIND OUT What is the question you have to answer?
  - What do you know about the number of records sold?

- **CHOOSE A** Will guessing the answer help you to solve this problem?
- **STRATEGY** How can you use the information from an incorrect guess?

- **SOLVE IT** The mystery number will be a multiple of what number?
  - What is your guess? Keep the 500 number in mind.
  - What is  $\frac{4}{6}$  of the number you guessed? What is the result when you double that? What is the result when you subtract 500 from that? Is that the same number as your guess?
  - If your guess was wrong, how can you use the information to make your next
  - Continue guessing and checking until you find out how many records were sold.



### Teaching Plan

The Split Ends are hot! They have played 20 live shows this week, and every record store in town is sold out of their albums. Their manager told reporters that over a thousand records were sold in one hour after the last concert. He said, "If you take 1/6 of the number and double it, that's 500 more than the number of records sold." Give the reporters a break. How many records were sold?

FIND OUT

- What is the question you have to answer? How many records were sold?
- What do you know about the number of records sold? 1/6 of the number, doubled, is 500 more than the number of records sold.

CHOOSE A STRATEGY

- · Will guessing the answer help you to solve this problem? Yes.
- How can you use the information from an incorrect guess? The incorrect guess can help us decide if the next guess should be higher or lower.

SOLVE IT

- The mystery number will be a multiple of what number? 6
- What is your guess? Keep the 500 number in mind. 1200 (This is an example of a guess.)
- What is 4/6 of the number you guessed? 800 What is the result when you double that? 1600 What is the result when you subtract 500 from that? 1100 Is that the same number as vour quess? No.
- If your guess was wrong, how can you use the information to make your next guess? Our next guess should be lower.
- (Have students continue guessing and checking until they find a solution.) How many records were sold?

Solution: 1500

 $\frac{4}{6}$  × 1500 = 1000

 $1000 \times 2 = 2000$ 

2000 - 500 = 1500

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • How many records would have been sold if the manager had said, "Seven ninths of the number, doubled, is 675 more than the number of records sold."

PRACTICE • Similar Practice Problems: 74, 104, 115



### MAKE A PICTURE OR DIAGRAM

Name

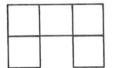
Nikko moves five pictures around on the table and then steps back to look at the arrangement. He is arranging the photographs for a page in the school yearbook. The five pictures that he is working with are all the same size square. If each photo has to share at least one side with another photo, what are all the different ways he could place the pictures on the page?

- FIND OUT What is the question you have to answer?
  - How many photos is Nikko trying to arrange on the page?
  - What shape are the photographs? What do you know about the sizes of the photos?
  - Will the photos be separated on the page?

## CHOOSE A STRATEGY

- Would it be helpful to try and "see" this problem by making a picture or diagram?
- Would it be helpful to use graph paper to record the different ways?

- **SOLVE IT** How many sides must each photo share with another photo?
  - Draw the squares or shade in squares on graph paper to show one way that Nikko could arrange the five photos.
  - Is there another way Nikko could arrange the pictures?
  - Continue to find and record arrangements of 5 squares, until you have made all possible arrangements.





## MAKE A PICTURE OR DIAGRAM

Teaching Plan

Nikko moves five pictures around on the table and then steps back to look at the arrangement. He is arranging the photographs for a page in the school yearbook. The five pictures that he is working with are all the same size square. If each photo has to share at least one side with another photo, what are all the different ways he could place the pictures on the page?

FIND OUT • What is the question you have to answer? What are all the different ways Nikko could place the pictures on the page?

How many photos is Nikko trying to arrange on the page? 5

- · What shape are the photographs? Square What do you know about the sizes of the photos? They are all the same size.
- Will the photos be separated on the page? No, each photo will share at least one side with another photo.

### CHOOSE A STRATEGY

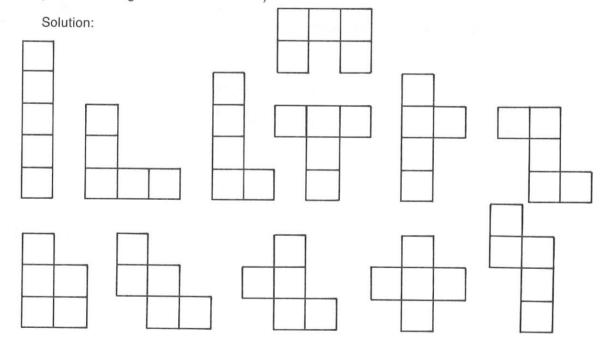
- Would it be helpful to try and "see" this problem by making a picture or diagram? Yes. We have to record all the different ways that Nikko could arrange the photos.
- · Would it be helpful to use graph paper to record the different ways? Yes.

### SOLVE IT

- How many sides must each photo share with another photo? 1
- Draw the squares or shade in squares on graph paper to show one way that Nikko could arrange the five photos.

Is there another way Nikko could arrange the pictures? Yes.

(Have students continue to draw arrangements of 5 squares, until they have made all possible arrangements shown below.)



LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your diagrams. Is your answer reasonable?

**EXTEND IT** • How many different ways could Nikko arrange the photos if they were equilateral triangles of the same size?



### MAKE A PICTURE OR DIAGRAM

Name

King Hoptumastep called his advisors together one hot, sunny day on the plains of Gonz. He said, "I want to build a structure that will tell all the world of my greatness, and that we can use as a communications base. I want the structure to be made up of six triangular pieces of quartz, which is an excellent conductor of supersonic waves. Each triangle must be equilateral, and must share at least one side with another triangle. All triangles must be the same size. I hereby command you, my advisors, to present to me as many different designs as possible. Be off with you now, and don't return until you have designs to show me!" With that, King Hoptumastep's advisors scurried off to discover the designs. What are all the different designs the advisors could present to the king?

- **FIND OUT** What is the question you have to answer?
  - How many pieces of quartz must the advisors use in the structure?
  - · What shape must the pieces be? Will the sides of the triangles be different lengths? What do you know about the sizes of the triangular pieces?
  - Will the pieces of quartz be separated in the structure?

### CHOOSE A STRATEGY

- Would it be helpful to try and "see" this problem by making a picture or
- Would it be helpful to use graph paper to record the different designs?

- **SOLVE IT** How many sides must each triangle share with another triangle?
  - Draw the triangles or shade in triangles on graph paper to show one design that the advisors would present to the king.
  - Is there another way the advisors could arrange the triangles?
  - Continue to draw arrangements of 6 triangles until you have made all possible arrangements.

King Hoptumastep called his advisors together one hot, sunny day on the plains of Gonz. He said, "I want to build a structure that will tell all the world of my greatness, and that we can use as a communications base. I want the structure to be made up of six triangular pieces of quartz, which is an excellent conductor of supersonic waves. Each triangle must be equilateral, and must share at least one side with another triangle. All triangles must be the same size. I hereby command you, my advisors, to present to me as many different designs as possible. Be off with you now, and don't return until you have designs to show me!" With that, King Hoptumastep's advisors scurried off to discover the designs. What are all the different designs the advisors could present to the king?

### FIND OUT

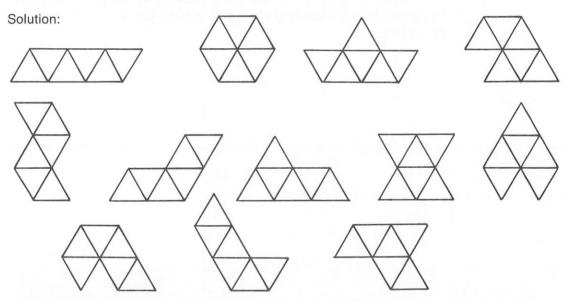
- What is the question you have to answer? What are all the different designs the advisors could present to the king?
- How many pieces of quartz must the advisors use in the structure?
- What shape must the pieces be? Equilateral triangles Will the sides of the triangles be different lengths? No, all of the sides must be the same length. What do you know about the sizes of the triangular pieces? They must all be the same size.
- Will the pieces of quartz be separated in the structure? No. each piece will share at least one side with another piece.

### CHOOSE A STRATEGY

- Would it be helpful to try and "see" this problem by making a picture or diagram? Yes. We have to record all the different designs that the advisors could make.
- · Would it be helpful to use graph paper to record the different ways? Yes.

### SOLVE IT

- How many sides must each triangle share with another triangle? 1
- · Draw the triangles or shade in triangles on graph paper to show one design that the advisors would present to the king.
- Is there another way the advisors could arrange the triangles? Yes.
- (Have students continue to draw arrangements of 6 triangles, until they have made all possible arrangements shown below.)



# LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your diagrams. Is your answer reasonable?

## EXTEND IT

 How many different designs would be possible if the structure was made of six rhombuses?

PRACTICE • Similar Practice Problems: 72, 102, 113



Name

"This is Dino, your dynamite D.J. from radio station KLOT," announced the familiar voice. "All you listeners out there, our Rock Trivia Contest begins today, and we're giving away a LOT of prizes. We're giving one first prize, two second prizes, three third prizes, and so on. Call in and be a winner. Good luck! Go for it!" How many winners will there be by the time the sixteenth-place prizes have been awarded?

- **FIND OUT** What is the question you have to answer?
  - How many winners will there be for first prize? For second prize? For third prize? What does "and so on" mean?
  - What is the highest number of the prizes to be given?

- **CHOOSE A** How can you organize the information in the problem?
- **STRATEGY** Is there a pattern in the way the numbers of winners for the prizes change?

- SOLVE IT Continue to fill in the table until you get to the 16th prize.
  - What sum do you get when you add the number of 1st-prize winners to the number of 16th-prize winners? What sum do you get when you add the number of 2nd-prize winners to the number of 15th-prize winners? If you continue in this way, how many paired sets of winners give a sum of 17? Is there a set of winners that you cannot pair with another set?
  - How many winners will there be by the time the 16th-place prizes have been awarded?

Prize	1 2	2	3
inners	1 2	2	3



Teaching Plan

"This is Dino, your dynamite D.J. from radio station KLOT," announced the familiar voice. "All you listeners out there, our Rock Trivia Contest begins today, and we're giving away a LOT of prizes. We're giving one first prize, two second prizes, three third prizes, and so on. Call in and be a winner. Good luck! Go for it!" How many winners will there be by the time the sixteenth-place prizes have been awarded?

- FIND OUT What is the question you have to answer? How many winners will there be by the time the sixteenth-place prizes have been awarded?
  - How many winners will there be for first prize? 1 For second prize? 2 For third prize? 3 What does "and so on" mean? The number of winners equals the number of the prize for each kind of prize.
  - What is the highest number of the prizes to be given? 16

### CHOOSE A STRATEGY

- How can you organize the information in the problem? We can make a table.
- Is there a pattern in the way the numbers of winners for the prizes change? Yes, the numbers keep increasing by 1.

### SOLVE IT

- Continue to fill in the table until you get to the 16th prize.
- What sum do you get when you add the number of 1st-prize winners to the number of 16th-prize winners? 17 What sum do you get when you add the number of 2nd-prize winners to the number of 15th-prize winners? 17 If you continue in this way, how many paired sets of winners give a sum of 17? 8 Is there a set of winners that you cannot pair with another set? No.
- How many winners will there be by the time the 16th-place prizes have been awarded? 8 × 17

Solution: 136

Prize	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Winners	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	1	1					45	47							1	1
	_			-		-2 - -1 -	+ 15	= 17						:	'_	i

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern. Is your answer reasonable?

**EXTEND IT** • What number prize will the 75th winner receive?



Name

34

The fierce army of ants marches forth to battle. The general makes up the whole first row. Two colonels make up the second row. Three lieutenant colonels form the third row. Four majors make up the fourth row. Five soldiers form the fifth row, and so on, row after row. If the army consists of 525 ants, how many rows are there, and is the last row full?

- **FIND OUT** What is the question you have to answer?
  - How many ants are there in the first row? In the second row? In the third row? In the fourth row? In the fifth row? In the tenth row?
  - · How many ants are there in the army?

- **CHOOSE A** How can you organize the information in the problem?
- STRATEGY Is there a pattern in the way the numbers of ants in the rows change?

- **SOLVE IT** Continue to fill in the table through row 10.
  - · What sum do you get when you add the number of ants in the first row to the number of ants in the tenth row? What sum do you get when you add the number of ants in the second row with the ants in the ninth row? If you continue in this way, how many paired sets of ants give a sum of 11? Is there a set of ants that you cannot pair with another set? How many ants are there in 10 rows?
  - If you added 10 more rows, how many rows would there be in all? What sum would you get if you added the ants from row 1 to the ants in row 20? How many ants would there be in all?
  - Continue to add rows until you find out how many rows the 525 ants formed, and whether or not the last row was full.

Row	1	2	3	4	5	
Ants	1	2	3	4	5	



Teaching Plan

The fierce army of ants marches forth to battle. The general makes up the whole first row. Two colonels make up the second row. Three lieutenant colonels form the third row. Four majors make up the fourth row. Five soldiers form the fifth row, and so on, row after row. If the army consists of 525 ants, how many rows are there, and is the last row full?

- What is the question you have to answer? How many rows of ants are there, and is the last row full?
  - How many ants are there in the first row? 1 In the second row? 2 In the third row? 3 In the fourth row? 4 In the fifth row? 5 In the tenth row? 10
  - How many ants are there in the army? 525

### CHOOSE A STRATEGY

- How can you organize the information in the problem? We can make a table.
- Is there a pattern in the way the numbers of ants in the rows change? Yes, the numbers keep increasing by 1.

### SOLVE IT

· Continue to fill in the table through row 10.

- · What sum do you get when you add the number of ants in the first row to the number of ants in the tenth row? 11 What sum do you get when you add the number of ants in the second row with the ants in the ninth row? 11 If you continue in this way, how many paired sets of ants give a sum of 11? 5 Is there a set of ants that you cannot pair with another set? No. How many ants are there in 10 rows?  $5 \times 11 = 55$
- If you added 10 more rows, how many rows would there be in all? 20 What sum would you get if you added the ants from row 1 to the ants in row 20? 21 How many ants would there be in all?  $20 \div 2 \times 21 = 210$
- (Have students continue to add rows.) How many rows did the 525 ants form, and was the last row full?

Solution: 32 rows: the 32nd row was not full

Row	1	2	3	4	5	6	7	8	9	10	 32
Ants	1	2	3	4	5	6	7	8	9	10	 32

(You may wish to show students an algorithm than can be used for solving this type of problem. Multiply the number of rows by the sum of the first and last sets of ants, and divide by 2.  $32 \times 33 \div 2$ 

### LOOK BACK

• Read the problem again. Look at the data, conditions, and the main question. Review your table and the pattern. Is your answer reasonable?

### EXTEND IT

• The army of ants increased to 40 rows. Then the first 10 rows broke into a faster stride and moved out ahead. How many ants were left in the last 30 rows?

PRACTICE • Similar Practice Problems: 82, 106, 116

Jana, Garth, Ian, Elsa, Hak, and Adriana are all employed in these twenty-first century jobs: bionic engineer, used space shuttle sales, robot repair, intergalactic weather analyst, laser technician, and robot relations counselor. Jana, Elsa, and Adriana did not take robot relations courses in college. The bionic engineer and laser technician often ride the bus with lan, Hak, and Adriana. The used shuttle salesperson, weather analyst, and bionic engineer often have lunch with Jana, Garth, and Adriana. Hak and the weather analyst work on different floors in the same building. Can you match each person with a iob?

- **FIND OUT** What is the question you have to answer?
  - Who are the people employed in twenty-first century jobs?
  - · What jobs are they employed in?
  - What do you know about Jana, Elsa, and Adriana? What else do you know about Adriana? What do you know about Ian and Hak? What else do you know about Hak? What do you know about Jana and Garth?

# STRATEGY

- **CHOOSE A** The information in this problem is given in a set of clues. If you use a series of "If...then" statements to solve this problem, what kind of thinking will you be using?
  - Is there another strategy you can use to organize the information?

- **SOLVE IT** When you make a table, how many columns will you need? How many rows?
  - What will you label the columns? The rows?
  - If Jana, Elsa, and Adriana did not take robot relations courses in college, then what job can you assume they don't have? In which boxes can you write N for
  - If Ian, Hak, and Adriana have lunch with the bionic engineer and laser technician, then which jobs can you assume lan, Hak, and Adriana do not have? In which boxes can you write N?
  - What else do you know about Adriana? If you take all the information you have about her and put N in the boxes, can you figure out what job she must have? In which box will you put a Y? If you match her with a job, you can write N for all the other boxes in the row for that job.
  - Continue going through the clues and writing N where you can, until you match everyone with a job.

	Jana	Elsa	Garth	lan	Hak	Adriana	
Robot relations	N	N				N	
Bionic engineer							
Laser technician							
Shuttle sales							
Weather analyst							
Robot repairs							



### **USE LOGICAL REASONING**

### Teaching Plan

Jana, Garth, Ian, Elsa, Hak, and Adriana are all employed in these twenty-first century jobs: bionic engineer, used space shuttle sales, robot repair, intergalactic weather analyst, laser technician, and robot relations counselor. Jana, Elsa, and Adriana did not take robot relations courses in college. The bionic engineer and laser technician often ride the bus with lan, Hak, and Adriana. The used shuttle salesperson, weather analyst, and bionic engineer often have lunch with Jana, Garth, and Adriana. Hak and the weather analyst work on different floors in the same building. Can you match each person with a job?

- FIND OUT What is the question you have to answer? Can you match each person with a job?
  - Who are the people employed in twenty-first century jobs? Jana, Elsa, Garth, lan, Hak, and
  - What jobs are they employed in? Robot relations, bionic engineer, laser technician, used shuttle sales, weather analyst, robot repair
  - · What do you know about Jana, Elsa, and Adriana? They did not take robot relations courses in college. What else do you know about Adriana? She rides the bus with the bionic engineer and laser technician, and she goes to lunch with the shuttle salesperson, weather person, and bionic engineer. What do you know about Ian and Hak? They ride the bus with the bionic engineer and laser technician. What else do you know about Hak? He and the weather analyst work on different floors. What do you know about Jana and Garth? They have lunch with the used shuttle salesperson, weather person, and bionic engineer.

### CHOOSE A STRATEGY

- The information in this problem is given in a set of clues. If you use a series of "If . . . then" statements to solve this problem, what kind of thinking will you be using? Logical reasoning
- Is there another strategy you can use to organize the information? Yes, we can make a

## SOLVE IT

- When you make a table, how many columns will you need? 6 How many rows? 6
- What will you label the columns? With names of people The rows? Names of jobs
- If Jana, Elsa, and Adriana did not take robot relations courses in college, then what job can you assume they don't have? Robot relations counselor In which boxes can you write N for No? Under their names in the row for robot relations counselor
- If lan, Hak, and Adriana have lunch with the bionic engineer and laser technician, then which jobs can you assume lan, Hak, and Adriana do not have? They must not be the bionic engineer or laser technician. In which boxes can you write N? In rows for bionic engineer and laser technician, under lan, Hak, and Adriana
- What else do you know about Adriana? She rides the bus with the used shuttle person, weather person, and bionic engineer. If you take all the information you have about her and put N in the boxes, can you figure out what job she must have? Yes, she must be in robot repairs. In which box will you put a Y? Under robot repairs If you match her with a job, you can write N for all the other boxes in the row for that job.
- (Have students continue going through the clues and filling in the boxes.) Which person has which job?

Solution: Jana—laser technician, Elsa—bionic engineer, Garth—robot relations, lan-weather analyst, Hak-shuttle sales, Adriana-robot repairs

	Jana	Elsa	Garth	lan	Hak	Adriana
Robot relations	N	N	Y	N	N	N
Bionic engineer	N	Y	N	N	N	N
Laser technician	Y	N	N	N	N	N
Shuttle sales	N	N	N	N	Y	N
Weather analyst	N	N	N	Υ	N	N
Robot repairs	N	N	N	N	N	Y

- LOOK BACK Read the problem again. Look at the data, conditions, and the main question. Review the clues and your table. Is your answer reasonable?
- **EXTEND IT** Make up a similar type of problem with 7 different types of jobs that might exist in the twenty-first century.

36

Poor Alex! He recently began his job at the zoo and today he can't find his morning feeding instructions for the ocelot, owl, otter, and elephant. He is trying to remember what Percy, Louise, Roger, and Rita eat. The possibilities are mice, raw meat, hay, or fish. This is all that Alex can remember: Louise is to be fed before the otter and the animals having mice or hay. Percy is next to the ocelot's cage and across from the animal having fish. Roger, who isn't the owl, is across from the otter and the animal that is eating raw meat. Rita and the owl are next to the animals that are eating hay and red meat. The ocelot doesn't eat hay. What was the name of each kind of animal and what was Alex supposed to feed each one?

- FIND OUT What is the question you have to answer?
  - · Which animals are assigned to Alex?
  - · What are the names of the animals?
  - · What are the choices for food?
  - What do you know about Louise? Percy? Roger? Rita? The ocelot?

# STRATEGY

- CHOOSE A The information in this problem is given in a set of clues. If you use a series of "If...then" statements to solve this problem, what kind of thinking will you be using?
  - Is there another strategy you can use to organize the information?

- **SOLVE IT** When you make a table, how many columns will you need? How many rows?
  - What will you label the columns? The rows?
  - If Louise is to be fed before the otter, then which animal is not named Louise? If Louise is fed before the animals having mice and hay, then what foods can you eliminate for Louise? In which boxes can you put Ns?
  - Use the remaining clues to help you put N or Y in each box of the table.

	Percy	Louise	Roger	Rita	Hay	Mice	Meat	Fish	
Owl									
Ocelot									
Otter									
Elephant									

Poor Alex! He recently began his job at the zoo and today he can't find his morning feeding instructions for the ocelot, owl, otter, and elephant. He is trying to remember what Percy, Louise, Roger, and Rita eat. The possibilities are mice, raw meat, hay, or fish. This is all that Alex can remember: Louise and the otter are to be fed before the animals having mice or hay. Percy is next to the ocelot's cage and across from the animal having fish. Roger, who isn't the owl, is across from the otter and the animal that is eating raw meat. Rita and the owl are next to the animals that are eating hay and red meat. The ocelot doesn't eat hay. What was the name of each kind of animal and what was Alex supposed to feed each one?

- FIND OUT What is the question you have to answer? What was the name of each kind of animal and what was Alex supposed to feed each one?
  - Which animals are assigned to Alex? Ocelot, owl, otter, and elephant
  - · What are the names of the animals? Louise, Percy, Rita, Roger
  - · What are the choices for food? Mice, raw meat, hay, fish
  - · What do you know about Louise? She is to be fed before the otter and the animals having mice or hay. Percy? He is next to the ocelot's cage and across from the animal having fish. Roger? He is across from the otter and the animal eating raw meat. Rita? She and the owl are next to the animals eating hay and red meat. The ocelot? The ocelot doesn't eat hay.

### CHOOSE A STRATEGY

- · The information in this problem is given in a set of clues. If you use a series of "If. . . then" statements to solve this problem, what kind of thinking will you be using? Logical reasoning
- Is there another strategy you can use to organize the information? Yes, we can make a

## SOLVE IT

- When you make a table, how many columns will you need? 8 How many rows? 4
- What will you label the columns? With the names and the different kinds of food The rows? The different kinds of animals
- If Louise is to be fed before the otter, then which animal is not named Louise? The otter If Louise is fed before the animals having mice and hay, then what foods can you eliminate for Louise? Mice and hay In which boxes can you put Ns? In column for Louise and row for otter, also in columns for mice and hay and in row for otter because the information says that Louise and the otter are to be fed before the animals having mice or hay.
- (Have students continue to use the clues to help them fill in the table.) What is the name of each animal and what does it eat?

Solution: Percy-owl, mice; Louise-ocelot, red meat; Rita-otter, fish; Roger-elephant, hay

	Percy	Louise	Roger	Rita	Hay	Mice	Meat	Fish
Owl	Υ	N	N	N	T N	Υ	N	N
Ocelot	N	Y	N	N	N	N	Y	N
Otter	N	N	N	Υ	N	N	N	Y
Elephant	N	N	Υ	N	Y	N	N	N

- LOOK BACK Read the problem again. Look at the data, conditions, and the main question. Review the clues and your table. Is your answer reasonable?
- EXTEND IT
- Make up a similar type of problem with 6 different kinds of animals and 6 different kinds of food.
- PRACTICE Similar Practice Problems: 73, 87, 100



King Karl was in his counting house, counting out his money. "Four thousand five hundred sixty-four dollars, four thousand five hundred sixty-four dollars and fifty cents," counted the king as he dropped the last coin into the bag. The king closed the door tightly behind him, and made his way to the parlor where Queen Kathleen was eating bread and honey. "My dear," he announced, "There's a total of \$4,564.50 in our travel fund. I counted 8 times as many half-dollars as quarters, and 6 times as many dimes as nickels." The queen asked, "Then how many coins were there of each kind?" What was the king's reply?

- FIND OUT What is the question you have to answer?
  - How much money was in the fund?
  - What do you know about the number of half-dollars and quarters?
  - What do you know about the number of dimes and nickels?

### CHOOSE A STRATEGY

- Might it be helpful to begin with a smaller amount of money than \$4,564.50?
- How can you organize the information in the problem?

- **SOLVE IT** Try \$40 in place of \$4,564.50.
  - If the king counted 8 times as many half-dollars as quarters, what is the least amount of money he could have counted in half-dollars and quarters?
  - If the king counted 6 times as many dimes as nickels, what is the least amount of money he could have counted in dimes and nickels?
  - If you combine the sum of money for quarters and half-dollars with the sum of money for dimes and nickels, what is the total? That is the amount of money for set 1. How many sets are there in \$40? How many of each kind of coin would there be in that number of sets?
  - Now try \$4,564.50. How can you begin? How many sets do you get? What is the total of those sets? Can the remaining money be divided evenly by \$4.25 or \$.65?
  - How many coins of each kind were there in the king's and queen's travel fund?

Sets	Quarters	Half- dollars	Total	Nickels	Dimes	Total	Combined Total
1	1	8	\$4.25				
2							



### MAKE IT SIMPLER

### Teaching Plan

King Karl was in his counting house, counting out his money. "Four thousand five hundred sixty-four dollars, four thousand five hundred sixty-four dollars and fifty cents," counted the king as he dropped the last coin into the bag. The king closed the door tightly behind him, and made his way to the parlor where Queen Kathleen was eating bread and honey. "My dear," he announced, "There's a total of \$4,564.50 in our travel fund. I counted 8 times as many half-dollars as quarters, and 6 times as many dimes as nickels." The queen asked, "Then how many coins were there of each kind?" What was the king's reply?

FIND OUT • What is the question you have to answer? How many coins were there of each kind in the roval travel fund?

How much money was in the fund? \$4,564.50

- What do you know about the number of half-dollars and quarters? There were 8 times as many half-dollars as quarters.
- What do you know about the number of dimes and nickels? There were 6 times as many dimes as nickels.

### CHOOSE A STRATEGY

- Might it be helpful to begin with a smaller amount of money than \$4,564.50? Yes, it would help to make the problem simpler.
- How can you organize the information in the problem? We can make an organized list.

- **SOLVE IT** Try \$40 in place of \$4,564,50.
  - If the king counted 8 times as many half-dollars as quarters, what is the least amount of money he could have counted in half-dollars and quarters? One half-dollar and 8 quarters,
  - · If the king counted 6 times as many dimes as nickels, what is the least amount of money he could have counted in dimes and nickels? One nickel and 6 dimes, or \$.65
  - If you combine the sum of money for quarters and half-dollars with the sum of money for dimes and nickels, what is the total? \$4.90 That is the amount of money for set 1. How many sets are there in \$40? 8 How many of each kind of coin would there be in that number of sets? 8 quarters, 64 half-dollars, 8 nickels, and 48 dimes
  - Now try \$4,564.50. How can you begin? Divide it by \$4.90 to find the number of sets. How many sets do you get? 931 What is the total of those sets? \$4,561.90 Can the remaining money be divided evenly by \$4.25 or \$.65? Yes, by \$.65.
  - How many coins of each kind were there in the king's and queen's travel fund?

Solution: 931 quarters, 7,448 half-dollars, 935 nickels, 5,610 dimes

Sets	Quarters	Half- dollars	Total	Nickels	Dimes	Total	Combined Total
1	1	8	\$4.25	1	6	\$ .65	\$4.90
2	2	16	\$8.50	2	12	\$1.30	\$9.80

- LOOK BACK Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?
- **EXTEND IT** If the king had said that he found 5 times as many half-dollars as quarters, and 7 times as many dimes as nickels, how many coins of each kind would he have found?

On a large planet in a distant galaxy lives Mr. Alphotec with his collection of 85,800 exotic animals. He has one-half as many auples as burfuls, one-third as many burfuls as curlaps, and one-fourth as many curlaps as drolops. Auples have soft patches of fur dotting their tubular bodies. Blue burfuls have gold spots all over their pyramidshaped bodies. Curlaps have metallic green plates covering their egg-shaped bodies, and drolops have thick, pointed hairs all over their short, stubby bodies. If an auple is worth \$43.50, a drolop sells for \$12.50, a curlap is valued at \$22.50, and a burful is worth \$30.00, what is the value of Mr. Alphotec's collection?

- FIND OUT What is the question you have to answer?
  - What do you know about the number of auples in the collection?
  - What do you know about the number of burfuls in the collection?
  - What do you know about the number of curlaps in the collection?
  - · How many animals are there in the collection?
  - What is an auple worth? What is a burful worth? A curlap? A drolop?

### CHOOSE A STRATEGY

Might it be helpful to begin with a smaller number than 85,800?

- **SOLVE IT** Try 858 in place of 85,800.
  - Which kind of animal does Mr. Alphotec have the most of?
  - If you make a guess about how many drolops there are in the collection, can you figure out how many of the other animals there are? What is your guess for drolops? Then how many curlaps are there? How many burfuls are there? How many auples are there? How can you check your guess? Was your guess too high or too low?
  - Now try 85,800. How many auples, burfuls, curlaps, and drolops were in Mr. Alphotec's collection?
  - What is the value of the collection?

38

On a large planet in a distant galaxy lives Mr. Alphotec with his collection of 85,800 exotic animals. He has one-half as many auples as burfuls, one-third as many burfuls as curlaps, and one-fourth as many curlaps as drolops. Auples have soft patches of fur dotting their tubular bodies. Blue burfuls have gold spots all over their pyramid-shaped bodies. Curlaps have metallic green plates covering their egg-shaped bodies, and drolops have thick, pointed hairs all over their short, stubby bodies. If an auple is worth \$43.50, a drolop sells for \$12.50, a curlap is valued at \$22.50, and a burful is worth \$30.00, what is the value of Mr. Alphotec's collection?

### FIND OUT

- What is the question you have to answer? What is the value of Mr. Alphotec's collection?
- What do you know about the number of auples in the collection? There are half as many auples as burfuls.
- What do you know about the number of burfuls in the collection? There are one-third as many burfuls as curlaps.
- What do you know about the number of curlaps in the collection? There are one-fourth as many curlaps as drolops.
- How many animals are there in the collection? 85,800
- What is an auple worth? \$43.50 What is a burful worth? \$30.00 A curlap? \$22.50 A drolop? \$12.50

# CHOOSE A STRATEGY

 Might it be helpful to begin with a smaller number than 85,800? Yes, it would help to make the problem simpler.

### SOLVE IT

- Try 858 in place of 85,800.
- · Which kind of animal does Mr. Alphotec have the most of? Drolop
- If you make a guess about how many drolops there are in the collection, can you figure out how many of the other animals there are? Yes. What is your guess for drolops? (This is an example of a guess.) 300 Then how many curlaps are there?  $\frac{1}{4} \times 300$ , or 75 How many burfuls are there?  $\frac{1}{3} \times 75$ , or 25 How many auples are there?  $\frac{1}{2} \times 25$ , or 12.5 How can you check your guess? Find the sum of the numbers for the four kinds of animals: 300 + 75 + 25 + 12.5 = 412.5 Was your guess too high or too low? Too low.
- Now try 85,800. How many auples, burfuls, curlaps, and drolops were in Mr. Alphotec's collection? 2,600 auples, 5,200 burfuls, 15,600 curlaps, and 62,400 drolops
- What is the value of the collection?

Solution: \$1,400,100 (\$113,000 + \$156,000 + \$351,000 + \$780,000)

### LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

### EXTEND IT

- Write your own problem about new kinds of exotic animals. Use a different number for the total animals in the collection, and use a different amount of money for each kind of animal.
- PRACTICE
- Similar Practice Problems: 83, 105, 117

Albert, Bernard, Gina, Denise, and Elroy had all made it to the finals of the Chili Championship Cook-off. Their recipes were all top secret concoctions, some passed down from generation to generation. Three of their recipes called for ground beef, and two called for stew meat. Three of them used hot peppers and two used mild peppers. Bernard and Elroy used the same kind of peppers. Denise and Gina used the same kind of meat. Albert and Bernard didn't use the same kind of meat in their chili. Gina and Albert used different kinds of peppers. Who won the cook-off with a chili recipe that used stew meat and mild peppers?

- FIND OUT What is the question you have to answer?
  - Who were the contestants in the finals of the Chili Cook-off?
  - How many recipes used ground beef? How many used stew meat?
  - How many recipes used hot peppers? How many used mild peppers?
  - What do you know about the kind of peppers used by Bernard and Elroy? By Gina and Albert?
  - What do you know about the kind of meat used by Denise and Gina? By Albert and Bernard?

### CHOOSE A STRATEGY

- To solve this problem you need to look at the different pieces of information and use a series of "If this is true. . . then this must be true" type of statements. What do you call this kind of thinking?
- What other strategy would be helpful?

- SOLVE IT If you make an organized list, how many headings do you need? What are they?
  - How many names will go under Ground Beef? How many under Stew Meat? How many under Hot Peppers? How many under Mild Peppers?
  - If Bernard and Elroy used the same kind of peppers, and Gina and Albert used different kinds of peppers, which names would you put under each kind of pepper? Where would you put Denise's name?
  - If Denise and Gina used the same kind of meat, and Albert and Bernard used different kinds of meat, where would you put each of their names? In which list would you put Elroy's name?
  - What can you deduce now from looking at each list?

Ground beef	Stew meat	Hot peppers	Mild peppers		
Albert/Bernard	Albert/Bernard				



## LOGICAL REASONING

stew meat and mild peppers?

## Teaching Plan

Albert, Bernard, Gina, Denise, and Elroy had all made it to the finals of the Chili Championship Cook-off. Their recipes were all top secret concoctions, some passed down from generation to generation. Three of their recipes called for ground beef, and two called for stew meat. Three of them used hot peppers and two used mild peppers. Bernard and Elroy used the same kind of peppers. Denise and Gina used the same kind of meat. Albert and Bernard didn't use the same kind of meat in their chili. Gina and Albert used different kinds of peppers. Who won the cook-off with a chili recipe that used

### FIND OUT

- What is the question you have to answer? Who won the cook-off with a chili recipe that used stew meat and mild peppers?
- Who were the contestants in the finals of the Chili Cook-off? Albert, Bernard, Gina, Denise,
- How many recipes used ground beef? 3 How many used stew meat? 2
- How many recipes used hot peppers? 3 How many used mild peppers? 2
- What do you know about the kind of peppers used by Bernard and Elroy? They used the same kind. By Gina and Albert? They used different kinds.
- What do you know about the kind of meat used by Denise and Gina? They used the same kind. By Albert and Bernard? They used different kinds.

### CHOOSE A STRATEGY

- · To solve this problem you need to look at the different pieces of information and use a series of "If this is true. . .then this must be true" type of statements. What do you call this kind of thinking? Logical reasoning
- What other strategy would be helpful? We can make an organized list.

- SOLVE IT If you make an organized list, how many headings do you need? 4 What are they? Ground beef, stew meat, hot peppers, mild peppers
  - How many names will go under the Ground Beef? 3 How many under Stew Meat? 2 How many under Hot Peppers? 3 How many under Mild Peppers? 2
  - If Bernard and Elroy used the same kind of peppers, and Gina and Albert used different kinds of peppers, which names would you put under each kind of pepper? We would put Gina and Albert under both the hot peppers and the mild peppers. We know that if either Gina or Albert belongs under hot peppers, then Bernard and Elroy must go under hot peppers because there should be three names in this list. Where would you put Denise's name? Her name has to go under mild peppers, if there are three under the hot peppers.
  - If Denise and Gina used the same kind of meat, and Albert and Bernard used different kinds of meat, where would you put each of their names? We would put Albert and Bernard in both lists for meat. Then that would mean that Denise and Gina must go in the list for ground beef, to make three in that list. In which list would you put Elroy's name? His name must go in the list for stew meat, as the other list has three in it.
  - What can you deduce now from looking at each list? If we are looking for someone that used both stew meat and mild peppers, then the only name that appears on both lists is Albert.

Solution: Albert

Ground beef	Stew meat	Hot peppers	Mild peppers		
Albert/Bernard	(Albert) Bernard	Gina/Albert	Gina Albert		
Denise	Elroy	Bernard	Denise		
Gina		Elroy			

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • The year before there were 4 people who used stew meat, 3 that used ground beef, 4 used hot peppers, and 3 used mild peppers. William, Denise, and Bernard used the same kind of meat. Bernard and Angelo used different kinds of peppers. Albert, Gina, and Elroy used the same kind of peppers. Angelo and Elroy used different kinds of meat. Who won, using ground beef and mild peppers?



### LOGICAL REASONING

Name

40

Seven friends are standing by Loch Ness, gazing into the lake. Angie, Bess, Carl, Darcy, Edwin, Frank, and Gary are discussing Nessie, the legendary monster. Three of them think the creature should be captured and put into an aquarium, but the rest of them believe Nessie should remain in the lake. Three of them think the creature is a direct descendant of the dinosaurs, but four of them believe it is just a huge fish. Angie, Bess, and Carl don't want to capture Nessie. Gary and Frank have different opinions on that issue. Gary, Darcy, and Frank think Nessie is a huge fish. Bess and Angle have an argument about what Nessie is. Which one of the friends wants to put Nessie in an aquarium so people can see what ancient dinosaurs were like?

- **FIND OUT** What is the question you have to answer?
  - Who are the friends standing by Loch Ness?
  - How many think the creature should be captured? How many think it should stay in the lake?
  - How many think Nessie is a direct descendant of the dinosaurs? How many think it is a big fish?
  - What do you know about Angie, Bess, and Carl and their opinions about whether or not to capture Nessie? What about Gary and Frank?
  - What do you know about what Gary, Darcy, and Frank think Nessie is? What about Bess and Angie?

# STRATEGY

- **CHOOSE A** To solve this problem you need to look at the different pieces of information and use a series of "If this is true...then this must be true" type of statements. What do you call this kind of thinking?
  - What other strategy would be helpful?

- **SOLVE IT** If you make an organized list, how many headings do you need? What are they?
  - How many names will go under Aquarium? How many under Lake? How many under Dinosaur? How many under Fish?
  - If Angie, Bess, and Carl don't want to capture Nessie, and Gary and Frank have different opinions about this, where would you put each of their names? Where would you put Edwin's and Darcy's names?
  - Continue to use the clues to list the names.

Lake	Lake Aquarium		Dinosaur		
Gary/Frank	Gary/Frank				



### LOGICAL REASONING

### Teaching Plan

Seven friends are standing by Loch Ness, gazing into the lake. Angle, Bess, Carl, Darcy, Edwin, Frank, and Gary are discussing Nessie, the legendary monster. Three of them think the creature should be captured and put into an aquarium, but the rest of them believe Nessie should remain in the lake. Three of them think the creature is a direct descendant of the dinosaurs, but four of them believe it is just a huge fish. Angie, Bess. and Carl don't want to capture Nessie. Gary and Frank have different opinions on that issue. Gary, Darcy, and Frank think Nessie is a huge fish. Bess and Angie have an argument about what Nessie is. Which one of the friends wants to put Nessie in an aguarium so people can see what ancient dinosaurs were like?

- FIND OUT What is the question you have to answer? Which of the friends wants to put Nessie in an aquarium so people can see what ancient dinosaurs were like?
  - · Who are the friends standing by Loch Ness? Angle, Bess, Carl, Darcy, Edwin, Frank, and
  - · How many think the creature should be captured? 3 How many think it should stay in the lake? 4
  - · How many think Nessie is a direct descendant of the dinosaurs? 3 How many think it is a big fish? 4
  - What do you know about Angie, Bess, and Carl and their opinions about whether or not to capture Nessie? They think Nessie should stay in the lake. What about Gary and Frank? They disagree.
  - . What do you know about what Gary, Darcy, and Frank think Nessie is? They think it is a huge fish. What about Bess and Angie? They have an argument about this.

### CHOOSE A STRATEGY

- To solve this problem you need to look at the different pieces of information and use a series of "If this is true...then this must be true" type of statements. What do you call this kind of thinking? Logical thinking
- What other strategy would be helpful? We can make an organized list.

### SOLVE IT

- If you make an organized list, how many headings do you need? 4 What are they? Lake. aquarium, fish, dinosaur
- How many names will go under Aquarium? 3 How many under Lake? 4 How many under Dinosaur? 3 How many under Fish? 4
- If Angie, Bess, and Carl don't want to capture Nessie, and Gary and Frank have different opinions about this, where would you put each of their names? Gary and Frank would go under both aquarium and lake, and the slash between their names would indicate that this means either/or. Then Angie, Bess, and Carl would have to go under Lake because there should be 4 in this list. Where would you put Edwin's and Darcy's names? Under Aquarium
- (Have students continue to use the clues and list names.) Who wanted to put Nessie in an aquarium so people can see what ancient dinosaurs were like?

Solution: Edwin

Lake	Aquarium	Fish	Dinosaur	
Gary/Frank	Gary/Frank	Bess/Angie	Bess/Angie	
Bess	(Edwin)	Darcy	(Edwin)	
Carl	Darcy	Gary	Carl	
Angie		Frank		

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • Make up a problem similar to this one.

**PRACTICE** • Similar Practice Problems: 86, 99, 114



Asa was reeling after his first day at the Cactus Farm. He would never learn all the plants and their prices! First he sold some tan terps, then twice as many raspberry ribbits as lavender listers, three times as many heavy-footed fungi as pipples, 1.5% more morps than listers, and twice the number of listers as pipples. He sold 26 pipples, which was 6.5% of his total sales for the day. How many cactus plants did Asa sell?

- FIND OUT What is the question you have to answer?
  - What do you know about the number of terps Asa sold? Raspberry ribbits? Lavender listers? Heavy-footed fungi? Pipples? Morps?
  - The number of pipples was what percent of his total sales?

### CHOOSE A STRATEGY

• Where is the only specific number of plants sold given in the problem? If you begin with this information, how can you work your way through the problem?

- SOLVE IT If you begin with the number of pipples, then can you figure out what 1% of the total number of cactus sold would be?
  - If you work backwards, the next type of cactus mentioned is listers. The number of listers is what percent of the total? How many listers did he sell?
  - Do you need to keep track of the percent of each type of cactus?
  - · Working backwards, the next type of cactus is morps. The number of morps sold is what percent of the total? How many morps were sold?
  - Keep working backwards, until you figure out how many plants of each type of cactus Asa sold.

Pipples — 
$$6.5\% = 26 + 26 \div 6.5 = 4$$
, so  $1\% = 4$   
Listers —  $2 \times 6.5 = 13 \times 4 = 52$ 

Asa was reeling after his first day at the Cactus Farm. He would never learn all the plants and their prices! First he sold some tan terps, then twice as many raspberry ribbits as lavender listers, three times as many heavy-footed fungi as pipples, 1.5% more morps than listers, and twice the number of listers as pipples. He sold 26 pipples, which was 6.5% of his total sales for the day. How many cactus plants did Asa sell?

### FIND OUT

- What is the question you have to answer? How many cactus plants did Asa sell?
- What do you know about the number of terps Asa sold? Only "some" Raspberry ribbits? Twice as many as lavender listers Heavy-footed fungi? 3 times as many as pipples Morps? 1.5% more than listers Listers? Twice the number of pipples Pipples? 26
- The number of pipples was what percent of his total sales? 6.5%

### CHOOSE A STRATEGY

• Where is the only specific number of plants sold given in the problem? At the end of the problem If you begin with this information, how can you work your way through the problem? We need to work backwards.

### SOLVE IT

- If you begin with the number of pipples, then can you figure out what 1% of the total number of cactus sold would be? Yes,  $26 \div 6.5 = 4$
- · If you work backwards, the next type of cactus mentioned is listers. The number of listers is what percent of the total? 13% How many listers did he sell? 52
- Do you need to keep track of the percent of each type of cactus? Yes, because we'll need to find out what percent of the total the terps are, in order find how many were sold,
- Working backwards, the next type of cactus is morps. The number of morps sold is what percent of the total? 14.5% How many morps were sold? 58
- (Have the students continue to work backwards until they find a number for each kind of cactus.) How many cactus plants did Asa sell?

Solution: 400 plants

Pipples — 6.5% = 26 $26 \div 6.5 = 4$ Listers —  $2 \times 6.5 = 13\% \times 4 = 52$ Morps — 1.5% more than listers –  $14.5\% \times 4 = 58$ Fungi —  $3 \times \text{pipples} = 19.5\% \times 4 = 78$ Ribbits  $-2 \times \text{listers} = 26\% \times 4 = 104$ Total = 79.5%, 100 - 79.5 = 20.5Terps = 20.5% = 82 26 + 52 + 58 + 78 + 104 + 82 = 400

### LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • What if Asa had sold some terps, then 2% fewer ribbits than listers, 100 more fungi than morps, 1.6% more morps than pipples, and twice the number of listers as pipples. If he sold 37 pipples that equaled 7.4% of his total sales, how many plants did he sell?

Fran was demonstrating her specialty on KRTB, Fruit-Bat Soup for a crowd. First she threw in her prime ingredient, giant fruit bats. Then she added 6 times as many pounds of frogs legs as chicken wings. She added  $\frac{2}{9}$  as many pounds of chicken wings as potatoes,  $1\frac{1}{2}$  times as many pounds of potatoes as carrots, and 6 times as many pounds of carrots as mudflat mushrooms. The mudflat mushrooms were  $\frac{3}{128}$  of the total ingredients, which amounted to 6 pounds. How many pounds of each ingredient did she add?

- **FIND OUT** What is the question you have to answer?
  - What do you know about the number of fruit bats that Fran put in? Frogs legs? Chicken wings? Potatoes? Carrots? Mudflat mushrooms?
  - The mushrooms are what fractional part of the total ingredients?

### CHOOSE A STRATEGY

 Where is the only specific number of pounds given in the problem? If you begin with this information, how can you work your way through the problem?

- **SOLVE IT** If you begin with the pounds of mushrooms, then can you figure out what  $\frac{1}{128}$  of the pounds would be?
  - If you work backwards, the next ingredient mentioned is carrots. The carrots are what fractional part of the total ingredients? How many pounds of carrots did she add?
  - Do you need to keep track of what fractional part of the total ingredients each ingredient is?
  - Working backwards, the next ingredient is potatoes. The potatoes are what fractional part of the total ingredients? How many pounds of potatoes did she
  - · Keep working backwards, until you figure out how many pounds of each ingredient she added.

$$\frac{3}{128} = 6$$
 6 ÷ 3 = 2, so  $\frac{1}{128} = 2$  6 × mushrooms (6) = 36 ÷ 2 = 18, mushrooms =  $\frac{18}{128}$ 

Fran was demonstrating her specialty on KRTB, Fruit-Bat Soup for a crowd. First she threw in her prime ingredient, giant fruit bats. Then she added 6 times as many pounds of frogs legs as chicken wings. She added 3/9 as many pounds of chicken wings as potatoes, 11/2 as many pounds of potatoes as carrots, and 6 times as many pounds of carrots as mudflat mushrooms. The mudflat mushrooms were 3/128 of the total ingredients, which amounted to 6 pounds. How many pounds of each ingredient did she add?

- FIND OUT What is the question you have to answer? How many pounds of each ingredient did she add?
  - What do you know about the number of fruit bats that Fran put in? Just that she threw some in Frogs legs? 6 times as many pounds as chicken wings Chicken wings? 36 the amount of potatoes Potatoes? 11/2 times the amount of carrots Carrots? 6 times the amount of mushrooms Mudflat mushrooms? 6 pounds
  - The mushrooms are what fractional part of the total ingredients? 3/128

### CHOOSE A STRATEGY

• Where is the only specific number of pounds given in the problem? At the end If you begin with this information, how can you work your way through the problem? We need to work backwards.

- SOLVE IT If you begin with the pounds of mushrooms, then can you figure out what 1/128 of the pounds would be? Yes, that is 2 pounds.
  - If you work backwards, the next ingredient mentioned is carrots. The carrots are what fractional part of the total ingredients?  $6 \times 6 = 36 \div 2 = \frac{18}{128}$  How many pounds of carrots did she add? 36 pounds
  - · Do you need to keep track of what fractional part of the total ingredients each ingredient is? Yes, because then we can subtract the total of the other ingredients from 128/128 to find out what fractional part of the total the fruit bats are.
  - Working backwards, the next ingredient is potatoes. The potatoes are what fractional part of the total ingredients? 1.5  $\times$  36 = 54  $\div$  2 =  $\frac{27}{128}$  How many pounds of potatoes did she add? 54 pounds
  - (Have students continue to work backwards until they can find the pounds of fruit bats she added.) How much of each ingredient did she add?

Solution: 6 lb mushrooms, 36 lb carrots, 54 lb potatoes, 12 lb chicken wings, 72 lb frogs legs, 76 lb fruit bats

mushrooms:  $\frac{3}{128} = 6$  6 ÷ 3 = 2, so  $\frac{1}{128} = 2$ carrots:  $6 \times \text{mushrooms}$  (6) =  $36 \div 2 = \frac{18}{128}$ potatoes: 1.5  $\times$  carrots = 54  $\div$  2 =  $\frac{27}{128}$ chicken wings:  $\frac{2}{9}$  of potatoes =  $12 \div 2 = \frac{6}{128}$ frogs legs:  $6 \times$  chicken wings =  $72 \div 2 = \frac{36}{128}$ fruit bats: 38/128 = 76

- **EXTEND IT** When Fran made soup for the Thanks Alive concert, she began with fruit bats, added ½ as many frogs legs as chicken wings, 1/2 as many chicken wings as potatoes, 1.5 times as many potatoes as carrots, 7 times as many carrots as mushrooms, and 40 pounds of mushrooms, which was \$\\^256\$ of the total ingredients. How many pounds of each item did she add?
- PRACTICE Similar Practice Problems: 84, 108, 119



### MAKE IT SIMPLER

Name

Pegasus is flying in today for a checkup. He needs to have his coat brushed and his wings cleaned. He's logged in 4,740,000 minutes of flying time since his last checkup at 4 P.M. on April 12, 2026. What is the present checkup time and date?

- FIND OUT What is the question you have to answer?
  - · How many minutes has Pegasus been flying since his last checkup?
  - When was his last checkup?

### CHOOSE A STRATEGY

 Would it be easier to try to solve the problem using fewer minutes to begin with?

- SOLVE IT If you substitute 10,000 minutes for 4,740,000 minutes, how could you divide up the minutes?
  - Now how can you divide up the hours?
  - You could add on from April 12 at 4 P.M. to see when this would be. Now go back to the 4,740,000 minutes. How many hours is this?
  - How many days would this be?
  - How many years would this be? Do we have the same number of days every
  - Count on from April 12, 2026 to find out the present checkup time and date.

Pegasus is flying in today for a checkup. He needs to have his coat brushed and his wings cleaned. He's logged in 4,740,000 minutes of flying time since his last checkup at 4 P.M. on April 12, 2026. What is the present checkup time and date?

- What is the question you have to answer? What is the date and time of Pegasus' present checkup?
  - How many minutes has Pegasus been flying since his last checkup? 4.740.000
  - When was his last checkup? April 12, 2026 at 4 P.M.

### CHOOSE A STRATEGY

• Would it be easier to try to solve the problem using fewer minutes to begin with? Yes, we can make it simpler to get started.

### SOLVE IT

- If you substitute 10,000 minutes for 4,740,000 minutes, how could you divide up the minutes? Divide 10.000 by 60, which is 166 hours and 40 minutes
- Now how can you divide up the hours? Divide by 24 to get days, which is 6 days, 22 hours, and 40 minutes
- You could add on from April 12 at 4 P.M. to see when this would be. Now go back to the 4,740,000 minutes. How many hours is this? 79,000
- How many days would this be? 3291 days and 16 hours
- How many years would this be? 9 Do we have the same number of days every year? No. every 4 years we have leap year which means 366 days instead of 365 days.
- (Have the students continue to figure out when leap years would be and how many days 9 years would be, and then count on from the date given.) When is Pegasus' present checkup?

Solution: April 17, 2035, at 8 A.M.

3291 days and 16 hours leap year 2028 and 2032, so  $(7 \times 365) + (2 \times 366) = 3287$  days, leaving 4 days and 16 hours

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • If Pegasus had been flying for 9,845,630 seconds since his last checkup, when would he arrive for his present appointment?



### MAKE IT SIMPLER

Name

Jacques, pirate chief of Plunder Island, was slightly diabolical. He left this note for Potter, his partner in crime: "Friday, October 2, 10:00 A.M. I've gone to survey the other side of the island. I will travel 601,920 inches due west. Please meet me there by 5:30 P.M. sharp so we can bury some treasure." Potter found the note 3 hours later. If he left immediately and walked 2 miles an hour, would he make it or was he in big trouble?

- **FIND OUT** What is the question you have to answer?
  - How far did Jacques go?
  - When did Potter find the note? When was he supposed to meet Jacques?
  - How fast did Potter walk?

# STRATEGY

**CHOOSE A** • Would it be easier to try to solve the problem using fewer inches to begin with?

- SOLVE IT If you substitute 1,000 inches for 601,920 inches, how could you divide up the
  - Now how can you divide up the feet? How many miles would this be?
  - How long would this take Potter?
  - Now go back to the 601,920 inches. How many feet is this?
  - · Continue to figure out the number of miles and then how many hours it would take him.

 $601,920 \div 12 =$ 



### MAKE IT SIMPLER

### Teaching Plan

Jacques, pirate chief of Plunder Island, was slightly diabolical. He left this note for Potter, his partner in crime: "Friday, October 2, 10:00 A.M. I've gone to survey the other side of the island. I will travel 601,920 inches due west. Please meet me there by 5:30 P.M. sharp so we can bury some treasure." Potter found the note 3 hours later. If he left immediately and walked 2 miles an hour, would he make it or was he in big trouble?

### FIND OUT

- What is the question you have to answer? If Potter walked 2 miles an hour, would he make it or was he in big trouble?
- How far did Jacques go? 601,920 inches due west
- When did Potter find the note? 3 hours later When was he supposed to meet Jacques? At 5:30 P.M. sharp
- How fast did Potter walk? 2 miles an hour

### CHOOSE A STRATEGY

 Would it be easier to try to solve the problem using fewer inches to begin with? Yes, we can make the problem simpler.

### SOLVE IT

- If you substitute 10,000 inches for 601,920 inches, how could you divide up the inches? Divide by 12 into feet, which is 8331/3 feet
- Now how can you divide up the feet? Divide by 5,280 feet into miles How many miles would this be? Not even a mile
- How long would this take Potter? Less than ½ hour
- Now go back to the 601,920 inches. How many feet is this? 50,160
- (Have students figure out how many miles this would be, and then how long this would take him.) Would Potter make it, or would he be in big trouble?

Solution: He wouldn't make it!

 $601.920 \div 12 = 50160 \div 5280 = 9.5$ If he walks 2 miles/hour, then he would arrive 7 hours, 45 minutes later, or 5:45, 15 minutes late

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**EXTEND IT** • If the note said that the chief had traveled 1,393,920 inches and to meet him before midnight, would Potter make it if he again found the note 3 hours later and traveled 2 miles an hour?

**PRACTICE** • Similar Practice Problems: 90, 98, 112

A giant brown and white dog named Bernie runs the Mountain Rescue Agency. The agency claims to be ready instantly to handle any rescue mission. Today Bernie is sending Howie on a difficult mission that will take him eight days. Howie can carry food for only five days, so Bernie is looking for volunteers to help Howie reach his destination. Each helper can also carry provisions for only five days. How many dogs need to begin the journey so that Howie will reach his destination?

- **FIND OUT** What is the question you have to answer?
  - How long will Howie's mission take him?
  - How much food can Howie carry?
  - How much food can each volunteer carry?

### CHOOSE A STRATEGY

- You can use a series of "if...then" statements to help you solve this problem. What kind of thinking do we call this?
- · Is there another strategy you could use for this problem?

- SOLVE IT Begin a diagram by writing the days across the page. Then put Howie's name in the second row under Days.
  - If each person can only carry provisions for 5 days, then you have to start thinking about how they could exchange food. Does each helper/volunteer have to go the whole distance?
  - · How much food can Howie take from a helper, if he can only carry enough for 5 days? How much can the helper give away, if the helper has to return to the starting point?
  - Is one helper enough? Can a second helper give food to Howie as well as to the first helper?
  - Continue to experiment with different ways that Howie and the helpers could exchange food until you find a way to get Howie to his destination and the helpers safely back to the starting point.

Days	1	2	3	4	5	6	7	8	
Howie (5)	-1								
Helper A (5)	-1								



### **USE LOGICAL REASONING**

### Teaching Plan

A giant brown and white dog named Bernie runs the Mountain Rescue Agency. The agency claims to be ready instantly to handle any rescue mission. Today Bernie is sending Howie on a difficult mission that will take him eight days. Howie can carry food for only five days, so Bernie is looking for volunteers to help Howie reach his destination. Each helper can also carry provisions for only five days. How many dogs need to begin the journey so that Howie will reach his destination?

- FIND OUT What is the question you have to answer? How many dogs need to begin the journey so that Howie will reach his destination?
  - · How long will Howie's mission take him? 8 days
  - How much food can Howie carry? Food for 5 days
  - How much food can each volunteer carry? Food for 5 days

### CHOOSE A STRATEGY

- · You can use a series of "if...then" statements to help you solve this problem. What kind of thinking do we call this? Logical thinking
- Is there another strategy you could use for this problem? We can make a diagram.

### SOLVE IT

- Begin a diagram by writing the days across the page. Then put Howie's name in the second row under Days.
- If each person can only carry provisions for 5 days, then you have to start thinking about how they could exchange food. Does each helper/volunteer have to go the whole distance?
- How much food can Howie take from a helper, if he can only carry enough for 5 days? He can only take enough that makes a total of 5, 1 after the first day, 2 after the second day, and so on. How much can the helper give away, if the helper has to return to the starting point? The helper could give away 3 after the first day, and only 1 after the second day because the helper would need provisions for 2 days' travel back after the end of the second
- Is one helper enough? No Can a second helper give food to Howie as well as to the first helper? Yes
- (Have the students continue to experiment with different ways that Howie and the helpers could exchange food. Have them diagram their ideas.) How many dogs need to begin the journey to get Howie to his destination?

Solution: 4 dogs

Days	1	2	3	4	5	6	7	8
Howie (5)	-1,+1	-1,+1-	-1,+1-	(5)-1	(4)-1	(3)-1	(2)-1	(1)-1
Helper A(5)	<del>-1,+</del> 1-	<u>-1,+1</u> -	2					
Helper B(5)	<u>-1,+1</u> -	_3_						
Helper C(5)	_4_							



# **USE LOGICAL REASONING**

Name

46

Randall, an outstanding rider for the pony express, braved all forms of adversity, believing that "The mail must go through!" One day Randall and his faithful pony got an important letter to deliver. The trip would take ten days over rocky terrain. Randall could only carry enough food for six days, so he requested help from the other riders. Each helper could only carry provisions for six days as well. How many riders were needed to begin the trip so that Randall would get his letter delivered?

- **FIND OUT** What is the question you have to answer?
  - · How long will Randall's trip take?
  - · How much food can Randall take with him?
  - How much food can each of the other riders take?

# STRATEGY

- **CHOOSE A** You can use a series of "if...then" statements to help you solve this problem. What kind of thinking do we call this?
  - Is there another strategy you could use for this problem?

- SOLVE IT Begin a diagram by writing the days across the page. Then put Randall's name in the second row under Days.
  - If another rider helps out, does he have to go the whole distance?
  - How much food can Randall take from a helper, if he can only carry enough for 6 days? How much can the helper give away, if the helper has to return to the starting point?
  - Is one helper enough? Can a second helper give food to Randall as well as to the first helper?
  - Continue to experiment with different ways that Randall and some helpers could exchange food until you find a way to get Randall to his destination and the helpers safely back to the starting point.

Days	1	2	3	4	5	6	7	8	9	10	
Randall (5)	-1										
Helper A (5)	-1										

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?



# USE LOGICAL REASONING

# Teaching Plan

Randall, an outstanding rider for the pony express, braved all forms of adversity, believing that "The mail must go through!" One day Randall and his faithful pony got an important letter to deliver. The trip would take ten days over rocky terrain. Randall could only carry enough food for six days, so he requested help from the other riders. Each helper could only carry provisions for six days as well. How many riders were needed to begin the trip so that Randall would get his letter delivered?

FIND OUT • What is the question you have to answer? How many riders need to begin the trip so that Randall will get his letter delivered?

How long will Randall's trip take? 10 days

- How much food can Randall take with him? Enough for 6 days
- How much food can each of the other riders take? Supplies for 6 days

# CHOOSE A STRATEGY

- You can use a series of "if...then" statements to help you solve this problem. What kind of thinking do we call this? Logical thinking
- Is there another strategy you could use for this problem? We can make a diagram.

# SOLVE IT

- · Begin a diagram by writing the days across the page. Then put Randall's name in the second row under Days.
- If another rider helps out, does he have to go the whole distance? No, but he has to return to where he started from.
- How much food can Randall take from one helper, if he can only carry enough for 6 days? Only supplies for 1 day at the end of the first day, supplies for 2 days at the end of the second day, and so on. How much can the helper give away, if the helper has to return to the starting point? 3 after the first day, and only 1 after the second day in order to leave enough to return with.
- Is one helper enough? No Can a second helper give food to Randall as well as to the first helper? Yes
- (Have students continue to experiment with different ways that Randall and some helpers could exchange food until they find a way to get Randall to his destination and the helpers safely back to the starting point.) How many riders need to start the trip?

Solution: 5 riders

Days	1	2	3	4	5	6	7	8	9	10
Randall (6)	-1,+1	-1,+1	-1,+1	-1,+1	6-1	5-1	4-1	3-1	2-1	1–1
Helper A (6)	<u>-1,+1</u> -	<u>−1,+</u> 1-	<u>-1,+</u> 1-	-2-						
Helper B (6)	<u>-1,+1</u> -	<u>-1,+1</u> -	_3_							
Helper C (6)	<u>-1,+1</u> -	_4_								
Helper D (6)	_5_									

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE • Similar Practice Problems: 85, 109, 118



Name

Lotario picked up the scissors and started to cut the fabric. Lucia held the cloth down on the table so that it wouldn't slip and make Lotario cut crooked. Lotario and Lucia were making banners for the Pumpkin Festival. The cloth was 45 yards long, and they were going to cut it into 5-yard pieces for the banners. It took Lotario two minutes to make each cut, because he had to cut very carefully. How many minutes did it take him to cut the fabric into 5-yard pieces?

- FIND OUT What is the question you have to answer?
  - · What were Lotario and Lucia doing?
  - How long was the piece of fabric they began with?
  - How long were the pieces they cut from the cloth? How many pieces could they cut from the cloth?
  - How long did it take Lotario to make each cut?

- CHOOSE A What strategy can you use to help you solve this problem?
- STRATEGY When the strategies you know about don't appear to be helpful for solving a problem, and you don't know where to begin, relax and open up your mind to any and all possibilities. Explore anything you think of, reasonable or unreasonable. We call this "brainstorming."

- **SOLVE IT** What is your first reaction to this problem?
  - Do you think this is a straightforward math problem?
  - How long do you think it took Lotario to cut the long fabric into pieces?

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?



# Teaching Plan

Lotario picked up the scissors and started to cut the fabric. Lucia held the cloth down on the table so that it wouldn't slip and make Lotario cut crooked. Lotario and Lucia were making banners for the Pumpkin Festival. The cloth was 45 yards long, and they were going to cut it into 5-yard pieces for the banners. It took Lotario two minutes to make each cut, because he had to cut very carefully. How many minutes did it take him to cut the fabric into 5-yard pieces?

- FIND OUT What is the question you have to answer? How many minutes did it take Lotario to cut the fabric into 5-yard pieces?
  - What were Lotario and Lucia doing? Cutting a long piece of cloth into smaller pieces to make banners.
  - How long was the piece of fabric they began with? 45 yards
  - How long were the pieces they cut from the cloth? 5 yards How many pieces could they cut from the cloth? 9
  - · How long did it take Lotario to make each cut? 2 minutes

# CHOOSE A STRATEGY

• What strategy can you use to help you solve this problem?

When the strategies you know about don't appear to be helpful for solving a problem, and you don't know where to begin, relax and open up your mind to any and all possibilities. Explore anything you think of, reasonable or unreasonable. We call this "brainstorming."

# SOLVE IT

• What is your first reaction to this problem? Easy!

- Do you think this is a straightforward math problem? Yes, we just multiply.
- · How long do you think it took Lotario to cut the long fabric into pieces?

Solution: 16 minutes, because it took 8 cuts to make 9 pieces.

# LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?



Name

The Beachcomber Cafe is having a contest. Winners get a free milkshake. The contest question is: When you multiply 5 times 115469 on the calculator, what do you find that is ordinarily found at the beach? You are a winner! What is your answer?

**FIND OUT** • What is the question you have to answer?

STRATEGY

**CHOOSE A** • When the strategies you know about don't appear to be helpful, what can you

- **SOLVE IT** What is your first reaction to the question?
  - When you multiply the two numbers in the problem, what do you get?
  - How could the number be related to the beach?
  - If you think about the beach, what ideas pop into your mind? Is the number connected with any of those ideas?
  - Continue brainstorming about the beach and the number.

**LOOK BACK** • Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?



# Teaching Plan

The Beachcomber Cafe is having a contest. Winners get a free milkshake. The contest question is: When you multiply 5 times 115469 on the calculator, what do you find that is ordinarily found at the beach? You are a winner! What is your answer?

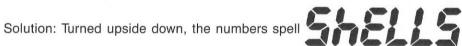
FIND OUT • What is the question you have to answer? When you multiply 5 times 115,469 on the calculator, what do you find that is ordinarily found at the beach?

# CHOOSE A STRATEGY

• When the strategies you know about don't appear to be helpful, what can you do? We can brainstorm.

# SOLVE IT

- What is your first reaction to the question? We don't know how to answer it.
- When you multiply the two numbers in the problem, what do you get? 577,345
- · How could the number be related to the beach? We don't know!
- If you think about the beach, what ideas pop into your mind? Water, fish, etc. Is the number connected with any of those ideas? It doesn't seem to be.
- (Encourage students to continue brainstorming about the beach and the number.) What is the answer to the problem?

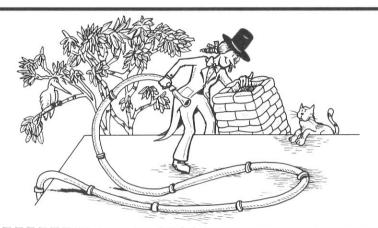


LOOK BACK

· Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE • Similar Practice Problems: 97, 110, 120

Sarah, the chimney sweep, cleans fireplaces. She uses a super vacuum cleaner to remove the soot that she brushes down the chimney. Sarah doesn't want to take the noisy cleaner inside a customer's home, so she leaves the cleaner outdoors and runs a hose from it to the fireplace. The distance from the cleaner to the fireplace is different at each home, so she has sections of hose that she can snap together to make the length of hose she needs. She has 3 sections each of these lengths: 8 feet, 10 feet, 12 feet, 16 feet, and 24 feet. Sarah is ready to clean a fireplace and needs a 100-foot hose. How many different combinations of hose sections could she use?



## Name

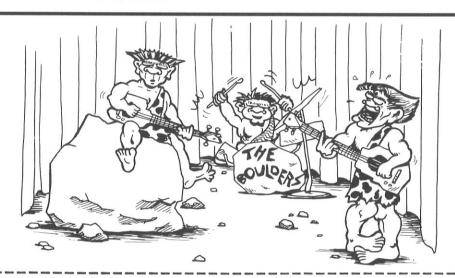
Lara, Lola, and Pam work out at the Fitness Center after school. Lara goes every 3 days, Lola every 4 days, and Pam plays in the band, so she can only go every 8 days. On what days in the next six weeks will Lola and Lara, and Lola and Pam, be at the Fitness Center on the same day?

Craig and his friends are on a 2-week backpacking trip in the mountains. They took their time traveling the 35 miles from the park ranger's office to Black Hawk Summit. They stopped along the way to fish and to explore old mines. Now they are on their way back to the ranger's office, and they can travel faster because their packs are lighter. The hikers have only three days left and they still have to cover 24 miles. They don't plan to cover more than 12 miles or less than 1 mile a day, but they will cover a whole number of miles every day. What are all the different ways they could divide the 24 miles for their three days of hiking?

# Name

- Rose is in charge of putting the cartons of ice cream into the large freezer every night when the 45 Fabulous Flavors Ice Cream Shop closes. She also puts them back into the display case every morning. The flavors she is putting into the case today are Chewey Chocolate, Very Vanilla, Ruby Raspberry Swirl, Blueberry Cheesecake, Luscious Lime, Outrageous Orange, Creamy Coconut, Pineapple Parfait, and Walnut Fudge Supreme. Can you arrange the cartons the way Rose wants them? Here are her instructions:
  - 1. Do not put Blueberry Cheesecake in the row in front of Very Vanilla.
  - 2. Put Ruby Raspberry Swirl between Chewy Chocolate and Very Vanilla, but not behind Luscious Lime.
  - 3. Put Luscious Lime to the right of Creamy Coconut.
  - 4. Put Outrageous Orange to the left of Blueberry Cheesecake in the row behind Chewy Chocolate.
  - 5. Do not put Walnut Fudge in the row with Creamy Coconut.

Ticket lines are huge around the Colossal Arena. The Boulders, one of the most popular rock groups ever, are giving a concert on Saturday. When the tickets went on sale Monday, 500 were sold in the first hour, 560 tickets were sold in the second hour, and 640 tickets were sold in the third hour. After the second hour, the increase in ticket sales every hour was 20 more than the increase of the previous hour. If sales continue at that rate, in what hour will 10,000 tickets be sold for the Boulders concert?



### Name

Gina and some friends are gathering information for a paper on astrological signs. For the 477 classmates born under Virgo, Gemini, Leo, Pisces, and Aquarius, they discovered that there were six times as many Virgos as Geminis;  $\frac{3}{8}$  as many Leos as Virgos;  $\frac{1}{2}$  as many Pisces as the combined total of Leos and Virgos; and 5 more than  $\frac{1}{3}$  the number of Pisces were Aquarius. How many of the 477 classmates were born under Virgo and Pisces?

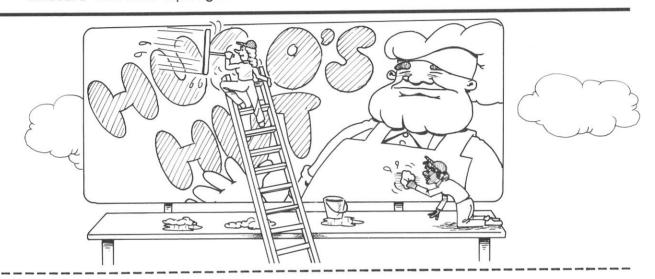
## Name

In the afternoon 75 swimmers competed in breaststroke events at the Jamesboro Swim Championships. There were 84 swimmers that competed in butterfly events and 94 swimmers who swam in freestyle events. There were 28 divers. There were 23 swimmers that competed in the breaststroke and butterfly, 12 in the freestyle and butterfly, 14 in the freestyle and breaststroke, and 18 that swam all three strokes. How many swimmers and divers competed in the afternoon at the swim meet?

### Name

Dr. A. B. Loney is a marine biologist charting the number of sharks, whales, jellyfish, and stingrays found in the Pacific Ocean. On the first day he found 7 sharks, 3 whales, 2 jellyfish, and 4 stingrays. The second day he counted 6 sharks, 5 whales, 8 jellyfish, and 8 stingrays. On the third day he counted 8 sharks, 4 whales, 5 jellyfish, and 7 stingrays. The fourth day he counted 7 sharks, 6 whales, 11 jellyfish, and 11 stingrays; and on the fifth day 9 sharks, 5 whales, 8 jellyfish, and 10 stingrays. If he continues to spot the animals at the same rates, on what day will he spot 75 animals?

Bella and Jana's billboard business is really taking off. They just got a new assignment, cleaning the billboard for Hugo's Hut. The billboard is 45 feet by 90 feet and they decide to each do  $\frac{1}{2}$  the job. Bella and Jana take turns with a squeegee  $2\frac{1}{2}$  feet wide, beginning at an outside edge and going all the way around, working their way toward the center. When it gets dark and they have to quit, they have completed  $\frac{4}{5}$  of the billboard. How many trips have they made around the billboard with their squeegee?



# Name

Omar is working at the fruitstand after school. One afternoon he decided to get some help with his homework. With the next customer, he made this offer: "If you can correctly solve this riddle, I'll give you 2 pounds of peaches free. Six oranges weigh as much as 4 apples. Five bananas weigh the same as 3 pears. Ten bananas weigh the same as 8 oranges. How many pears and apples would you need to have their weights balance?"

Some friends decided to make a week-long bike trip, with some of them leaving or joining the trip along the way. Two-thirds of the group wanted to try and go the entire distance. After the first day  $\frac{1}{3}$  of the bikers left. On the second day they got 8 new bikers. At the end of the second day they lost  $\frac{3}{7}$  of the group. On the third day they gained 2 new bikers and at the end of the day  $\frac{2}{5}$  of the group left. Five new bikers joined up on the fourth day and they lost 3 at the end of the day. On the fifth day they picked up 4, and then lost  $\frac{1}{4}$  of the total group. On the sixth day they got 4 new ones and at the end of the day 6 departed. On the seventh day, 7 bikers finished the trip. Did the group that wanted to go the whole way make it?

# Name

Although many people are aware that Indians used shells in their trading, few people know that Indians, especially West Coast Indians, also used feathers. To the traders, it was important that the birds from whom the feathers came still be soaring among the clouds. So the feathers had to be collected from dangerous perches along the beach cliffs. The most popular feathers came from birds that were common along the coast: seagulls, pelicans, and sandpipers. The feathers were all given equal value. A blanket or a piece of jewelry that cost 11 feathers could be paid for with any combination of 11 feathers from one or more of the three kinds of birds. How many different combinations of feathers could the traders have used for that purchase?

Shelly sells sunglasses at the shopping center. One Saturday she decided to keep track of her sales. She noticed that for every 46 pairs of sunglasses she sold, 22 had light sensitive lenses, 9 had mirror lenses, 11 had wire rims, 3 were rimless, and 1 had blue rims. If she sold a total of 552 pairs of sunglasses, how many of each type did she sell?

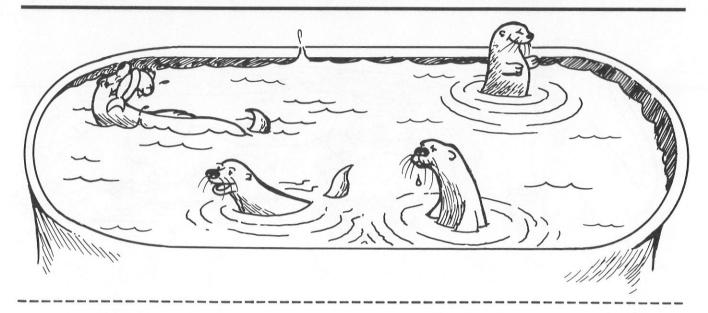


# Name

- Mrs. Swoople is making a seating chart for her students: Landen, Janet, Brenda, Lee, John, Gilda, Joy, Judy, Andrew, Sieu, Cindy, Rachel, Rene, Ray, Tim, and Roger. Using the clues below, fill in Mrs. Swoople's seating chart.
  - 1. Students who have names with the letter *a* as the second letter sit in the corner seats.
  - 2. Students who have names that begin with R sit in row 1.
  - 3. Sieu sits to the left of Cindy, who sits to the left of Andrew and Tim.
  - 4. Students who have names that begin with J sit next to one another.
  - 5. The total number of letters in students' names is the same for all columns and rows.

1		
2		
3		
4		

Otto B. Wright is responsible for feeding the otters at Sea Circus. He has to feed them one pound of fish each day. He can feed them smelt, perch, sardines, and squid. The smelt weigh 3 ounces each, the perch weigh 4 ounces each, the sardines weigh 1 ounce each, and the baby squid weigh 1½ ounces each. How many different combinations of fish can Otto use to feed the otters?



# Name

BULLETIN: All citizens are asked to be on the lookout for a dangerous animal-like plant that has escaped from the Interplanetary Botany Exhibit. The plant, named Jupiter Tietrap, consumes all kinds of ties: long ties, bow ties, shoe ties, and others. Evidence of the Tietrap's appetite has already been seen at four shops in Mars Park Plaza. There are 26 ties missing from the first shop, 38 from the second shop, 62 from the third shop, and 98 from the fourth shop. The increase in ties taken is always 12 more than the previous increase. If the Tietrap continues to eat ties at that rate, how many ties will it devour in the 12th shop?

Gary is delivering mail at Twin Towers East. He begins in the morning and takes the elevator to the 17th floor where he leaves  $\frac{5}{6}$  of his bag and then picks up 22 new letters, memos, and packages. Next stop is the 4th floor to leave  $\frac{3}{7}$  of his bag, and then pick up 14 pieces of mail. Next he goes to the 30th floor to drop off  $\frac{1}{3}$  of his bag and then pick up 4 items. Up again to the 43rd floor to deliver  $\frac{3}{4}$  of his bag and pick up 10 items. Now down to the 10th floor to deliver  $\frac{1}{2}$  his bag and then collect 2 new items. Up to the 24th floor to leave  $\frac{3}{5}$  of his bag. Then he picks up 3 new items. He goes to the 7th floor where he delivers everything he has left — 7 items, and then takes a break. How many pieces of mail did he deliver?

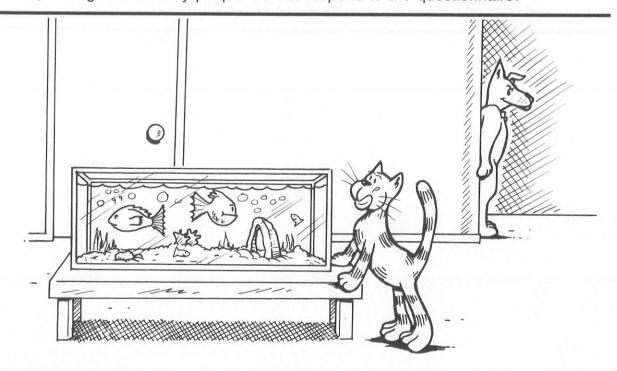
# Name

Mombasa is on the swim team. He swims his first 3 laps without weights and the next 5 laps with weights. Jamie is also on the swim team, but she swims her first 5 laps with weights and her next 3 laps without. If Mombasa and Jamie each swim 40 laps, using their own patterns of carrying weights, during how many laps will they both be using weights at the same time?

As Ginger and her friends were leaving Yulip at 10:00 this morning, they spotted some river rafters who were just arriving. The rafters said they had started out from Rogue Rapids 6 hours before. Ginger and her friends were just beginning their 7-hour hike along the South Fork of the Tubonia River to Rogue Rapids. They saw many rafters along the way, because a new group left Rogue Rapids every hour on the hour, from 4 o'clock in the morning to 8 o'clock at night. How many groups of rafters did Ginger and her friends see along their way?

## Name

The local pet store sent out 2500 questionnaires. They found out that 657 people had fish, 592 had cats, and 402 had dogs. They found that 124 had fish and dogs, 93 had dogs and cats, and 219 had fish and cats. Only 174 didn't have fish, cats, or dogs. How many people did not respond to the questionnaire?

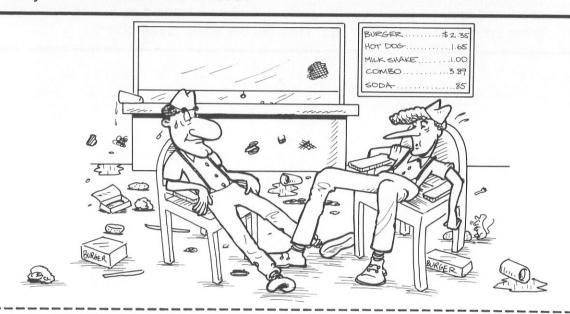


With the spring dance coming up, the dance committee did a survey to see what kind of band to get. They talked to 123 students and found that twice as many people wanted soul music as jazz; the number wanting heavy metal was  $\frac{2}{7}$  as many as the number wanting rock;  $\frac{1}{2}$  as many wanted the big band sound as heavy metal;  $\frac{1}{3}$  the combined total of jazz and soul wanted country western; 5 more than the combined total for jazz, country western, and soul wanted rock; and there were 2 fewer votes for reggae than jazz. How did the votes break down?

## Name

In a faraway galaxy, on the planet Ortem, meteorologists are practicing climate control. They are producing four kinds of weather and have worked out a pattern of 3 hot days, then 2 mild days, followed by 4 cloudy days, and then 4 warm days. If there are 9 days to a week on Ortem, how many weeks would it be before they get back to a week beginning with three hot days?

They sold hot dogs for \$1.65, hamburgers for \$2.35, sodas for \$.85, and the combo plate with fries, salad, a hamburger, and soda for \$3.89. They sold 80 items in an hour for \$163.24. How many of each kind of food did they sell, and how much did they make on each kind of food?



# Name

The employees at Roland's Records are rearranging the records by category. They have found that six cube-like boxes will hold all of the records for one category. For example, all of the rock music records will fit into one set of six boxes. There are many categories of records, and Roland wants the six boxes to be arranged in a different way for each category. How many different ways can six boxes be arranged so that one side of a box will be touching at least one side of another box?

- Leon, Mitch, Ray, Julio, and Ben have started baseball practice. Their positions include first base, second base, third base, pitcher, and catcher. Look at the following clues:
  - 1. Ben and Ray usually throw the ball to the pitcher or the catcher to help make the outs.
  - 2. The first baseman practices with Ben until the third baseman is ready.
  - 3. Leon can get the ball to first base faster than Ray can.
  - 4. Julio and the pitcher think the first baseman is a great guy.

Which position does each person play?

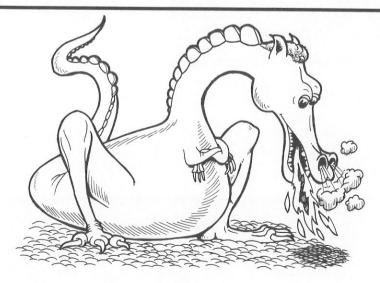
## Name

Prenda Beetle and Marcia Mosquito followed the trail of other bugs to the Registration Center of Ivy Leaf High School. Brenda and Marcia felt frightened by the large number of bugs at the center, but they did their best to encourage one another. Brenda whispered nervously, "How many bugs do you think there are here?" Marcia, trying to take Brenda's mind off her fears, said, "Take \frac{7}{9} of the total number of bugs, double it, and you will have 6 more than 1\frac{1}{2} times the number of bugs in this crowd." How many bugs were at the Ivy Leaf Registration Center?

The university recently purchased an enormous electric polisher for cleaning their radio satellite dish. The company selling the polisher has guaranteed clearer messages and less static or the entire purchase price is refunded. The polisher is on a long pole which is attached to the center of the dish. The mop-like polishing device has a 4-foot-wide-cleaning head that reaches to the outside edge of the dish. The polisher goes around in circles, moving toward the middle of the dish. If the dish is 234 feet across, how many trips around are needed to clean  $\frac{1}{2}$  of the area of the dish?

# Name

Sir Dwayne is in charge of dragon designs at the King's Toy Factory. He hopes that his new designs will please the king. "Your Highness," he says. "I have the plans for the toy dragons. If it pleases you, every 6th dragon shall have glowing eyes, every 4th dragon shall breathe fire, every 3rd dragon shall billow clouds of smoke through its nostrils, and every 8th dragon shall have a tail with spikes on it. I shall make 100 dragons." The king asks, "Sir Dwayne, how many dragons shall have more than one of these characteristics?" Sir Dwayne doesn't have the answer. Can you help him answer the king before he is thrown into the dungeon?



On Grand Opening Day, Toofer's Tape Store opened at 8:00 A.M. In the first hour Tom Toofer sold 4 tapes. In the second hour he sold 8 tapes, twice as many as in the first hour. In the third hour, customers bought 16 tapes, twice as many as the previous hour. In the next hour Tom sold three-fourths as many as in the previous hour, or 12 tapes. Tom kept selling tapes in the same way all morning: twice as many as the previous hour, twice as many as the previous hour, then three-fourths as many as the previous hour. And Tom gave customers one free tape for every tape they bought. During the last hour Toofer's was open, customers left the store with 1,296 tapes. How late did the store stay open?

## Name

78 Franky Flightright invented the flingwinger, a flying machine. The flingwinger has a rectangular body, made up of 10 compartments in this arrangement:

С	D	F	
В			G
Α	J	1	Н

In order to fly, the combined weight of the passengers must be the same in compartments A-B-C as in compartments C-D-E-F, as in compartments F-G-H, and as in compartments H-I-J-A. There must be at least 1 person in each compartment. On the first flight of the flingwinger, there were 56 passengers, who all weighed the same. How could Franky have seated the passengers in the flingwinger?

The monorail at Gateway to the Galaxy delivers people to different rides, and gives them a view of the entire park. Today the first car left the main station with some passengers on board and then stopped at the water rides where it left  $\frac{1}{3}$  of the people. Then 16 new riders got on. Next stop was the food area where  $\frac{2}{8}$  of the riders got off. Then 20 new riders got on and rode as far as the roller coaster. Here 12 riders got off before 10 more got on. At the fourth stop  $\frac{1}{4}$  of the passengers got off and then 3 new ones got on. At the fifth stop  $\frac{3}{8}$  of the riders got off before 4 new ones got on. At the sixth stop 4 people got off, then 3 new ones got on. At the seventh stop  $\frac{2}{3}$  of the passengers got off, then 2 new people got on. Finally at the last stop, all 13 of the passengers left got off to hear a concert. How many people got on the monorail in the beginning at the main station?

# Name

lrving the Invisible is beginning to reappear! Help! He desperately needs more of his invisibility potion. He must drink two gallons of the potion to remain invisible for the next 24 hours. He has 24-ounce, 32-ounce, 48-ounce, and 64-ounce bottles of the liquid. How many different combinations of bottles could Irving drink to prevent his reappearance?



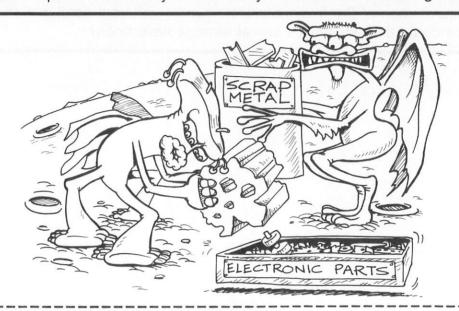
#### Name

Alfred was an astounding alchemist, but he was never well known. Alfred worked with aluminum, iron, lead, and zinc. He believed that if he combined the correct amounts of those metals, in the correct order, the result would be gold! Alfred's secret formula has been lost forever, but it is known that his formula called for a total of 19 ounces of metal, including not less than 3 ounces and not more than 9 ounces of each of the four metals. How many different combinations of the four metals could the alchemist's secret formula have been?

# Name

The Tripoli Triathalon has ended. The contestants, who had trained long for this event, all came hoping to win. But only the best remained in the competition to the end. The judges noticed something very interesting as the contestants crossed the finish line. There was one first-place winner, two second-place winners, and three third-place winners. If that pattern continued, in what place was the 200th person who crossed the finish line?

Barok and Zanna opened their B & Z Recycling Station one month ago, and already they have 7,000 tons of galactic debris for sale. They have collected five times as many 4-ton bins of asteroid fragments as 2-ton bins of meteorite dust. They have in storage four times as many 10-ton bins of scrap metal as 6-ton bins of electronic parts. How many bins do they have of each kind of galactic debris?



# Name

The manager of Majestic Ski Slopes was counting the number of skiers at the resort one day. He discovered that there were twice as many people in the lodge as on the expert slopes. The same number of people were on the expert slopes and the advanced slopes. There were  $\frac{1}{3}$  as many advanced skiers as intermediate skiers. He found that there were  $\frac{6}{7}$  as many intermediates as beginners. There were twice as many beginners as racers. There were 84 racers that day, exactly  $\frac{21}{147}$  of the total number of skiers. How many people were skiing at the Majestic Ski Slopes that day?

Maurice, an avid mountain climber, is preparing his assault on Mt. Rebus. New supplies will be dropped on the mountain top, so he is working out the logistics of having enough supplies to get him through his six-day climb to the top. He can only carry food for four days. How many helpers, each only able to carry a four-day supply as well, will Maurice need?

#### Name

The representatives of Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune are having a meeting. They are considering whether satellites should be subject to intergalactic traffic regulations and whether the asteroids should be divided up among the planets for additional areas of expansion. Four of the representatives are against satellite traffic regulation and three are for regulation. Four representatives want to split up the asteroids and three planet representatives want the asteroids undivided. Earth, Venus, and Mars agree on the traffic regulation issue, while Jupiter and Saturn do not agree on this. Neptune, Jupiter, and Uranus all have the same opinion about the asteroids, but Mars and Saturn are on opposite sides on this issue. The representative that wants traffic regulation and doesn't want to divide the asteroids is hosting the next meeting. Where are the planets meeting next?

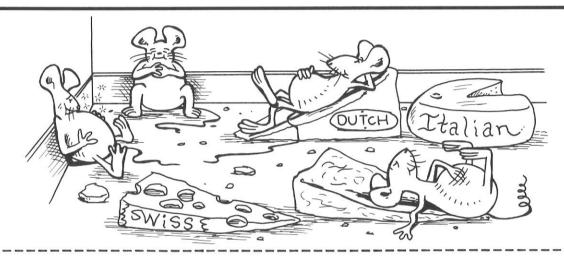
# Name

King Lorn gathered six of his knights together: Sir Guar, Sir Ruan, Sir Korm, Sir Meist, Sir Lawster, and Sir Fost. He told them he wanted them to manage the five countries he recently had gained control of: Carmelop North, which was cold and icy but loaded with gold; Carmelop South, warm and sunny; Nesselot, land of forests; Sandelot, rich in oil; Fasselmoor, which contained precious metals; and Ganderheath, which had extensive diamond mines. King Lorn gave them a choice. After they had divided up the countries, the knights made arrangements to get building supplies from Sir Ruan, and heating supplies from Sir Fost. Sir Guar is especially worried about getting adequate heating supplies. Sir Meist is already planning to go to the sunny and warm kingdom for a beach vacation, followed by a buying spree in Ganderheath. Meanwhile, Sir Lawster is getting some mining experts in to advise him. Which kingdom is being ruled by which knight?

#### Name

Serena, locked in a castle dungeon, is digging her way to freedom. On the other side of the castle wall, her sister Stephanie is also digging. It is a distance of 30 feet from Serena's cell to Stephanie. Sometimes they dig a few feet, only to lose ground because of the walls falling in. On the first day Stephanie digs 3 feet and Serena 2 feet. At the end of the second day both Serena and Stephanie have tunnels only 1 foot long. On the third day Serena has a tunnel 3 feet long, and Stephanie 4 feet. On the fourth day, Serena's tunnel is 2 feet and Stephanie's 2 feet. On the fifth day, Serena has gone 4 feet and Stephanie 5 feet. If they keep constructing their tunnels at this rate, how many days will it be before Serena makes her escape?

Ralph and his friends were ecstatic. The exhibition hall where they lived was having an International Cheese Convention. When the people departed, Ralph and the other mice wandered through the halls filled with hundreds of varieties of cheese. They finally collapsed in a corner, stuffed to the whiskers from cheese tasting. Ralph noticed that out of every 34 pieces of cheese he tasted, 12 were American, 3 were English, 6 were Austrian, 9 were Swiss, 3 were Dutch, and 1 was Italian. After Ralph had tasted 81 pieces of Swiss cheese, how many pieces of cheese had he tasted altogether?



# Name

Rob Hoop and his Merry Hoods were playing a marathon concert. They began with great enthusiasm, but finally collapsed at 10:00 P.M., Friday, October 13 after playing for 150,000 seconds. When did the concert begin?

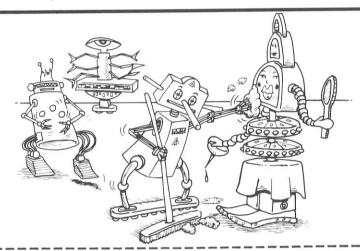
Jeremy and Josh had just gotten admitted to the SWIS Program (Summer Work in Space). The alien in charge told them they would need skypacks and checked to make sure they weren't afraid of flying! Their job was to clean the front window wall of the First Interspace Congressional Building. The wall measured 700 feet by 800 feet. Using their skypacks they decided to go around the wall beginning at one corner and cleaning a 10-foot area, working their way toward the center of the wall. How many trips around the wall did they make before they cleaned \( \frac{1}{4} \) of it?

# Name

- The Intergalactic Inventors Convention is meeting next week. Representatives from Mars, Jupiter, Saturn, and Venus will bring their newest inventions to sell. There is one small problem: each planet has its own money system and the price for the inventions will be in the currency of the inventor's planet. If only there were some way to figure out how many Jupiter jorgs equaled what number of Saturn skeeps! Luckily, Mr. Orz, the convention organizer, has developed the following chart:
  - 5 Mars pims = 6 Jupiter jorgs
  - 2 Venus firps = 7 Saturn skeeps
  - 8 Venus firps = 4 Mars pims

What number of Jupiter jorgs would be equivalent to the fewest Saturn skeeps?

Leo and Castor were off to the robotics convention. They wandered around, trying to take in some of the 905 robots on display. There were 50% as many do-it-yourself robots as new robots;  $\frac{4}{5}$  as many entertainment robots as do-it-yourself kinds;  $\frac{1}{3}$  as many cleaning robots as new ones;  $\frac{1}{6}$  as many robots with programmable modules as ones for entertainment;  $\frac{1}{4}$  as many robots for special tasks as with programmable modules; and twice as many business robots as the combined total for cleaning and special task robots. If Leo and Castor covered all the displays at the convention, how many of each type of robot did they see?



#### Name

Abdul is new to the caravan trade, so there is still much he does not know. He is young, but his brothers have great confidence in him. That is why they have invited him to come along on this important trip to Bahra. During the trip, Abdul and his brothers will be meeting many merchants coming from the city of Bahra. A different company of merchants leaves the gate of Bahra every day at the same time, and begins the 15-day journey to Abdul's city. If it takes Abdul and his brothers 15 days to travel from their city to Bahra, how many companies of merchants from Bahra will they see during their journey?

## Name

It was election time in Foxboro County. The residents were bombarded with ads in the newspapers, in the mail, and on TV. There were 193 candidates that used the mail, 167 that took out spots on TV, and 149 that had newspaper ads. Only 31 candidates had radio ads. There were 35 candidates that used the mail and the newspaper, 38 that used the mail and TV, and 24 that used TV and the newspaper, and 30 that used all three. How many candidates were running for office in Foxboro County?

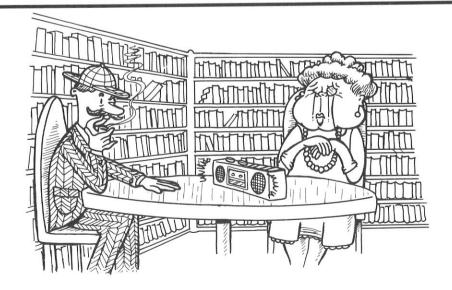
# Name

On the planet Pluto, the heater salesman has a very lucrative business. This year, Mr. Hurox has added to his heater line and now carries the jumbo heater for 16 trogs that is the equivalent of \$23.40 in our currency. He also has the giant heater for \$18.35, the large-sized heater for \$13.50, the medium-sized heater for \$8.65, and the small heater at \$3.77. So far this month Mr. Hurox has sold 72 heaters for a total number of trogs equal to \$974.38. How many heaters of each size has he sold?

"What does MIX have to do with 733 plus 276?" posed the announcer on Charlie's favorite TV station, channel 33, sponsored by the MIX Corporation. Channel 33 was always coming up with something new and unusual instead of commercials. Sometimes it was cartoons, sometimes lessons in cartooning, and sometimes questions that listeners could answer to win prizes. If Charlie can answer this question correctly, he will win a party for himself and 20 friends. Can you help him?

### Name

Sherlock Homes was listening to a strange tape that had been left at the home of Lady Webberly: "At the sound of the tone the time is 7:00 p.m. exactly, Monday, May 3, 1987. In precisely 16,700 minutes you must be under the oak tree behind the Somerset library, or you will be in grave danger." Sherlock instructed Lady Webberly to follow the instructions and that he would follow behind at a discreet distance. On what day and at what time was Lady Webberly to be under the oak tree behind the library?



99

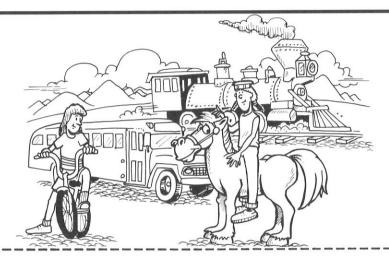
Monty, Nelson, Phil, Megan, Elise, Reggie, Salesi, Troy, and Vic are getting their tickets for the upcoming concerts by the Madison Shuttle and Frodo and the Fronds. They can only afford one concert, so they are deciding which one to go to and whether to attend the early or late show. Five of them are going to see Frodo and four of them are going to see the Madison Shuttle. Five have decided on the late show and four on the early show. Monty, Phil, Megan, and Salesi are going to the same show. Troy and Vic finally decided on different shows. Nelson, Reggie, Troy, and Elise are going at the same time. Vic and Phil wanted to go together but have to go at different times because of their jobs. Who is going to the Madison Shuttle's early show?

# Name

- 100
- At Skylake Camp this summer the head counselors for the five camper groups are Tim, Nancy, Wally, Rosa, and Marco. The campers are divided into Muskrats, Coyotes, Gray Foxes, Bobcats, and Hummingbirds. The groups also each have a number from one to five. Look at the following clues:
- 1. Tim and Wally's areas are on the other side of camp from the Hummingbirds, Rosa's area, and camp #4.
- 2. The counselor for the Gray Foxes, the counselor for camp #3, and Rosa all have campers on the same volleyball team, across the net from Marco's and Wally's groups.
- 3. The counselor for the Muskrats, the counselor for camp #2, and Nancy are meeting with Rosa and the counselor for the Coyotes to plan the next hike.
- 4. Tim, Rosa, and Marco are not the counselors for the Hummingbirds, the Muskrats, or camp #5.

Which counselor and which number is assigned to the Coyotes, the Gray Foxes, the Muskrats, the Hummingbirds, and the Bobcats?

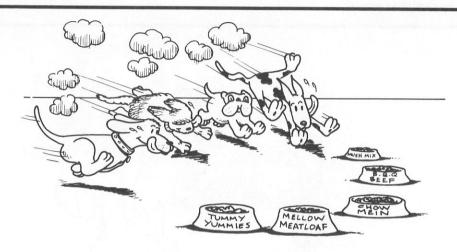
Four friends are traveling by different forms of transportation. They are trying to compare their travel times, so that they can get together during their vacation. Elena is traveling by horse, Carlotta is traveling by bus, Gina is traveling by train, and Joan is traveling by bicycle. So far the girls have figured out that 20 minutes on horseback is equal to 14 minutes by bus, 12 minutes by train is equal to 32 minutes by bicycle, and 18 minutes by train is equal to 30 minutes on horseback. What is the shortest amount of time that a bus and a bicycle would take to travel the same distance?



# Name

Selena Spider is an especially skillful web-builder, and she frequently experiments with different designs for her web. She has discovered that a web design of seven equilateral triangles, with each triangle sharing at least one side with another triangle, traps many more flies than other designs. Find as many different webs as you can, using Selena's design.

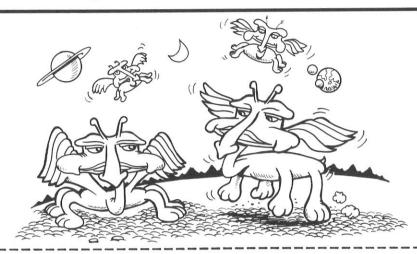
A recent survey at the Tummy Yummies Dog Food Company revealed that for every package of Beef Stew sold, doggie diners devoured 7 packages of Mellow Meatloaf, 6 packages of Chopped Chicken Chow Mein, and 9 packages of Barbequed Beef. They discovered that for every 2 packages of Beef Stew sold, one package of Mush Mix was purchased. If Tummy Yummies Dog Food Company sold 84 packages of chicken dinners one day, how many dinners did they sell altogether?



### Name

The eighth-grade class is preparing the food for the all-school Halloween party, and Gary is making the ice cream. Garlic ice cream. He flips the cookbook open to the recipe, but finds a chocolate stain covering the number that tells how many cloves of garlic he is supposed to use. There's a note at the end of the recipe that says \frac{1}{8} teaspoon of garlic powder may be substituted for 1 clove of garlic. If \frac{9}{15} of the number of teaspoons of garlic powder, added to 48, equals the total number of teaspoons needed, how many cloves of garlic must Gary use?

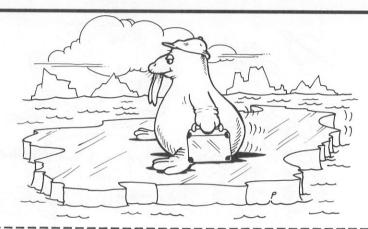
The air above planet Zarz is thick with 258,750 soaring fi-lees. Unfortunately, filees must pay a yearly fee for using airspace. Infant fi-lees travel up to 10 miles per day and pay a \$1 fee; children travel up to 20 miles per day and pay a \$2 fee; teenagers fly 31 to 50 miles per day and pay a \$3 fee; adults travel over 50 miles per day and pay a \$4 fee. The fi-lee fee collector says there are three times as many teenagers as children, one-half as many adults as teenagers, and one-fourth as many infants as children this year. How much money will the collector get from the fi-lees this year?



#### Name

Exotic plant life covers the planet Botone in the distant galaxy of Hydroxia. One of the strangest of trees on the planet produces a delicious fruit with a creamy texture and a chocolate flavor. Every year the tree produces one more pound of the fruit than the previous year. In the first year it produced one pound, in the second year two pounds, in the third year three pounds, and so on. This year was the first time that one tree bore over 700 pounds of the wonderful fruit. How many years old is that tree?

Nano, the walrus, travels for 9 days to get from his tundra home on an ice floe to Dumbarton Island. He nods to the seals as they swim past him in the icy waters, because they are usually swimming south when he is swimming north. One day he and Nanette, another walrus, got to talking about the seals. Nanette said that it took the seals 13 days to travel from Dumbarton Island to Nano's ice floe, and that they traveled in small groups. As a matter of fact, she said that a different group left the island every day at noon. Well, that made Nano start wondering how many groups he would see the next time he traveled from his ice floe to Dumbarton Island. Can you tell him?



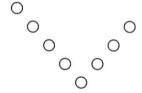
# Name

At Now You See It, Now You Don't, Jill sold magic kits to aspiring magicians. Last week she sold a lot of card tricks; 20.6% more rope tricks than rabbits-in-a-hat kits; \frac{1}{3} as many mice-in-the-pocket kits as the rabbit kits; 10 times as many rabbit kits as trick boxes; twice as many disappearing-dollar tricks as trick boxes; and 9 trick boxes. The boxes were 1.8% of the total number of items that she sold. How many magic kits, trick boxes, and card tricks did Jill sell?

The Star Sports Company is organizing a canoe trip. When Blaine signs up for the trip he discovers a small problem. Each canoeist can only carry supplies for eight days, but it will take 14 days to reach the trip destination. Blaine has asked some friends to help out. Each friend can only carry supplies for eight days as well. How many friends will need to begin the trip with Blaine, so he can finish the trip?

Name

110 Suzy took 8 pennies out of her backpack and arranged them in a V-shape on the table:

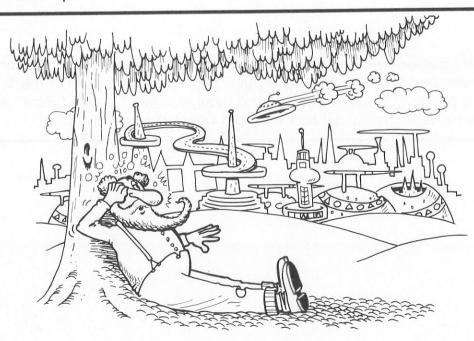


Suzy purposely made one side of the V longer than the other. Then she challenged Vladimir, "Can you put 5 pennies in each side of the V, by moving just one penny?" Vladimir thought and thought. Then he said "Sure" and moved one penny. What penny did he move and where did he move it to?

111 Kara is playing Raider Robot, trying to beat the best score. Finally, she sets a new record after 6 minutes and 8 seconds. For every blue robot that Kara is able to turn red, she gets points, depending on whether it's an R1, R2, R3, R4, R5, or Raider Robot. R1 is worth 2 points, R2 is worth 4 points, R3 is 11 points, R4 is 17 points, R5 38 points, and Raider Robot is worth 170 points. Kara changed a total of 92 robots from blue to red for a total of 2,244 points. How many robots of each kind did she turn to red, and how many points did she get for each kind?

#### Name

Willard Van Winkle (distant cousin of Rip) woke up slowly and looked around. "These things don't exist," he said, "wait a minute...." Suddenly a voice said, "No, wait 6,000,000 minutes!" And so it was — Willard had in fact been asleep for 6,000,000 minutes, having fallen asleep on January 1, 1990 at 8 P.M. What time did Willard wake up?



The president of Chipper Chocolate Company asked the employees of the company to suggest new designs for candy bars. One day, Chico was eating a Chocochunk and doodling on paper when an idea popped into his mind: Why not put 3 square Chocochunks together with 2 triangular Chippernips to make a whole candy bar. Each piece of chocolate would have to stick to at least one other piece of chocolate in the candy bar. That would not be hard, because the sides of Chocochunks and Chippernips were the same length. Find as many different designs as you can, that Chico could suggest for the candy bar.

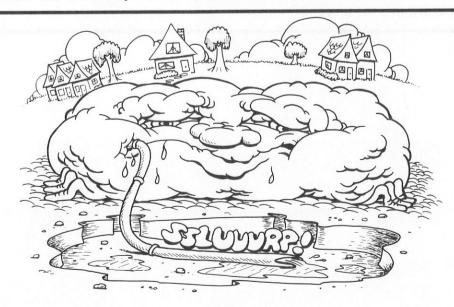
#### Name

Spring break was coming up and Janet, Kim, Laurie, Veronica, Cheryl, Rochelle, Rumiko, and Kimiko are planning trips. Five of them are going to the beach and four of them are going on camping trips. Five of them are going on a three-day trip and four of them are going for a week. Rochelle, Cheryl, Veronica, and Rumiko are going to the same place. Janet and Laurie are going to different places. Kim, Marie, Janet, and Kimiko are going to be away for the same length of time, while Cheryl and Laurie will be gone for different time periods. Who was going camping for a week?

One day an eagle and a hummingbird happened to settle not far from each other on the same tree branch. The eagle had been soaring around on the air currents, and the hummingbird had been exploring new territory for sweet nectars. The birds struck up a conversation. The hummingbird said, "You have big, powerful wings that carry you very high, and they beat so seldom. I admire the grace and ease of your flight." The eagle responded, "That is true. It does not take many beats for me to travel. I have large wings, and I do enjoy soaring. But you possess an ability that I do not have. You are able to fly in reverse, even though your wings must beat many times. How many times per minute do your wings beat?" The hummingbird replied, "You are wise, so I will give you the information to find the answer. Two-fifths the number of beats, plus 368, equals one-half the number of beats my wings make in a minute." The wise eagle thought for a few moments before saying the number. What number did the eagle tell the hummingbird?

#### Name

The town is buzzing with the news: The Teah has returned! Everyone groans. The Teah is a water monster with an enormous thirst. It slips unseen into the favorite lake of the townsfolk and begins its work. On its first day in residence, it politely drinks only one gallon of water. The second day, it drinks two gallons. The third day, it consumes three gallons. The summer goes on and the Teah keeps drinking until it has consumed 1,830 gallons of water. The next day it slips away as quietly as it came. On what day did the Teah leave the lake?

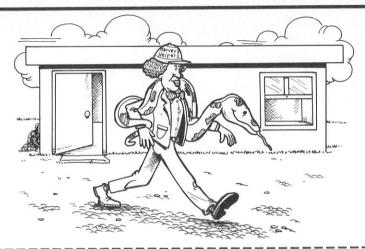


117 "Twinkle Twinkle Little Star," sing the Earthlings, not knowing that starbops deliver 510,000 dars of fuel per hour to Sirius, Arcturus, Rigel, and Betelgeuse. The brighter the star, the more fuel is needed to keep it shining. Starbops deliver 1.5 times as much fuel to Sirius as to Arcturus, and 2 times as much fuel to Arcturus as to Rigel. They carry 1.25 times as much fuel to Rigel as to Betelgeuse. The cost of one dar of star fuel is \$15. What is the cost per minute for fueling each star?

#### Name

Celia had to cross the swamps of Morundi, a trip that took 16 days. She could only carry food for 9 days, due to other provisions she needed to carry in the swamp country. Some of Celia's friends offered to help, and each friend could only carry food for 9 days as well. How many friends would have to leave with Celia, so that she could cross the swamps?

Harvey's Heavenly Helpers supply aid and assistance for any job: large, small, or out of the ordinary. During the last year they had 1.4% fewer jobs searching for stray mountain lions than looking for stolen hamsters; twice as many requests for building fences as washing windows; 3.5 times more jobs moving pianos than searching for hamsters; \(\frac{1}{4}\) as many calls for finding hamsters as washing windows; twice as many jobs washing windows as locating lost snakes; and 52 lost snake cases which was 10.4% of their business. If they successfully completed each job, how many snakes, hamsters, and mountain lions did they locate?



#### Name

Dan said he'd pay for pizza if Sam could answer this tricky question: "Can you prove that ten is half of sixty?" Now, Sam and Dan were walking in a shopping mall, and a shop two doors away was baking pizza. Sam could smell the pizza. The more he smelled the pizza, the more he thought about Dan's question. Ten minutes later, Dan paid for pizza. What answer did Sam give for Dan's question?

## Solutions

- **1** 4 (15th, 18th, 20th, 30th)
- 2 Combination of 2 metals 9 objects; combination of 3 metals 1 object
- 3 29
- 4 35

5	shirt	beach towel	suntan lotion	beach bag	can of soda	Name and Address of the Owner, or other Designation of the Owner, where the Owner, while th
	shorts	swimsuit	sunglasses	sandals	magazine	

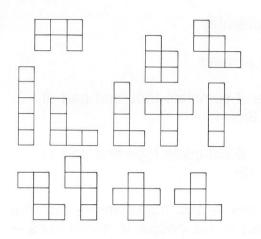
- 6 2 5 12 1 8 9 6 3 4 7
- 7 46
- 8 72

•	Denne											
	TV	R	5	TP	TV	R	5	TP	TV	R	5	TP
	2	3	4	8	2	4	5	6	5	6	2	4
	2	3	8	4	2	4	6	5	5	6	4	2
	2	4	8	3	2	5	4	6	5	2	4	6
	2	4	3	8	2	5	6	4	5	2	6	4
	2	8	4	3	2	6	4	5	5	4	2	6
	2	В	3	4	2	6	5	4	5	4	6	2
	3	4	8	2	4	5	Ь	2	6	2	4	5
	3	4	2	8	4	5	2	6	6	2	5	4
	3	8	2	4	4	6	2	5	6	4	2	5
	3	В	4	2	4	6	5	2	6	4	5	2
	3	2	4	8	4	2	5	6	6	5	2	4
	3	2	8	4	4	2	6	5	ь	5	4	2
	TV	R	5	TP	TV	R	5	TP	Tv	R	5	TP
	5	7	2	3	4	8	2	3	2	3	5	7
	5	7	3	2	4	8	3	2	2	3	7	5
	5	2	3	7	4	2	3	8	2	5	3	7
	5	2	7	3	4	2	8	3	2	5	7	3
	5	3	7	2	4	3	2	8	2	7	3	5
	5	3	2	7	4	3	8	2	2	7	5	3
	7	2	3	5	8	2	3	4	3	5	7	2
	7	2	5	3	8	2	4	3	3	5	2	7
	7	2	5 2			2	4	<i>3</i>	3	5 7	2 5	7 2
				3	8							
	7	3	2	3 5	8	3	2	4	3	7	5	2
	7	3	2 5	3 5 2	8 8	3	2 4	4	3	7	5	2 5

- 9 10th
- **10** 14th
- 11 Wednesday
- **12** 478 animals
- **13** After 2 complete trips and part of the 3rd
- **14** After 6 complete trips and part of the 7th
- **15** Clowns 140, magicians 420, fireeaters — 35, people in gorilla suits — 140, jugglers — 560
- **16** Radishes 129, zucchini 387, heads of lettuce 215, potatoes 349, artichokes 258, carrots 43
- **17** 40 orders
- 18 61 fossils
- 19 1451 pairs of shoes
- **20** 14,570 customers
- 21 16 pounds of sand = 22 pounds of fuel
- 22 8 oak tree votes = 13 maple tree votes
- 23 606 items
- 24 812 vehicles
- **25** 11
- **26** 19
- **27** Trilobat 13, 39 oz, Hepplefoot 14, 98 oz, Rippersnout 14, 406 oz, Fizzlenit 15, 570 oz, Furkadoo 15, 660 oz

- **28** Banana yogurt = 216, Bran oatmeal = 156, Chocolate raisin = 270
- 29 64
- 30 1500

31



- **33** 136
- 34 32 rows, the 32nd row is not full
- 35 Jana laser technician, Elsa bionic engineer, Garth robot relations, Ian weather analyst, Hak shuttle sales, Adriana robot repairs
- **36** Percy owl, mice; Louise ocelot, red meat; Rita otter, fish; Roger elephant, hay
- **37** 931 quarters, 7,448 half-dollars, 935 nickels, 5,610 dimes
- 38 \$1,400,100
- 39 Albert
- **40** Edwin

- 41 400 plants
- **42** 6 lb mushrooms, 36 lb carrots, 54 lb potatoes, 12 lb chicken wings, 72 lb frogs legs, 76 lb fruit bats
- 43 April 17, 2035, at 8 A.M.
- 44 He wouldn't make it!
- **45** 4 dogs
- 46 5 riders
- **47** 16 minutes, because it took 8 cuts to make 9 pieces.
- 48 Turned upside down, the numbers spell ShELLS

# SHELLS

**49** 20

24'	16'	12'	10'	8'	24'	16'	12'	10'	8'
3	1	1	0	0	1	3	1	0	2
3	0	1	0	2	1	3	0	2	1
3	0	0	2	1	1	2	3	0	,
2	2	1	0	,	1	2	2	2	0
2	2	0	2	0	1	2	0	2	3
2	1	3	0	0	/	1	3	0	3
2	1	1	0	3	/	1	2	2	z
2	1	0	2	2	0	3	3	0	2
2	0	3	0	2	0	3	2	2	1
2	0	2	2	1 -	0	2	2	2	3

**50** Lola and Lara — days 12, 24, 36; Lola and Pam — days 8, 16, 24, 32, 40

	1	2.	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
LA			×			X			X			X			×			×			×
				X				X				X				×				×	
								X								×					
	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	36	39	40	41	42
			×			X			X			X			×			×			×
			×				×				×				×				×		
			×								*								×		

#### 51 89

9												
	12	//	1		9	3	12		12	6	6	
	12	1	11		3	9	12		6	12	6	
	11	ı	12		3	12	9		6	6	12	
	11	12	1		12	8	4		П	11	2	
	1	11	12		12	4	8		н	2	11	
	1	12	11		8	4	12		2	71	11	
	12	10	2		8	12	4		11	10	3	
	12	2	10		4	8	12		11	3	10	
	10	2	12		4	12	8		10	3	11	
	10	12	2		12	7	5		10	it	3	
	2	12	10		12	5	7		3	11	10	
	2	10	12		7	5	12		3	10	11	
	12	9	3		7	12	5		11	7	4	
	12	3	9		5	12	7		11	4	9	
	9	12	3		5	7	12		9	4	11	
		Г	Ι.	١	Γ	_	1 .	1	· ·		-	1
	9	11	4		10	10	4		10	10	7	
	4	9	11		10	4	10		7		7	
	4	11	9		4	10	10		7	7	10	
	11	8	5		10	9	5		9	9	6	
	11	5	8		10	5	9		9	6	9	
	8	5	//		9	5	10		6	9	9	
	8	11	5		9	10	5		9	8	7	
	5	11	8		5	9	10		9	7	8	١
	5	8	11		5	10	9		8	1	9	
	и	7	6		10	8	6		8	9	7	
	11	6	7		10	6	8		7	9	8	
	7	6	11		8	6	10		7	8	9	
	7	11	6		8	10	6		8	8	8	
	6	//	7		6	10	8					
	6	7	11		6	8	10					

**52** 

Orange	Blueberry	Walnut
Chocolate	Raspberry	Vanilla
Pineapple	Coconut	Lime

#### **53** 30th

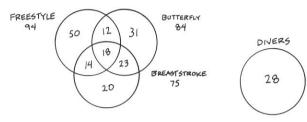
HOUR	/	2	3	4	5	6	7	8	9	10
INCREASE		60	80	100	120	140	160	180	200	220
TOTAL SALES	500	560	640	740	960	1,000	1,160	1,340	1,540	1,760
HOUR	11	/2	13	14	15	/6	17	18	19	20
NCREASE	240	260	280	300	320	340	360	380	400	420
TOTAL SALES PER HOUR	2,000	2,260	2,540	2,840	3,160	3,500	3,860	4,240	4,640	5,060
HOUR	21	22	23	24	25	26	27	28	29	30
NCREASE	440	460	480	500	520	540	560	580	600	620
TOTAL SALES	5,500	5,160	6,440	6,940	7,460	8,000	8,560	9,140	9,740	10,360

**54** 324 Pisces — 132 Gemini — 32 Leo — 72 Aquarius — 49

Virgo — 192

Total = 477

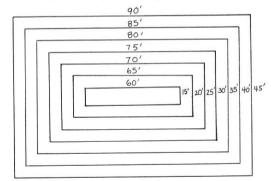
### **55** 196



#### **56** 14th

	1	2	3	4	5	6	7	8	9	ID	11	12	13	14
SHARK -1 +2	7	6	გ	٦	9	8	10	9	11	10	12	11	13	12
WHALE '	3	5	4	6	5	7	6	8	7	9	8	10	9	11
JELLYFISH +6 -3	2	8	5	11	გ	14	11	17	14	20	17	23	20	26
STINGRAY	4	8	7	11	10	14	13	17	16	20	19	23	22	26
TOTAL +11 -3	16	27	24	35	32	43	40	51	48	59	56	67	64	75

### 57 6 complete trips and part of the 7th



90 × 45 = 4050 SQ. FT ÷ 5 = 810 IF 810 = Vs, THEN 4/5 = 3240 SQ. FT. (LEAVING 810 SQ. FT.)

AFTER 1: 85 × 40 = 3400 3a. FT. LEPT
2: 80 × 35 = 2800 3a. FT. LEFT
3: 75 × 30 = 22.50 3a. PT. LEFT
4: 70 × 25 = 1750 5a. PT. LEFT
5: 65 × 20 = 1800 5a. FT. LEPT
6: 60 × 15 = 900 5a. FT. LEPT

### **58** 8 apples = 9 pears

6 ORANGES = 4 APPLES 5 BANANAS = 3 PEARS 10 BANANAS = 8 ORANGES

24 (4×6) ORANGES = 16(4×4) APPLES 30 (6×5) BANANAS = 18(6×3) PEARS 30 (3×10) BANANAS = 24 (3×8) ORANGES

16 APPLES = 18 PEARS

59 Yes (If 9 started out, 2/3 or 6 wanted to finish, there were never fewer than 6, and 7 finished, then the 6 would have made it.)

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	LEAVE	DAY 1	DAY 2	DAY 3
6+5=11	9 1	9-(1/30=9)=6	14-(3/7 0+ 14)	10-(45 0=10)
11-3=8 12-(40+12) 7	DAY 4	DAY 5	DAY 6	ARRIVE
11-5-0 12 (4012) 12-1-57	6+5=11	8+4=12	9+4=13	
	11-3=8		13-6=7	=

**60** 78

	54	P	SP	54	P	SP		54	P	SP
	0	0	11	9	0	2		7	0	4
	0	11	0	9	2	0		0	5	6
	11	0	D	D	3	8	-	0	6	5
1	D	1	10	0	8	3		5	6	0
	0	10	1	3	8	0		5	0	6
	1	0	10	3	0	8		6	0	5
	1	10	0	8	0	3		6	5	0
	10	0	1	8	3	0		1	1	9
	10	1	0	0	4	7		1	9	1
	0	2	9	0	7	4		9	1	1
	0	9	2	4	7	0		1	2	8
	2	9	0	4	0	7		1	B	2
	2	D	9	7	4	0		2	8	1

54	P	SP	54	P	SP	54	P	SP
2	1	8	6	1	4	6	3	2
8	1	2	6	4	1	2	4	5
8	2	1	1	5	5	2	5	4
1	3	7	5	1	5	4	5	2
1	7	3	5	5	1	4	2	5
3	7	1	2	2	7	5	2	4
3	1	7	2	7	2	5	4	2
7	1	3	7	2	2	3	3	5
7	3	1	2	3	6	3	5	3
1	4	6	2	6	3	5	3	3
1	6	4	3	6	2	3	4	4
4	6	1	3	2	6	4	3	4
4	1	6	6	2	3	4	4	3

**61** 264 light sensitive, 108 mirror lenses, 132 wire rims, 36 rimless, 12 with blue rims (table)

## 61 (CON'T)

LIGHT SENSITIVE	22	44	66	88	110	132	154	176	198	220	242	264
MIRROR	9	18	27	36	45	54	63	72	81	90	99	108
WIRE	11	22	33	44	55	66	77	86	99	110	121	132
RIMLESS	3	6	9	12	15	18	21	24	21	30	33	36
BLUE RIMS	1	2	3	4	5	6	7	8	9	10	11	12
TOTAL	46	92	138	184	230	276	322	368	414	460	506	552

62	Ray	Rene	Roger	Rachel
	Sieu	Cindy	Andrew	Tim
	Gilda	Brenda	Joy	Judy
	Landen	Lee	Jose	Janet

### **63** 46

	NUO	CES				0 0	NCE	5			OUN	685				001	Jees	
4	3	1/2	1		4	3	1/2	1		4	3	13	1		4	3	11/2	- 1
4	0	0	0	9	1	3	0	3		1	0	0	12			- 1	6	4
3	1	0	1		1	2	4	0			5	0	1			1	4	7
3	0	2	1		1	2	2	3			4	2	1			1	2	10
3	0	0	4		1	2	0	6			4	0	2	- 1		1	0	13
2	2	0	2		1	1	6	0			3	4	L				10	1
2	1	2	2		1	1	4	3			3	2	4				8	4
2	1	0	5		1	1	2	6	N.F.		3	0	7				6	7
2	0	4	2		1	1	0	9			2	6	1				4	ID
2	0	2	5		1	0	8	0			2	4	4				2	13
2	0	0	8		1	0	6	3			2	2	7				0	16
1	4	D	0		1	0	4	6			2	0	10					
I	3	2	0	H	1	0	2	9			1	8	1					

### 64 818

SHOP	1	2	3	4	5	6	7	8	9	ID	11	12
INCREASE		+12	+24	+36	+48	+60	+72	+84	+96	+108	+120	+132
TIES	26	38	62	98	146	206	278	362	458	566	686	818

### **65** 91

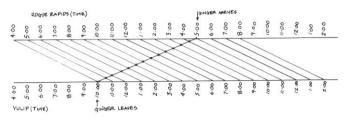
17th FLOOR	4TH FLOOR	30TH FLOOR	43 RO FLOOD
36 - (5/6 × 36)	28-(3/7×28)	30-(43 × 30)	24 -(3/4 ×24)
=6	=16	=20	=6
6+22=28	16+14=30	20+4=24	6+10=16

10TH FLOOR	24TH FLOOR	7TH FLOOR
16-(1/2×16) =8	10-(3/5 ×10) =4	DELIVER
8+2=10	4+3=7	

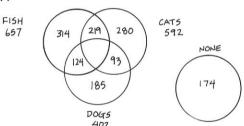
### **66** 10 laps

LAP	i	2	3	4	5	6	7	В	9	10	71	12	13	14	15	16	17	18	19	20
MOMBASSA				W	W	W	W	W				W	W	W	W	W				W
JAMIE	W	W	W	W	W				W	W	W	W	W				W	W	W	W
LAP	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	3%	31	40
MOMBASSA	W	W	W	W				W	W	W	W	W				W	W	W	W	W
JAMIE	W				W	W	W	W	W				W	W	W	W	W			

#### **67** 14



#### 68 1111



**69** Jazz — 11

Country western — 11

Rock — 49

Reggae — 9

Soul - 22

Heavy metal — 14

Big band — 7

Total = 123

#### **70** 14 weeks

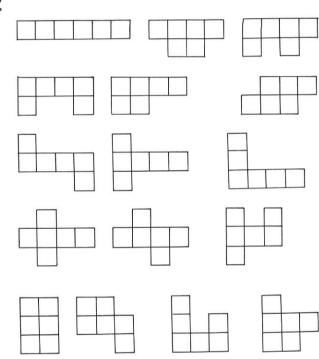
	1	2	3	4	5	6	7	8	9
1	Н	Н	Н	М	М	С	С	С	c
2	W	W	W	W	н	Н	Н	М	M
3	C	С	С	C	V	W	W	W	Н
4	Н	Н	М	М	С	С	C	С	W
5	W	W	W	Н	Н	Н	М	М	C
6	С	c	С	W	W	W	W	н	н
7	Н	м	М	С	С	С	۷	W	W
B	W	W	Н	Н	Н	М	М	С	c
9	С	C	W	W	W	W	Н	Н	Н
10	М	М	C	C	c	C	W	W	W
11	W	Н	Н	Н	М	M	С	C	c
12	C	W	W	W	W	н	H	н	M
13	М	С	۷	C	۷	W	W	W	W
14	Н	Н	Н						

71 16 Combos 17 Hot dogs 22 Hamburgers 25 Sodas

	50DA 4 .85	нот рост \$ 1.65	нин выявея \$ 2.35	\$ 3.89
1	.85	1.65	2.35	3.89
2	1.70	3.30	4.70	7.78
3	2.55	4.95	7.05	11.67
4	3.40	6.60	9.40	15.56
5	4.25	8.25	11.75	19.45
6	5.10	9.90	14.10	23.34
7	5.95	11.55	16.45	27. 23
8	6.80	13.20	18.80	31.12
9	7.65	14.85	21.15	35.01
ID	8.50	16.50	23.50	38.90
11	9.35	18.15	25.85	42.19
12	10.20	19.80	28.20	46.68
13	11.05	21.45	30.55	50.57

	50DA \$ .85	\$ 1.65	HAMBURGER \$ 2.35	\$ 3.89
14	11.90	23.10	32.90	54.46
15	12.75	24.75	35.25	58.35
16	13.60	26.40	37.60	62.24
17	14.45	28.05	39.95	66.13
18	15.30	29.70	42.30	70. 02
19	16.15	31.35	44.65	73.91
20	17.00	33.00	47.00	77.80
21	17.85	34.65	49.35	81.69
21	18.70	36.30	(51.70)	85.58
23	19.55	37.95	54.05	
24	20.40			
25	(21.25)			

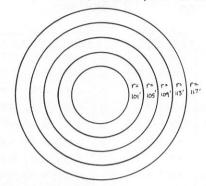
72



73 Leon — pitcher, Mitch — 1st base, Ray — 3rd base, Julio — catcher, Ben — 2nd base

**74** 108  $\frac{7}{9} \times 108 = 84$  84  $\times$  2 = 168  $\frac{11}{2} \times 108 = 162$  168 - 6 = 162

## 75 8 complete trips and part of the 9th



15T T: 113 - 40,094.66 50. FT. LEFT 15T 113 - 40,044.66 50. FT. LEFT 3RP 1: 105 - 34,618.50 50. FT. LEFT 3RP 1: 105 - 34,618.50 50. FT. LEFT 4TH 1: 101 - 52,031.14 50. FT. LEFT 5TH 1: 97 - 29,544.26 50. FT. LEFT 6TH 1: 97 - 27,157.86 50. FT. LEFT 7TH 1: 89 - 24,871.94 50. FT. LEFT 9 TH 1: 85 - 22,686.50 50. FT. LEFT 9 TH 1: 81 - 20,601.54 50. FT. LEFT

AREA = 42,983.46 50. PT. 1/2= 21, 491.73 30.FT.

### **76** 24 dragons

SPIKES	(8)	(b)	24)	32	40	48	(56)	64)	72	80)	<b>₩</b>	96	
EYES	6	2	(8)	24)	30)	36	42	48	<i>5</i> 4	60	66)	12	78
FIRE	4	8	(12)	(6)	20	24	28	32)	36)	(40)	44	(48)	52
SMOKE	3	6	9	(12)	15	(18)	21	24)	27	30)	33	36)	39

		_			_							
SPIKES		ka i										
EYES	84)	10	96									
FIRE	56	60)	64)	68	12	76	8	84	(%)	92	96)	100
SMOKE	(42)	45	(48)	51	54)	57	60)	63	66	69	(72)	

### **77** 10:00 P.M.

HOUR	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TAPES BOUGHT	4	8	16	12	24	48	36	72	144	108	216	432	324	648
TAPES FREE	4	8	16	12	24	48	36	72	144	108	216	432	324	648
TAPES	8	16	32	24	48	96	12	144	288	216	432	864	648	1296

**78** 7 6 4 11 9 2 5 8 3

## **79** 60 passengers



STOP 1 60-(1/3×60) =40 40+16=56

STOP 2 56-(2/8×56) = 42 42+20= 62

STOP 3 62-12=50 50+10= 60

STOP4 60-(1/4×60) =45 45+3=48

STOP 5 48-(3/8×48) =30 30+4=34

STOP 6 34-4=30 30+3=

STOP 7 33-(2/3 ×33) =11 11+2=13

LAST STOP 13-13

#### **80** 23

	DUNG	ES			OUN	CES	
64	48	32	24	64	48	32	24
4	0	0	0	1	0	3	4
3	0	2	0	1	0	0	8
2	2	1	0		4	2	0
2	1	1	2		3	2	2
2	0	4	0		2	5	0
1	4	0	0		2	2	4
I	3	0	2		1	5	2
1	2	3	0		1	2	6
1	2	0	4			8	0
1	1	3	2			5	4
1	1	D	6			2	8
1	0	6	0				

2 GALLONS = 256 02. = 32 × 8

81 116

SAMPLE OF ARRANGEMENTS:

3	3	4	9		3	3	5	8	 3	3	6	7
3	3	9	4		3	3	8	5	3	3	7	6
3	4	9	3		3	5	8	3	3	6	7	3
3	4	3	9		3	5	3	8	3	6	3	7
3	9	3	4		3	8	3	5	3	7	3	6
3	9	4	3		3	8	5	3	3	7	6	3
4	9	3	3		5	8	3	3	6	7	3	3
4	3	9	3		5	3	8	3	6	3	7	3
4	3	3	9		5	3	3	8	6	3	3	7
9	3	3	4		8	3	3	5	7	3	3	6
9	3	4	3	-	8	3	5	3	7	3	6	3
9	4	3	3		8	5	3	3	7	6	3	3

**82** 20th place  $19 \times 20 \div 2 = 190$ ,  $20 \times 21 \div 2 = 210$ 

83 107 bins of meteorite dust, 535 bins of asteroid fragments, 101 bins of electronic parts, 404 bins of scrap metal

	M.D.	A.F.	Total	E.P.	S.M.	Total	Combined
Sets	2 tons	4 tons	tons	6 tons	10 tons	tons	Total
1	1	5	22	1	4	46	68

 $7,000 \div 68 = 102 \text{ r } 64;$  64 is not divisible by 22 or 46;  $101 \times 68 = 6868 \text{ r } 132;$   $132 \div 22 = 6$   $(101 + 6) \times 1 = 107 \text{ bins M.D.};$   $(101 + 6) \times 5 = 535 \text{ bins A.F.};$   $101 \times 1 = 101 \text{ tons E.P.};$  $101 \times 4 = 404 \text{ tons S.M.}$ 

#### **84** 588

Racers =  ${}^{21}/{}_{147}$  84 ÷ 21 = 4 Beginners = 2 × racers = 168 ÷ 4 =  ${}^{42}/{}_{147}$ Intermediate =  ${}^{6}/{}_{7}$  of beginners, 144 ÷ 4 =  ${}^{36}/{}_{147}$ Advanced = 48 ÷ 4 =  ${}^{12}/{}_{147}$ Expert = 48 ÷ 4 =  ${}^{12}/{}_{147}$ Lodge = 96 ÷ 4 =  ${}^{24}/{}_{147}$ 

### 85 2 helpers

DAYS	1	2	3	4	5	6
MAURICE (4)	-l,+l →	-1,+1-	(4)-1	(3) -1	(2)-	(1) -1
HELPER A	-1,+1-	-2				
HELPER B	-3					

### 86 Saturn

REGULATED	NOT REGULATED	DIVIDED	DIVIDED
NEPTUNE	EARTH	NEPTUNE	EARTH
URANUS JUPITER /SATURN	VENUS MARS		VENUS MARS/SATURN
	JUPITER/SATURA	MARS SATURN	

87 Carmelop North — Guar, Sandelot — Fost, Ganderheath — Lawster, Nesselot — Ruan, Fasselmoor — Meist, Carmelop South — Korm

#### 87 (CON'T)

	GUAR	RUAN	KORM	MEIST	LAWSLER	Fost
CARMELOP	Υ	7	7	7	4	7
SANDELOT	7	7	7	N	N	Y
GANDER - HEATH	7	7	7	7	Y	2
NESSELOT	7	Y	7	7	7	7
FASSEL MOOR	7	1	И	Y	И	7
CARMELOP	N	N	Υ	N	4	4

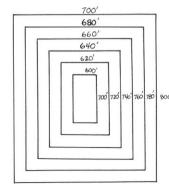
### **88** 27 days

DAY	1	2	3	4	5	6	7	8	9	ID	11	12	13	14
STEPHANIE	3	1	4	2	5	3	6	4	7	5	8	6	9	7
SERENA	2	1	3	2	4	3	5	4	6	5	7	6	8	7
TOTAL	5	2	7	4	9	6	11	8	13	10	15	12	17	14
DAY	15	16	17	18	19	20	21	22	23	24	25	26	27	
STEPHANIE	10	8	п	9	12	10	13	11	14	12	15	13	16	
SERENA	9	8	10	9	11	10	12	11	13	12	14	13	15	
TOTAL	19	16	21	18	23	20	25	22	27	24	29	26	31	

### 89 1530 pieces of cheese

	1	2	3	4	5	6	7	8	9
AMERICAN	12	24	36	48	60	72	84	96	108
ENGLISH	3	6	9	12	15	18	21	24	27
AUSTRIAN	6	12	18	24	30	36	42	48	54
5W155	9	18	27	36	45	54	63	12	(81)
DUTCH	3	6	9	12	15	18	21	24	27
ITALIAN	1	2	3	4	5	6	7	8	9
TOTAL	34	68	102	136	170	204	238	272	306

- **90** Thursday, October 12, 4:20 A.M. (1 day, 17 hours, 40 minutes)
- 91 5 trips (diagram)



AREA = 560,000 Sa.PT. +4 = 140,000 Sa.FT.

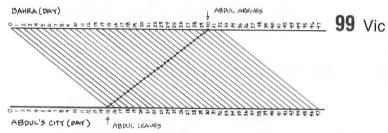
1. 530, 400 SQ. PT. LEPT 2. 501, 600 SQ. PT. LEPT 3. 473, 600 SQ. PT. LEPT 4.446, 400 SQ. PT. LEPT. 5. 420, 000 SQ. PT. LEPT

### **92** 6 Jorgs

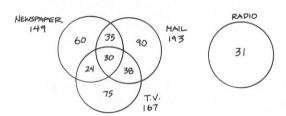
5 MARS PIMS = 6 JUPITER JORGS 2 VENUS FIRPS = 7 SATURN SKEEPS 8 VENUS FIRPS = 4 MARS AMS 20 (4×5) PIMS = 24 (4+6) JOR45 40(20+2) FIRPS = 140 (20+7) SKEEPS 40(5+8) FIRPS = 20(5+4) PIMS 24 JORGS = 140 SKEEPS 6 JOR45 = 35 SKEEPS

93 Do-it-yourself — 150 New - 300 Business - 210 Entertainment — 120 Programming modules — 20 Cleaning — 100 Special task — 5 Total = 905

### 94 31



### 95 383 candidates



**96** Jumbo — 14 Giant - 15 Large — 14 Medium - 15 Small - 14

	JUMBO	GIANT	LARGE	HEDUM	SMALL
1	23.40	18.35	13.50	8.65	3.77
2	46.80	36.10	27.00	17.30	7.54
3	70.20	55.05	40.50	25.95	11.31
4	93.60	73.40	54.00	34.60	15.08
5	117.00	91.75	67.50	43.25	18.85
6	140.40	110.10	81.00	51.90	22.62
7	163.80	128.45	94.50	60.55	26.39
В	187.20	146.60	108.00	69.20	30.16
					5

#### 96 (CON'T)

	JUMBO	GIANT	LARGE	MEDIUM	SMALL
9	210.60	165.15	121.50	77.85	33.93
10	234.00	183.50	135.00	86.50	37.70
11	257.40	201.85	148.50	95.15	41.47
12	280.80	220.20	162.00	103.80	45.24
13	304.20	238.55	175.50	112.45	49.01
14	(327.60)	256.90	(189.00)	121.10	(52.78)
15	351.00	215.25	202.50	(129.75)	56.55
16	374.40			138.40	

97 MIX = 1009733 + 276 = 1009

98 Saturday, May 15, 9:20 A.M.

16,700 minutes = 278 hours, 20minutes = 11 days, 14 hours, 20 minutes

MADISON	FRODO	EARLY	LATE
NELSON	MONTY	MONTY	NELSON
elise Reggie	PHIL	SALESI MEGAN	REGGIE TROY
TROY / VIL	MEGAN TROY/VIC	VIC/PHIL	ELISE VIC/PHIL

**100** Tim — Fox — #2, Wally — Muskrat — #5,

Rosa — Bobcat — #1,

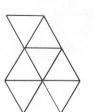
Nancy — Hummingbird — #3,

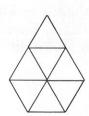
Marco — Coyote — #4

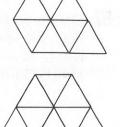
	COMOTE	BOBCAT	MUSKRAT	HUMMING- BIRD	rox	1	2	3	4	5
TIM	N	N	N	N	Y	N	Y	N	N	N
WALLY	N	N	Y	N	N	N	N	N	7	Y
ROSA	N	Y	N	N	N	Y	N	H	N	N
MANCY	N	N	N	Υ	N	N	N	Y	N	N
MARCO	Y	N	N	N	N	N	N	N	Y	N

**101** 7 minutes bus = 16 minutes bike

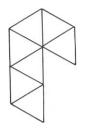
102 EXAMPLES:

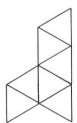


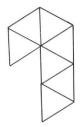


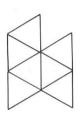


### 102 (CON'T)

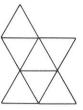


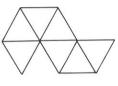


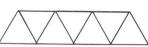












### **103** 329 dinners

STEW	1	2	3	4	5	6	7	8	9	10	11	12	13	14
MEAT	7	14	21	28	35	42	49	56	63	70	77	84	91	98
CHICKEN	6	12	18	24	30	36	42	48	54	60	66	72	78	84)
BARBEQUE	9	18	27	36	45	54	63	72	81	90	99	108	117	126
MUSH		1		2		3		4		5		6		7
TOTAL						Ġ.								220

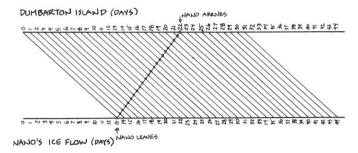
**104** 960 (The number of teaspoons is 120;  $\frac{9}{15} \times 120 = 72$ ; 72 + 48 = 120; 120 tsp.  $\times$  8 = 960 cloves)

### 105 \$776,250

 $(11,250 \text{ infants} \times \$1 = \$11,250;$   $45,000 \text{ children} \times \$2 = \$90,000;$   $135,000 \text{ teens} \times \$3 = \$405,000;$   $67,500 \text{ adults} \times \$4 = \$270,000;$  \$11,250 + \$90,000 + \$405,000 +\$270,000 = \$776,250)

**106** 37 years old  $(37 \times 38) \div 2 = 703$ 

#### **107** 23



### 108 500 magic kits, boxes, cards

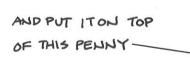
Trick boxes  $-9 \div 1.8 = 5$ , 1% = 5Disappearing  $-2 \times$  boxes, 3.6% = 18Rabbit-in-hat  $-10 \times$  boxes, 18% = 90Mice-in-pocket  $-\frac{1}{3}$  rabbits, 6% = 30Rope -20.6% more than rabbits =38.6% -193100% -68% = 32%, cards =32%, 160

#### **109** 6

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
BLAINE (8)	-1 +1 ◆	-! +! <del>*</del>	-1 +1 =7	-l +l *1	-1 +1 <sup>◆</sup>	-1 +1 <del>-</del> 7	(8)	(7) -1	(6)	(5) -I	(4) -1	(3) -1	(2) -1	(1) -1
FRIEND 1 (8)	-l ← +l	-1 -+	-l +l ◆	-1 - +1	-1 +	- 2								
FRIEND2 (8)	-1 -+ +1	-1 <b>+</b>	-1 +1	-1 <del>+</del> +1	-3									
FRIEND 3 (8)	-I +	-1 +1	-l +l	-4										
FRIEND4 (8)	-1 †1	-1 <del>-</del>	-5											
FRIEND 5 (8)	+1	-6												
FRIEND 6 (8)	-7													

#### 110

HE TOOK THE
PENNY THAT WAS
HERE

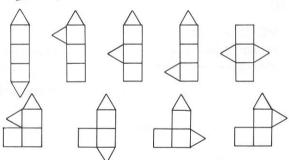


**111** R1 — 21, 42 points; R2 — 19, 76 points; R3 — 17, 187 points; R4 — 15, 255 points; R5 — 13, 494 points; Raider Robot — 7, 1190 points

	RI	R2	P.3	R4	R5	raiper robot
1	2	4	11	17	38	170
2	4	8	22	34	76	340
3	6	12	33	51	114	510
4	8	16	44	68	152	680
5	10	20	55	85	190	850
6	12	24	66	102	228	1020
1	14	28	71	119	266	(190)
8	16	32	88	136	304	1360
9	18	36	99	153	342	1530
10	20	40	110	170	380	1700
(1	22	44	121	187	418	1870
12	24	48	132	204	456	2040
13	26	52	143	221	(494)	
14	28	56	154	238	532	
15	30	60	165	255)		
16	32	64	176			
17	34	68	(187)			
18	56	72	198			
19	38	76)	209			
20	40	80	220			1 1
21	(42)	84				

**112** Noon, May 30, 2001 (11 years, 149 days, 16 hours; allow for leap years 1992 and 1996)

### 113 EXAMPLES:



### 114 Laurie

BEACH	CAMP	3 DAYS	WEEK
ROCHELLE	LETICIA	CHERYL/BIANCA	CHERYL/BIANCA
CHERYL VERONICA	KIMIKO	GITA	POCHELLE VERONICA
BIANCA JANET/GITA	JANET/GITA	LETICIA KIM UKO	LAURIE

- **115** 3,680 (½5 × 3680 = 1472; 1472 + 368 = 1840; 1840 × 2 = 3680)
- **116** Day 61 (60  $\times$  61)  $\div$  2 = 60; the next day is 61
- **117** Betelgeuse—\$15,000; Rigel—\$18,750; Arcturas—\$37,500; Sirius—\$56,250

 $510,000 \div 60 \text{ minutes} = 8,500$ Betelgeuse = 1,000 dars; Rigel = 1.25 × 1,000 = 1,250; Arcturas = 2 × 1,250 = 2500; Sirius = 1.5 × 1,250 = 3,750; 1,000 + 1,250 + 2,500 + 3,750 = 8,500

#### **118** 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
CELIA (4)	-1 +1	-1 +1 =	+1 4	+1	7)~	-l +l =	-1 +	(9) -1	(8)	(7)	(6)	(5) -1	(4) -1	(3)	(2) -1	(1)
FRIEND 1	-1 +1	-I -+	-1 +1	-  +	-  <del>-</del>	-1 -	-2									
FRIEND 2	-1 -	-1 -	-1 -	-1 +1	-1-	-3					_					
FRIEND 3	-l -	-1 + +1	-1 <del>-</del>	-1 -	4											
FRIEND 4	-1 +1	-1 -	-I +1	-5												
FRIEND 5	-1 +1	-1 - +1	-6													
FRIEND 6	-l + tl	-7														
FRIEND 7	-8															

**119** Snakes — 52, Hamsters — 26, Mountain lions — 19

Snakes —  $52 \div 10.4 = 5$ , 1% = 5Windows —  $2 \times \text{snakes} = 20.8\% \times 5 = 104$ Hamsters —  $\frac{1}{4}$  windows,  $5.2\% \times 5 = 26$ Pianos —  $3.5 \times \text{hamsters} = 18.2\% \times 5 = 91$ Fences —  $2 \times \text{windows} = 41.6\% \times 5 = 208$ Mountain lions — 1.4% less than hamsters =  $3.8\% \times 5 = 19$ 

**120** LX = 60 X is half of LX, so 10 is half of 60