The Problem Solver 6

Activities for Learning Problem-Solving Strategies

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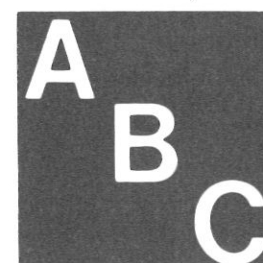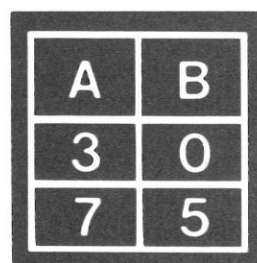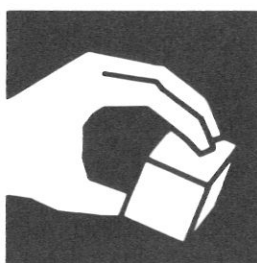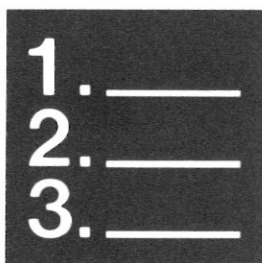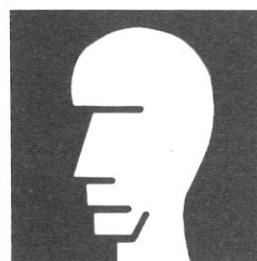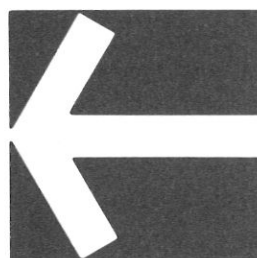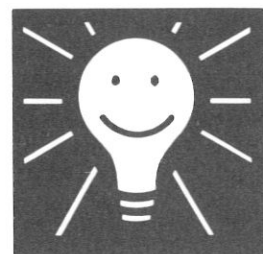
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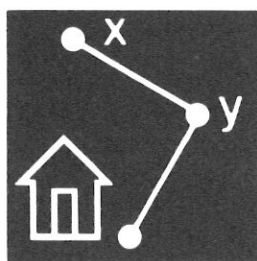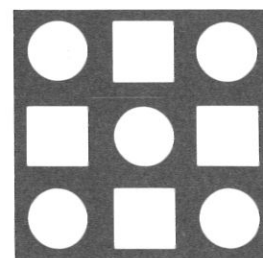| Problems	Pages	Strategy Focus
1, 2	T•1 – T•4	Use or make a table
3, 4	T•5 – T•8	Make an organized list
5, 6	T•9 – T•12	Act out or use objects
7, 8	T•13 – T•16	Make an organized list
9, 10	T•17 – T•20	Use or look for a pattern
11, 12	T•21 – T•24	Use or make a table
13, 14	T•25 – T•28	Make a picture or diagram
15, 16	T•29 – T•32	Guess and check
17, 18	T•33 – T•36	Work backwards
19, 20	T•37 – T•40	Make a picture or diagram
21, 22	T•41 – T•44	Use logical reasoning
23, 24	T•45 – T•48	Use or make a table
25, 26	T•49 – T•52	Make a picture or diagram
27, 28	T•53 – T•56	Make an organized list
29, 30	T•57 – T•60	Guess and check
31, 32	T•61 – T•64	Make a picture or diagram

THE PROBLEM SOLVER PROGRAM

The Problem Solver is a step-by-step instructional program designed to help your students become competent and confident problem solvers. It utilizes an easy-to-learn, four-step method combined with ten useful problem-solving strategies. A wide range of reproducible problems are carefully sequenced to systematically expand your students' problem-solving abilities.

CONTENTS AND ORGANIZATION OF *THE PROBLEM SOLVER 6*

Section 1: Teaching the Strategies

The first part of this section describes the four-step method and the ten solution strategies. Sample problems are used to illustrate how the method and strategies can be implemented.

The second part of this section presents 48 teaching problems. For each problem there is a reproducible student page and a one-page teaching plan. The problems are organized in groups of two that focus on the same strategy. The symbol of that strategy is shown in the upper left-hand corner of both the student page and the teaching plan page.

Section 2: Practice Problems

This section presents 72 reproducible problems similar to those in Section 1. But here, no strategies are recommended. The students must decide which strategy to use. There are two problems on a page and they have purposely been mixed up so that they do not follow the order of strategies presented in the teaching section. A reproducible recording sheet is provided for students to use with each problem. The ten strategy symbols are shown on the recording sheet. The students circle the appropriate symbol or symbols to indicate which strategy or strategies they used.

Section 3: Solutions

Here you will find a solution for every problem in Sections 1 and 2. For many problems, notes are also included describing the solution process. (Solutions for all the problems in Section 1 are also shown on their respective teaching plan pages.)

GENERAL TEACHING SUGGESTIONS

Presenting the Problems

Before you present any problems to the students, read through the descriptions of the four-step method and the ten solution strategies in Section 1. We recommend that you present the problems in the order they appear. The problems have been carefully sequenced so that they progress from easy to more challenging. The math skills required to solve even the most challenging problems are those which children have usually mastered by the middle of sixth grade.

Give the student a copy of the problem and a copy of the recording sheet. You may also want to give them calculators to use. Read the problem aloud to the students. Encourage the students to verbalize their thought processes as they choose strategies and find solutions. Develop an atmosphere in which the students feel comfortable expressing themselves. Let them know it's okay to make mistakes. In this setting, your students will become enthusiastic problem solvers and will begin to see problems as interesting challenges.

Use the teaching plans. The sequence of questions in each plan guides the students through the four-step method: FIND OUT, CHOOSE A STRATEGY, SOLVE IT, and LOOK BACK. You'll probably want to add some of your own questions to further clarify the process. The responses to the questions, shown in italics in the teaching plans, are only samples of the kinds of responses you want to draw from your students. Encourage them to risk giving incorrect answers. Remember, it is more important for them to take an active role in solving the problem, and enjoy doing it, than it is to respond with the right answer.

Since students learn in different ways, some students may wish to use a strategy other than the focus strategy for solving a problem given in Section 1. Encourage students to use methods that are best for them, when they can demonstrate a legitimate solution process with another strategy. (The goal is to equip your students with techniques for approaching future problems, but you also want them to be flexible in applying them.) Some students may even discover additional strategies and use them to solve the problems. Encourage the students to design new symbols for those strategies.

Additional Practice

For additional practice, you can give students the problem extensions which appear on many teaching plans in Section 1, and practice problems from Section 2. Since a problem extension restates a given problem with different data or a different question, you may wish to have students solve it immediately after solving the original problem. Each practice problem is similar to a group of two problems in Section 1. When students have solved both problems in that group, you can give them a similar practice problem from Section 2. However, you may wish to wait until the students have solved several groups of problems using different strategies before giving them the practice problems. That will make choosing a strategy more of a challenge.

The table below lists problems in *The Problem Solver 5* which are similar to the problems in *The Problem Solver 6*. The problems for fifth grade are less difficult and can be used with all students.

The Problem Solver 6		Similar Problems in The Problem Solver 5				
Problems	**Strategy Focus**	**Problems**				
1, 2	Use or make a table	1,	2,	51,	79,	103
3, 4	Make an organized list	3,	4,	49,	75,	99
5, 6	Act out or use objects	5,	6,	50,	78,	102
7, 8	Make an organized list	7,	8,	56,	82,	105
9, 10	Use or look for a pattern	9,	10,	53,	73,	97
11, 12	Use or make a table	11,	12,	59,	85,	108
13, 14	Make a picture or diagram	13,	14,	52,	77,	101
15, 16	Guess and check	15,	16,	54,	80,	100
17, 18	Work backwards	17,	18,	60,	81,	104
19, 20	Make a picture or diagram	19,	20,	57,	83,	98
21, 22	Use logical reasoning	21,	22,	55,	74,	106
23, 24	Use or make a table	23,	24,	64,	90,	112
25, 26	Make a picture or diagram	25,	26,	63,	88,	107
27, 28	Make an organized list	27,	28,	58,	84,	91
29, 30	Guess and check	29,	30,	61,	96,	113
31, 32	Make a picture or diagram	31,	32,	66,	93,	114
33, 34	Use or look for a pattern	33,	34,	62,	89,	111
35, 36	Use logical reasoning	35,	36,	65,	92,	115
37, 38	Act out or use objects	37,	38,	67,	94,	110
39, 40	Use logical reasoning	39,	40,	70,	109,	119
41, 42	Make it simpler	41,	42,	68,	95,	116
43, 44	Work backwards	43,	44,	69,	86,	118
45, 46	Make it simpler	45,	46,	72,	76,	117
47, 48	Brainstorm	47,	48,	71,	87,	120

If you wish to provide more practice for your students, the following books on problem solving are available from Creative Publications:

Brainstorming: Activities for Creative Thinking *Thinker Games*
Problem Solver Projects, Grade 6 *Think About It! Mathematic Activities of the Day*
Trivia Math: A Problem A Day *Thinker Tasks: Critical Thinking Activities*

TEACHING THE STRATEGIES

Part one of this section describes the four-step method and the ten solution strategies. Sample problems are used as examples to illustrate how the method and strategies can be implemented. Part two presents 48 teaching problems. For each problem there is a reproducible student page and a one-page teaching plan. The problems are organized in groups of two that focus on the same strategy. The symbol of that strategy is shown in the upper left-hand corner of both the student page and the teaching plan page. Please see pages vii –viii for some general teaching suggestions on presenting the problems to your students.

What Is the Four-Step Method?

The four-step method is a systematic approach to problem solving that can be used for solving any problem.

The first step is to FIND OUT what the problem means and what question you must answer to solve it. To find out what the problem means, you must understand the words and phrases used and what's happening in the problem. You must be able to identify the important information and the unimportant information, and determine if any necessary information is missing and what you must do to get that information. In some cases, the problem may need to be broken up into smaller problems before the larger problem can be solved. You should understand the problem well enough to say it in your own words. And finally, you must be able to state the question you have to answer to solve it.

The second step is to CHOOSE A STRATEGY that will help solve the problem. You will often find there is more than one strategy that can be used. The idea, however, is to find the strategy or strategies that will help you the most with a particular problem.

The third step is to SOLVE IT. Work through the problem until you find the answer to the question, using the strategy you selected. It is important that you record your work in a way that lets you see at a glance what you've completed. As you work to find the answer, you may find that the strategy you selected is not as helpful as you thought it might be. In that case, you will want to try a different strategy.

The fourth step is to LOOK BACK. Reread the problem and check the solution to see that it meets the conditions stated in the problem and that it answers the question. To review your solution and ask yourself if it's logical and reasonable is a very important step in problem solving.

What Are the Ten Solution Strategies?

ACT OUT OR USE OBJECTS

Some students may find it helpful to act out a problem or to move objects around while they are trying to solve a problem. It allows them to develop visual images of both the data in the problem and the solution process. By taking an active role in finding the solution, students are more likely to remember the process they used and be able to use it again for solving similar problems. The dramatizations and objects need not be elaborate: small scraps of paper and colored chips or counters will usually work quite well. This strategy is especially helpful when the problem solver wants to visualize relationships. For example:

Problem 5: Andre's Blue Ribbon Cars & Trucks is having a big sale. Mike and Don are setting up the lot. The boss gives them a diagram of the lot and these directions: "Put the 4-door car in front of the van. Put the jeep between the truck and the van. Put the sportscar to the left of the 2-door and 4-door cars." How did Mike and Don set up the lot?

truck	jeep	van
sportscar	2-door	4-door

or

sportscar	2-door	4-door
truck	jeep	van

MAKE A PICTURE OR DIAGRAM

For some children, it may be helpful to use an available picture or make a picture or diagram when trying to solve a problem. The pictures or diagrams need not be well drawn. It is most important that they help the problem solver understand and manipulate the data in the problem. Using diagrams is almost a necessity for some problems, particularly those which involve mapping. For example:

Problem 13: Gino and Mark had found all the things on the list for the treasure hunt and had only a few minutes to get to the finish point. But they were lost! Gino said, "When we were at the bridge, we were 2 blocks west of the finish point. Can you remember where we went after that?" Mark recalled that they had gone south 3 blocks, then they went to their left 5 blocks, left again for 2 blocks, then north for 1 block. What is the quickest route from where they are to the finish point?

Solution: West 3 blocks
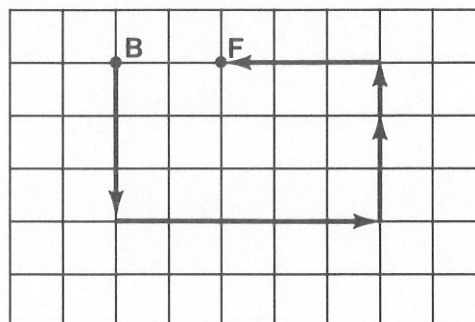
USE OR MAKE A TABLE

A table is an orderly arrangement of data, such as numbers. Problem solvers find that making tables helps them keep track of data, spot missing data, and identify data that is asked for in the problem. Because patterns often become obvious when data is organized in a table, this strategy is often used in conjunction with other strategies. In the example below, the table is used to keep track of data and could also be used for identifying a number pattern.

Problem 23: Travis works at the Fantasy in Flight Factory. He checks all the kites made in the factory before they are packaged. One day Travis discovered that for every 30 kites that passed inspection there were 7 kites that didn't pass: 4 kites without tails and 3 kites with the wrong colors. Of the 296 kites Travis examined, how many didn't have tails and how many had the wrong colors?

Solution: 32 without tails, 24 with the wrong colors

Pass	30	60	90	120	150	180	210	240
No tail	4	8	12	16	20	24	28	32
Wrong color	3	6	9	12	15	18	21	24
Total	37	74	111	148	185	222	259	296

MAKE AN ORGANIZED LIST

Making an organized list helps problem solvers organize their thinking about a problem. Recording work in an organized list makes it easy to review what has been done and to identify important steps that must yet be completed. It also provides a systematic way of recording computations made with given data or recording combinations of given items. For example:

Problem 3: James ran down the stairs to the subway and got in line for a ticket. If the line didn't move fast, he and Tony would be late for the movie. James fumbled in his pocket and finally found a dollar. He gave the dollar to the ticket man, got his change of 36 cents, and sprinted for the subway car. What are the possible combinations of coins that James could have received for his change of 36 cents?

Solution: 24

25¢	10¢	5¢	1¢		25¢	10¢	5¢	1¢
1	1	0	1			1	3	11
1	0	2	1			1	2	16
1	0	1	6			1	1	21
1	0	0	11			1	0	26
	3	1	1				7	1
	3	0	6				6	6
	2	3	1				5	11
	2	2	6				4	16
	2	1	11				3	21
	2	0	16				2	26
	1	5	1				1	31
	1	4	6					36

(continued)

?✓ GUESS AND CHECK

Guessing and checking is helpful when a problem presents large numbers or many pieces of data, or when the problem asks the solver to find one solution but not all possible solutions to a problem. When problem solvers use this strategy, they guess the answer, test to see if it is correct, and make another guess if the previous one was incorrect. In this way, they gradually come closer and closer to a solution by making increasingly more reasonable guesses. Problem solvers can also use this strategy to get started, and may then find another strategy which can be used. Guessing and checking is particularly helpful when a problem presents so many pieces of data that making an organized list becomes a major task. For example:

Problem 15: How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday? Joy came by skateboard, Heidi came by bike, and Saul came on the bus. It took Heidi twice as long as Joy to get there. It took Saul 10 minutes more than it took both the girls together. All three skaters together took 64 minutes to get to the rink.

Solution: Joy 9, Heidi 18, Saul 37

Guess: Joy 5
         Heidi 5 × 2 = 10
         Saul 10 + 10 + 5 = 25

Check: 5 + 10 + 25 = 40   Too low
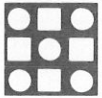
⬛ USE OR LOOK FOR A PATTERN

A pattern is a regular, systematic repetition. A pattern may be numerical, visual, or behavioral. By identifying the pattern, the problem solver can predict what will "come next" and what will happen again and again in the same way. Looking for patterns is a very important strategy for problem solving, and is used to solve many different kinds of problems. Sometimes students can solve a problem just by recognizing a pattern, but often they will have to extend a pattern to find a solution. Making a number table often reveals patterns, and for this reason is frequently used in conjunction with the "look for a pattern" strategy. For example:

Problem 9: On July 5, in the area around Center Village, there was great excitement. Six different people reported to the police that they had seen Bigfoot, the large hairy creature sometimes seen but never captured. The next day, twice as many people called the police, sure they had seen the creature. Each day the police received twice as many calls as the day before. After they got a total of more than 300 calls, the police took the phone off the hook! On what day did the police receive their 300th call?

Solution: 7

Day	1	2	3	4	5	6	7	8	
Number of calls	6	12	24	48	96	192	384		

◄ WORK BACKWARDS

To solve certain problems, the problem solver must make a series of computations, starting with data presented at the end of the problem and ending with data presented at the beginning of the problem. For example:

Problem 17: Kevin, Barbara, and their mother and father went backpacking in Yosemite National Park. On the first and second days, each hiker had a serving of food for breakfast, lunch, and dinner. A large, noisy, brown bear barged into camp the second night, got the food pack down from the tree where they had hung it, and ate one half of the food that was left. The next morning, after they all had breakfast, they found they had 4 food servings left. They decided they had better hike back to their car. How many servings of food did they begin the trip with?

Solution: 40 servings of food

Day 3 - 4 servings left
4 servings eaten by hikers

Day 2 - 8 servings eaten by bear
12 servings eaten by hikers

Day 1 - 12 servings eaten by hikers

USE LOGICAL REASONING

Logical reasoning is really used for all problem solving. However, there are types of problems that include or imply various conditional statements such as: "if . . . then," or "if . . . then . . . else," or "if something is true, then . . ." or "if something is not true, then . . . ." The data given in the problems can often be displayed in a chart or matrix. This kind of problem requires formal logical reasoning as the problem solver steps his or her way through the statements given in the problem. For example:

Problem 21: Bill, his sister Martha, and Ann are sitting down to learn a new game from Phil Hill. Each player has a partner and the partners are seated across the table from each other. Bill Dill is sitting to the right of Phil's sister. Phil Hill is sitting to the right of Bill's sister's partner. Where is each player sitting at the table?

Solution: Martha across from Bill, Phil on Bill's right and across from Ann

Martha Dill

Ann Hill   Phil Hill

Bill Dill
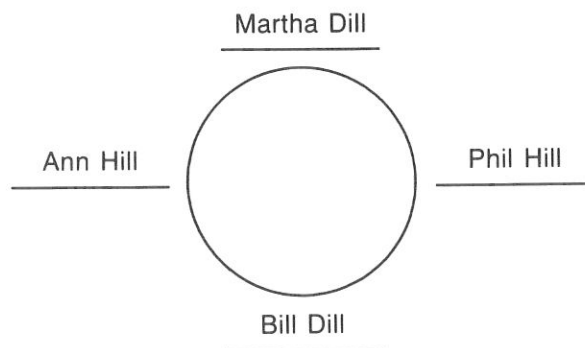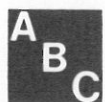
$\begin{array}{|c|}\hline A\\B\\C\\\hline\end{array}$ MAKE IT SIMPLER

Students will find it helpful to be able to make problems simpler, especially when they begin to solve complex problems. Making a problem simpler may mean reducing large numbers to small numbers, or reducing the number of items given in a problem. The simpler representation of the problem, then, may suggest what operation or process can be used to solve the more complex problem. The simpler representation may even reveal a pattern which can be used to solve the problem. For example:

Problem 41: A group of 13 friends were planning a trip. On the night before they left they made a lot of phone calls. Each friend talked to every other friend at least once. What is the fewest phone calls that could have been made?

Solution: 78

Number of friends	Calls
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66
13	78

BRAINSTORM

This strategy is often used when all else fails. When the problem solver cannot think of a similar problem that he or she has solved before, and cannot think of another strategy to use, brainstorming is a good strategy to try. Brainstorming means looking at a problem in new and inventive ways. There are always problems that stretch people beyond their experience and expertise. When students encounter problems that they cannot solve, they must be encouraged to open up, stretch, allow for inspiration, be creative, be flexible, and keep on trying until a light goes on! For example:

Problem 47: Juanita presented a problem to Karl. "If you can solve this," said Juanita, "I'll buy you the ice cream cone of your choice! Here's the problem: Show how one half of five is four." Karl got his ice cream cone. What was his answer?

Solution: F $\boxed{\text{I} \mid \text{V}}$ E

How Can You Teach the Four-Step Method and Solution Strategies?

Use the teaching plans. Read the problem aloud. Then begin the sequence of questions given in the teaching plan for the problem. Name each step as you begin it. Encourage the students to exchange a variety of ideas and opinions after each question is raised. The questions will lead the students through the four steps and serve as a model for them to follow when they must develop their own questions for solving later problems. The four-step method is illustrated below by the teaching plan for problem 5. Italicized responses following the questions are examples of the kinds of responses you want to elicit from students.

ACT OUT OR USE OBJECTS   Teaching Plan

5 Andre's Blue Ribbon Cars & Trucks is having a big sale. Mike and Don are setting up the lot. The boss gives them a diagram of the lot and these directions: "Put the 4-door car in front of the van. Put the jeep between the truck and the van. Put the sportscar to the left of the 2-door and 4-door cars." How did Mike and Don set up the lot?

FIND OUT
- What is the question you have to answer? *How did Mike and Don set up the lot?*
- How many cars and trucks are they setting up? *6* What are they? *4-door car, van, truck, 2-door car, jeep, sportscar*
- What do you know about the location of the 4-door car? *It goes in front of the van.*
- What do you know about the location of the jeep? *It is between the van and the truck.*
- What do you know about the location of the sportscar? *It is to the left of the 2-door and 4-door cars.*

CHOOSE A STRATEGY
- Would it help to have pieces of paper, or something to represent each car or truck, and be able to move them around? *Yes, using objects will let us change things easily when something doesn't work. We can experiment easily.*

SOLVE IT
- If you use pieces of paper, how would you label them? *Van, truck, 2-door, 4-door, jeep, sportscar*
- What goes in front of the van? *The 4-door car*
- What can you put on either side of the jeep? *The truck and the van*
- Where does the sportscar go? *To the left of the 4-door and 2-door cars*
- Now can you find a place for the 2-door car? *Between the 4-door car and sportscar*
- Is there more than one solution for this problem? *Yes, the rows can be reversed.*

Solution:

truck	jeep	van
sportscar	2-door	4-door

or

sportscar	2-door	4-door
truck	jeep	van

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your set-up of the lot. Is your answer reasonable?

EXTEND IT
- Make up a problem with 9 animals lined up in 3 rows.

When you have completed the four steps with the students, you can give them the problem extension to solve as further practice.

After your students have solved one or more groups of two problems, you can give them similar practice problems from Section 2. The practice problems related to a given group are identified in the teaching plan for the second problem in that group.

Recording Sheet

Name _____

Problem Number _____

FIND OUT
- What is the question you have to answer?
- What information does the problem give you?

CHOOSE A STRATEGY
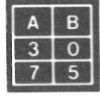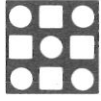
SOLVE IT

Solution:

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?
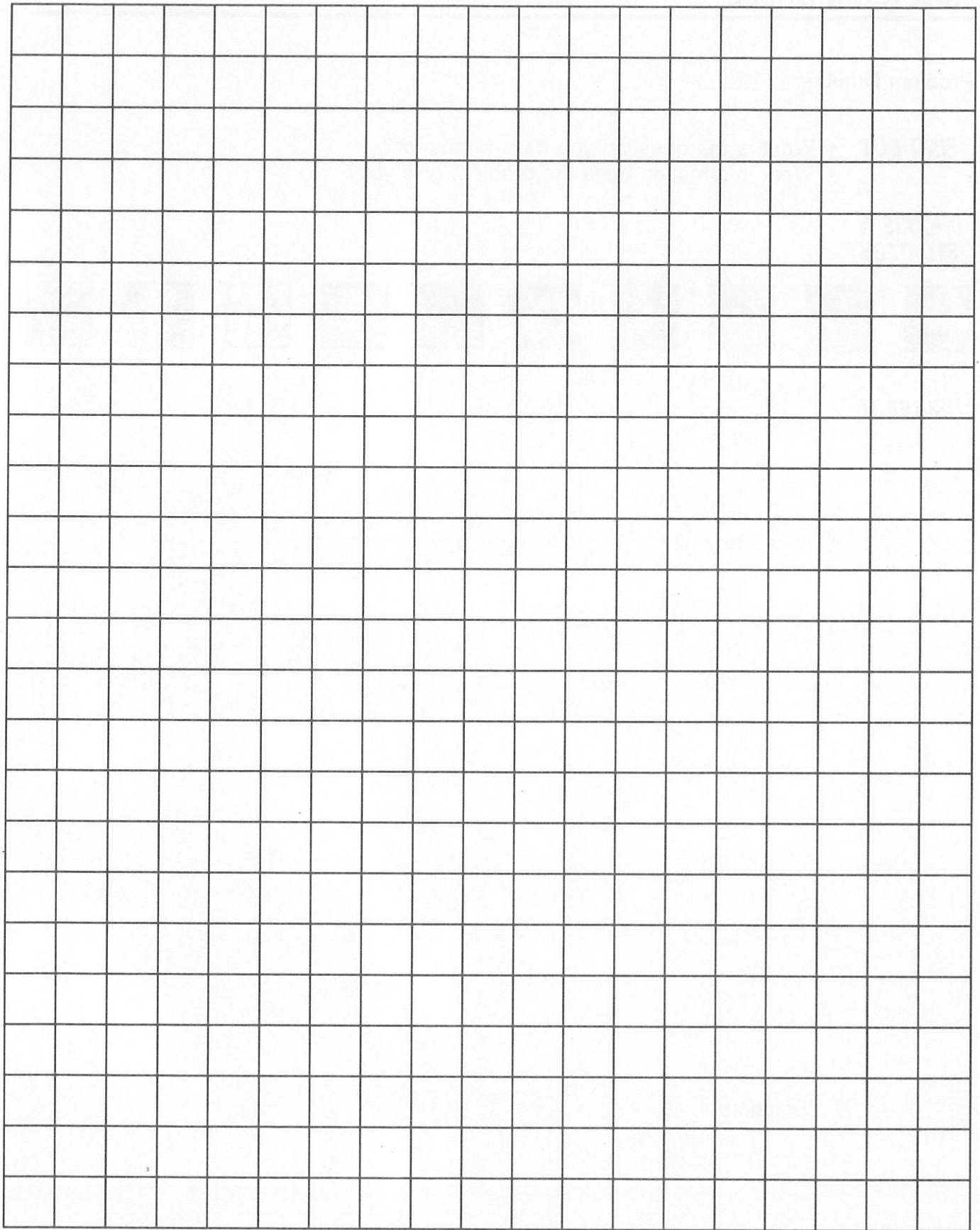
1 Ann and Makiko like to swim laps at the community center pool. They are swimming together today, but they are on different swim schedules. Ann swims every 3 days and Makiko swims every 5 days. How many times will they both be at the pool on the same day during the next ten weeks?

FIND OUT
- What is the question you have to answer?
- Are Makiko and Ann swimming laps together today? Do they always swim together?
- What is Ann's schedule?
- What is Makiko's schedule?

CHOOSE A STRATEGY
- What will be the next day that Ann is at the pool? What will be the next day that Makiko is at the pool?
- Is there a way to organize and lay out the information, to see which days Ann and Makiko are at the pool?

SOLVE IT
- When you make a table, what are you keeping track of in the first row?
- What are you keeping track of in the second row?
- What are you keeping track of in the third row?
- After day 3, what is the next day that Ann is at the pool? What is the next day that Makiko is at the pool after day 5?
- Do you need to add more days to the table?
- On which day are Ann and Makiko at the pool together again?
- How many days from this day will they be at the pool again on the same day? Do you see a pattern?
- How many days are there in ten weeks?
- How many times will this pattern repeat during ten weeks?
- How many times will they both be at the pool on the same day during the next ten weeks?

Day	1	2	3	4	5	6	7	8	9	10
Ann			X			X				
Makiko					X					

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

1   Ann and Makiko like to swim laps at the community center pool. They are swimming together today, but they are on different swim schedules. Ann swims every 3 days and Makiko swims every 5 days. How many times will they both be at the pool on the same day during the next ten weeks?

FIND OUT
- What is the question you have to answer? *How many times will they both be at the pool on the same day during the next ten weeks?*
- Are Makiko and Ann swimming laps together today? *Yes* Do they always swim together? *No*
- What is Ann's schedule? *She swims laps every 3 days.*
- What is Makiko's schedule? *She swims laps every 5 days.*

CHOOSE A STRATEGY
- What will be the next day that Ann is at the pool? *Day 3* What will be the next day that Makiko is at the pool? *Day 5*
- Is there a way to organize and lay out the information, to see which days Ann and Makiko are at the pool? *Yes, we can make a table.*

SOLVE IT
- When you make a table, what are you keeping track of in the first row? *The number of days*
- What are you keeping track of in the second row? *When Ann is at the pool*
- What are you keeping track of in the third row? *When Makiko is at the pool*
- After day 3, what is the next day that Ann is at the pool? *Day 6* What is the next day that Makiko is at the pool after day 5? *Day 10*
- Do you need to add more days to the table? *Yes, to find a day when they are there together*
- On which day are Ann and Makiko at the pool together again? *Day 15*
- How many days from this day will they be at the pool again on the same day? *Day 30* Do you see a pattern? *Yes, every 15 days they will be there together*
- How many days are there in ten weeks? *70*
- How many times will this pattern repeat during ten weeks? *4*
- How many times will they both be at the pool on the same day during the next ten weeks? *4*

Solution: 4

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ann			×			×			×			×			×
Makiko					×					×					×

| Day	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ann			×			×			×			×			×
Makiko					×					×					×

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT
- If Ann and Makiko keep the same schedules and are both at the pool on January 1, how many times will they be at the pool together in the first 6 months of the year?

2 It's here — the hottest item of the season, Rodney the Talking Robot! Rodney can keep anyone company, including the dog or cat. When the robots are made at the factory, they move along on a conveyor belt and every 8th robot gets a blue control panel, every 3rd robot gets blinking green eyes, and every 4th robot gets a square head. If 150 robots come off the conveyor belt, how many will have all three: a blue control panel, blinking green eyes, and a square head?

FIND OUT
- What is the question you have to answer?
- What is happening to the robots as they move along the conveyor belt? Does every robot get one of the three things on it?
- Which robots get blinking green eyes?
- Which robots get a blue control panel?
- Which robots get a square head?

CHOOSE A STRATEGY
- Does the first robot get a blue control panel, blinking green eyes, or a square head? Does the second robot? Does the third robot?
- Is there a way to organize and lay out the information, so you can "see" which robots get a blue control panel, blinking green eyes, or a square head?

SOLVE IT
- When you make a table, what are you keeping track of in the first row?
- What are you keeping track of in the second row?
- What are you keeping track of in the third row?
- What are you keeping track of in the fourth row?
- If you look at the table, which is the first robot with a blue control panel? Which robot has the next blue control panel?
- Which is the first robot with blinking green eyes? Which is the next robot with green eyes?
- Which is the first robot with a square head? Which is the next robot with a square head?
- Do you need to continue the table?
- Which robot is the first to have all three: a blue control panel, blinking green eyes, and a square head?
- Which robot will be the next one to have all three? Do you see a pattern?
- How many times will the pattern repeat for 150 robots?
- How many robots out of 150 will have a blue control panel, blinking green eyes, and a square head?

Robot	1	2	3	4	5	6
Control panel						
Green eyes			✕			✕
Square head				✕		

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

2

It's here — the hottest item of the season, Rodney the Talking Robot! Rodney can keep anyone company, including the dog or cat. When the robots are made at the factory, they move along on a conveyor belt and every 8th robot gets a blue control panel, every 3rd robot gets blinking green eyes, and every 4th robot gets a square head. If 150 robots come off the conveyor belt, how many will have all three: a blue control panel, blinking green eyes, and a square head?

FIND OUT
- What is the question you have to answer? *If 150 robots come off the conveyor belt, how many will have all three: a blue control panel, blinking green eyes, and a square head?*
- What is happening to the robots as they move along the conveyor belt? *They get a blue control panel, blinking green eyes, or a square head.* Does every robot get one of the three things on it? *No*
- Which robots get blinking green eyes? *Every 3rd robot*
- Which robots get a blue control panel? *Every 4th robot*
- Which robots get a square head? *Every 8th robot*

CHOOSE A STRATEGY
- Does the first robot get a blue control panel, blinking green eyes, or a square head? *No* Does the second robot? *No* Does the third robot? *Yes, it gets blinking green eyes.*
- Is there a way to organize and lay out the information, so you can "see" which robots get a blue control panel, blinking green eyes, or a square head? *Yes, we can make a table.*

SOLVE IT
- When you make a table, what are you keeping track of in the first row? *The number of the robot*
- What are you keeping track of in the second row? *The robots that get blue control panels*
- What are you keeping track of in the third row? *The robots that get blinking green eyes*
- What are you keeping track of in the fourth row? *The robots that get square heads*
- If you look at the table, which is the first robot with a blue control panel? *8* Which is the next robot with a blue control panel? *16*
- Which is the first robot with blinking green eyes? *3* Which is the next robot with green eyes? *6*
- Which is the first robot with a square head? *4* Which is the next robot with a square head? *8*
- Do you need to continue the table? *Yes, because there isn't a robot with all three things yet.*
- (Have the students continue to fill in the table until they find a robot with all three things.)
- Which robot is the first to have all three: a blue control panel, blinking green eyes, and a square head? *24*
- Which robot will be the next robot to have all three? *48* Do you see a pattern? *Yes, every 24th robot will have all three.*
- How many times will the pattern be repeated for 150 robots? *6*
- How many robots out of 150 will have all three: a blue control panel, blinking green eyes, and a square head? *6*

Solution: 6

Robot	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Control panel								×								×								×
Green eyes			×			×			×			×			×			×			×			×
Square head				×				×				×				×				×				×

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT
- If every 9th robot gets a blue control panel, every 3rd robot blinking green eyes, and every 5th robot a square head, how many robots out of 200 will have all three?

PRACTICE
- Similar Practice Problems: 49, 56, 91

3   James ran down the stairs to the subway and got in line for a ticket. If the line didn't move fast, he and Tony would be late for the movie. James fumbled in his pocket and finally found a dollar. He gave the dollar to the ticket man, got his change of 36 cents, and sprinted for the subway car. What are the possible combinations of coins that James could have received for his change of 36 cents?

FIND OUT
- What is the question you have to answer?
- What did James give the ticket man?
- What did the ticket man give to James?

CHOOSE A STRATEGY
- What is one possible combination of coins that makes 36 cents? Can you think of another combination?
- How can you systematically record all the possible combinations of coins that total 36 cents?

SOLVE IT
- What are the different kinds of coins that could have been in the change that James got?
- Make a list. What do you want to keep track of in the first column of your list? second column? third column? fourth column?
- Begin the list with 25 cents. In the first row of the list, 1 quarter is combined with 1 dime and 1 penny to make 36 cents. Is there any other way to combine 1 quarter with other coins to make 36 cents?
- Finish filling in the possible ways to combine a quarter with other coins to make 36 cents. Then move to the dimes. What is the largest number of dimes he could have received?
- Finish your list. What are the possible combinations of coins that James could have received for his change of 36 cents?

25¢	10¢	5¢	1¢
1	1	0	1

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?

3 James ran down the stairs to the subway and got in line for a ticket. If the line didn't move fast, he and Tony would be late for the movie. James fumbled in his pocket and finally found a dollar. He gave the dollar to the ticket man, got his change of 36 cents, and sprinted for the subway car. What are the possible combinations of coins that James could have received for his change of 36 cents?

FIND OUT
- What is the question you have to answer? *What are the possible combinations of coins that James could have received for his change of 36 cents?*
- What did James give the ticket man? *A dollar*
- What did the ticket man give to James? *36 cents*

CHOOSE A STRATEGY
- What is one possible combination of coins that makes 36 cents? *1 quarter, 1 dime, and 1 penny* Can you think of another combination? *1 quarter, 2 nickels, and 1 penny*
- How can you systematically record all the possible combinations of coins that total 36 cents? *We can make an organized list.*

SOLVE IT
- What are the different kinds of coins that could have been in the change that James got? *Quarter, dime, nickel, penny*
- Make a list. What do you want to keep track of in the first column of your list? *Quarters* second column? *Dimes* third column? *Nickels* fourth column? *Pennies*
- Begin the list with 25 cents. In the first row of the list, 1 quarter is combined with 1 dime and 1 penny to make 36 cents. Is there any other way to combine 1 quarter with other coins to make 36 cents? *Yes, 1 quarter, 2 nickels, and 1 penny*
- Finish filling in the possible ways to combine a quarter with other coins to make 36 cents. Then move to the dimes. What is the largest number of dimes he could have received? *3*
- Finish your list. How many different combinations of coins could have been in the change that James received? *24*

Solution: 24

25¢	10¢	5¢	1¢	25¢	10¢	5¢	1¢
1	1	0	1		1	3	11
1	0	2	1		1	2	16
1	0	1	6		1	1	21
1	0	0	11		1	0	26
	3	1	1			7	1
	3	0	6			6	6
	2	3	1			5	11
	2	2	6			4	16
	2	1	11			3	21
	2	0	16			2	26
	1	5	1			1	31
	1	4	6				36

(continued)

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?

EXTEND IT
- If the ticket man had given James 46 cents, how many combinations of coins could there have been?

4   Senta and Maurice are lab partners in science class. Today they have to weigh liquid. They have a tray full of 80 weights which they can use. The weights are of four different kinds: 50 grams, 25 grams, 15 grams, and 5 grams. The first liquid weighs 85 grams. How many different combinations of weights will balance the scale for the first liquid?

FIND OUT
- What is the question you have to answer?
- What are Senta and Maurice weighing?
- How many different kinds of gram weights do they have? What are the different kinds?
- What does the first liquid weigh?

CHOOSE A STRATEGY
- If you begin with a 50-gram weight, what can you add to this to make 85 grams? Is there another combination using a 50-gram weight?
- How can you systematically record all the possible combinations of weights that total 85 grams?

SOLVE IT
- Make a list. What do you want to keep track of in the first column of your list? second column? third column? fourth column?
- If you begin with a 50-gram weight, how many different combinations can you find that make 85 grams?
- What is the largest number of 25-gram weights you could use? How many combinations can you find using 25-gram weights with other weights?
- Finish your list. How many different combinations of weights balance the scale for the first liquid?

50	25	15	5
1	1	0	2
1	0	2	1

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?

4 Senta and Maurice are lab partners in science class. Today they have to weigh liquid. They have a tray full of 80 weights which they can use. The weights are of four different kinds: 50 grams, 25 grams, 15 grams, and 5 grams. The first liquid weighs 85 grams. How many different combinations of weights will balance the scale for the first liquid?

FIND OUT
- What is the question you have to answer? *How many different combinations of weights will balance the scale for the first liquid?*
- What are Senta and Maurice weighing? *Liquid*
- How many different kinds of gram weights do they have? *4* What are the different kinds? *50 gram, 25 gram, 15 gram, 5 gram*
- What does the first liquid weigh? *85 grams*

CHOOSE A STRATEGY
- If you begin with a 50-gram weight, what can you add to this to make 85 grams? *One 25-gram weight and two 5-gram weights* Is there another combination using a 50-gram weight? *Yes, with two 15-gram weights and one 5-gram weight*
- How can you systematically record all the possible combinations of weights that total 85 grams? *We can make an organized list.*

SOLVE IT
- Make a list. What do you want to keep track of in the first column of your list? *50-gram weights* second column? *25-gram weights* third column? *15-gram weights* fourth column? *5-gram weights*
- If you begin with a 50-gram weight, how many different combinations can you find that make 85 grams? *4*
- What is the largest number of 25-gram weights you could use? *3* How many combinations can you find using 25-gram weights with other weights? *9*
- Finish your list. How many different combinations of weights balance the scale for the first liquid? *19*

Solution: 19

50	25	15	5	50	25	15	5
1	1	0	2		1	2	6
1	0	2	1		1	1	9
1	0	1	4		1	0	12
1	0	0	7			5	2
	3	0	2			4	5
	2	2	1			3	8
	2	1	4			2	11
	2	0	7			1	14
	1	4	0				17
	1	3	3				

(continued)

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your organized list. Is your answer reasonable?

EXTEND IT
- If the first liquid weighed 105 grams, how many different ways could they combine the weights to balance the scale?

PRACTICE
- Similar Practice Problems: 60, 63, 113

5 Andre's Blue Ribbon Cars & Trucks is having a big sale. Mike and Don are setting up the lot. The boss gives them a diagram of the lot and these directions: "Put the 4-door car in front of the van. Put the jeep between the truck and the van. Put the sportscar to the left of the 2-door and 4-door cars." How did Mike and Don set up the lot?

FIND OUT
- What is the question you have to answer?
- How many cars and trucks are they setting up? What are they?
- What do you know about the location of the 4-door car?
- What do you know about the location of the jeep?
- What do you know about the location of the sportscar?

CHOOSE A STRATEGY
- Would it help to have pieces of paper, or something to represent each car or truck, and be able to move them around?

SOLVE IT
- If you use pieces of paper, how would you label them?
- What goes in front of the van?
- What can you put on either side of the jeep?
- Where does the sportscar go?
- Now can you find a place for the 2-door car?
- Is there more than one solution for this problem?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your set-up of the lot. Is your answer reasonable?

T•9

5   Andre's Blue Ribbon Cars & Trucks is having a big sale. Mike and Don are setting up the lot. The boss gives them a diagram of the lot and these directions: "Put the 4-door car in front of the van. Put the jeep between the truck and the van. Put the sportscar to the left of the 2-door and 4-door cars." How did Mike and Don set up the lot?

FIND OUT
- What is the question you have to answer? *How did Mike and Don set up the lot?*
- How many cars and trucks are they setting up? *6* What are they? *4-door car, van, truck, 2-door car, jeep, sportscar*
- What do you know about the location of the 4-door car? *It goes in front of the van.*
- What do you know about the location of the jeep? *It is between the van and the truck.*
- What do you know about the location of the sportscar? *It is to the left of the 2-door and 4-door cars.*

CHOOSE A STRATEGY
- Would it help to have pieces of paper, or something to represent each car or truck, and be able to move them around? *Yes, using objects will let us change things easily when something doesn't work. We can experiment easily.*

SOLVE IT
- If you use pieces of paper, how would you label them? *Van, truck, 2-door, 4-door, jeep, sportscar*
- What goes in front of the van? *The 4-door car*
- What can you put on either side of the jeep? *The truck and the van*
- Where does the sportscar go? *To the left of the 4-door and 2-door cars*
- Now can you find a place for the 2-door car? *Between the 4-door car and sportscar*
- Is there more than one solution for this problem? *Yes, the rows can be reversed.*

Solution:

truck	jeep	van
sportscar	2-door	4-door

or

sportscar	2-door	4-door
truck	jeep	van

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your set-up of the lot. Is your answer reasonable?

EXTEND IT
- Make up a problem with 9 animals lined up in 3 rows.

6 You have been hired to supervise 22 fourth-graders who are visiting a haunted house at Halloween. You have a diagram of the house showing the "control booth" where you will be sitting, hidden behind four two-way mirrors. One mirror is on each wall, and each mirror looks into three rooms. Mirror 1 looks into rooms A, B, C; mirror 2 looks into rooms C, E, H; mirror 3 looks into rooms F, G, H; mirror 4 looks into rooms A, D, F. What is a possible arrangement of the 22 students within the rooms, if you can see nine students through each of the four mirrors?

FIND OUT
- What is the question you have to answer?
- Where are you sitting?
- How many mirrors can you look through? How many rooms can you see through each mirror? Which rooms can you see through each mirror?
- How many students can you see through each mirror? How many students are there altogether?
- How many rooms are there altogether?

CHOOSE A STRATEGY
- Would it help to have pieces of paper to represent different numbers that you could move around?

SOLVE IT
- How many students are there? What is the total number of students you can see from any mirror?
- What are some combinations of numbers that add up to 9?
- When you cut up paper, do you want 22 pieces or would it be easier to have a piece represent one of the numbers from a combination that adds up to 9? How would you label the pieces of paper?
- Begin with the rooms you can see from mirror 1. What do the numbers in these rooms have to add up to? What is a possible arrangement?
- Now go to mirror 2. What number is already in room C? What can you put in rooms E and H?
- If you go to mirror 3, what number is in room H? What numbers are you going to put in rooms F and G?
- Now, what number has to go in room D? Do the numbers in all the rooms add up to 22?
- Keep experimenting by moving the numbers around, add new pieces of paper if necessary. Can you find an arrangement that works?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

6 You have been hired to supervise 22 fourth-graders who are visiting a haunted house at Halloween. You have a diagram of the house showing the "control booth" where you will be sitting, hidden behind four two-way mirrors. One mirror is on each wall, and each mirror looks into three rooms. Mirror 1 looks into rooms A, B, C; mirror 2 looks into rooms C, E, H; mirror 3 looks into rooms F, G, H; mirror 4 looks into rooms A, D, F. What is a possible arrangement of the 22 students within the rooms, if you can see nine students through each of the four mirrors?
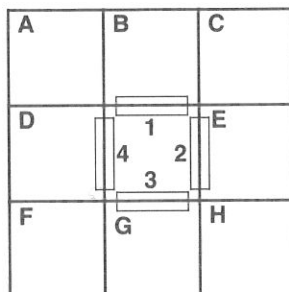
FIND OUT
- What is the question you have to answer? *What is a possible arrangement of the 22 students within the rooms, if you can see nine students through each of the four mirrors?*
- Where are you sitting? *In the control booth in the center of the house*
- How many mirrors can you look through? *4* How many rooms can you see through each mirror? *3* Which rooms can you see through each mirror? *1- A, B, C 2- C, E, H 3- F, G, H 4- A, D, F*
- How many students can you see through each mirror? *9* How many students are there altogether? *22*
- How many rooms are there altogether? *8*

CHOOSE A STRATEGY
- Would it help to have pieces of paper, or something to represent different numbers that you could move around? *Yes, if we can move objects around, then we can change things and experiment more easily.*

SOLVE IT
- (This is just an example of one way to begin this problem and some possible responses. You will get many different kinds of responses. Encourage experimentation!)
- How many students are there? *22* What is the total number of students you can see from any mirror? *9*
- What are some combinations of three numbers that add up to 9? *3 + 3 + 3, 4 + 3 + 2, 2 + 5 + 2*
- When you cut up paper, do you want 22 pieces or would it be easier to have a piece represent one of the numbers from a combination that adds up to 9? *It would be easier to have one piece of paper with a 2 or 3 on it, than 22 different pieces.* How would you label the pieces of paper? *Several of each of these numbers: 1, 2, 3, 4, 5*
- Begin with the rooms you can see from mirror 1. What do the numbers in these rooms have to add up to? *9* What is a possible arrangement? *3-3-3*
- Now go to mirror 2. What number is already in room C? *3* What can you put in rooms E and H? *E- 1, H- 5*
- If you go to mirror 3, what number is in room H? *5* What numbers are you going to put in rooms F and G? *G- 3, F- 1*
- Now, what number has to go in room D? *5* Why? *Because there are already numbers in A and F, and the three numbers have to add up to 9.* Do the numbers in all the rooms add up to 22? *No, 24*
- (Suggest that students keep experimenting by moving the numbers around. They may also want to make more pieces of paper. How many different ways can they do this?)

Solution:

4	2	3
2		2
3	2	4

3	1	5
3		1
3	3	3

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 51, 64, 89

7 Tanya and Mona are in the Big Beast Booth at the county fair. Tanya buys a ticket that gives her three throws with a bean bag at the bears. There are four bears, each with a different number: 1, 2, 3, and 4. Each time she hits a bear Tanya gets the number of points printed on the bear. After three throws the points are added up and she gets a prize for any score over 0. Tanya's score is 4. How many different ways could Tanya have scored 4 points?

FIND OUT
- What is the question you have to answer?
- How many bears are in the booth?
- How are the bears numbered?
- How many throws does Tanya get?
- What are the possible scores for one throw? What if Tanya doesn't hit any of the bears?
- What is Tanya's final score?

CHOOSE A STRATEGY
- If Tanya gets 1 on the first throw, 1 on the second throw, and 2 on the third throw, is it the same as getting 1 on the first throw, 2 on the second throw, and 1 on the third throw? Is there another way Tanya can score 4 points with the numbers 1, 1, and 2?
- Is there a systematic way to record all the possible ways Tanya can score 4 points?

SOLVE IT
- What are the possible scores for one throw?
- What are the possible combinations of three numbers that equal 4 points?
- One way to set up an organized list is to have three columns labeled Throw 1, Throw 2, and Throw 3. Begin with 0 for Throw 1, 1 for Throw 2, and 3 for Throw 3. Is there another way to arrange these numbers, using 0 for Throw 1 again?
- Now use 1 for Throw 1. How many ways can you list these same numbers, using 1 for Throw 1?
- Now use 3 for Throw 1. How many ways can you list the same numbers?
- Finish your list. How many ways did you find that Tanya could score 4 points?

Combinations of 4	Throw 1	Throw 2	Throw 3
0 + 1 + 3	0	1	3
0 + 4 + 0	0	3	1
	1	0	3

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

7 Tanya and Mona are in the Big Beast Booth at the county fair. Tanya buys a ticket that gives her three throws with a bean bag at the bears. There are four bears, each with a different number on it: 1, 2, 3, and 4. Each time Tanya hits a bear she gets the number of points printed on the bear. After three throws the points are added up and she gets a prize for any score over 0. Tanya's score is 4. How many different ways could Tanya have scored 4 points?

FIND OUT
- What is the question you have to answer? *How many different ways can Tanya score 4 points?*
- How many bears are in the booth? *4*
- How are the bears numbered? *1, 2, 3, or 4*
- How many throws does Tanya get? *3*
- What are the possible scores for one throw? *1, 2, 3, or 4* What if Tanya doesn't hit any of the bears? *She gets a 0.*
- What is Tanya's final score? *4*

CHOOSE A STRATEGY
- If Tanya gets 1 on the first throw, 1 on the second throw, and 2 on the third throw, is it the same as getting 1 on the first throw, 2 on the second throw, and 1 on the third throw? *No, the numbers are the same but in a different order.* Is there another way Tanya can score 4 points with the numbers 1, 1, and 2? *Yes, 1-2-1, or 2-1-1.*
- Is there a systematic way to record all the possible ways Tanya can score 4 points? *Yes, we can make an organized list.*

SOLVE IT
- What are the possible scores for one throw? *0, 1, 2, 3, or 4*
- What are the possible combinations of three numbers that equal 4 points? *0+1+3, 0+4+0, 1+1+2, 2+2+0*
- One way to set up an organized list is to have three columns labeled Throw 1, Throw 2, and Throw 3. Begin with 0 for Throw 1, 1 for Throw 2, and 3 for Throw 3. Is there another way to arrange these numbers, using 0 for Throw 1 again? *Yes, 0-3-1*
- Now use 1 for Throw 1. How many ways can you list these same numbers, using 1 for Throw 1? *1-0-3, 1-3-0*
- Now use 3 for Throw 1. How many ways can you list the same numbers? *2*
- (Have the students finish filling in their lists.) How many ways did you find that Tanya could score 4 points? *15*

Solution: 15

Combinations of 4
0 + 1 + 3
0 + 4 + 0
1 + 1 + 2
2 + 2 + 0

Throw 1	Throw 2	Throw 3
0	1	3
0	3	1
1	0	3
1	3	0
3	0	1
3	1	0
0	4	0
0	0	4

Throw 1	Throw 2	Throw 3
4	0	0
1	1	2
1	2	1
2	1	1
2	2	0
2	0	2
0	2	2

(continued)

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

EXTEND IT
- If there are 5 bears, numbered 1–5, how many different ways can Tanya score 5 points?

8   Dinner is over, and it is the twins' turn to clean up and do the dishes. Jaime grabs the dishtowel and waits for Josh to start washing the dishes. Instead of fighting about who has to wash, Josh suggests a game to settle the dispute. He hands Jaime a die whose six faces are marked 1, 2, 3, 4, 5, and 6. To play the game, Josh explains, they take turns rolling the die 3 times in a row. The first one to roll a total of 12 in 3 rolls gets to dry tonight. How many different ways can the twins combine 3 rolls of the die to total 12?

FIND OUT
- What is the question you have to answer?
- How many dice do they have?
- How is the die marked?
- How many rolls does each player get?
- What are the possible scores for one roll?
- What is the winning total for 3 rolls?

CHOOSE A STRATEGY
- If Jaime gets 2 on the first roll, 5 on the second roll, and 5 on the third roll, is it the same as getting 5 on the first roll, 2 on the second roll, and 5 on the third roll? Is there another way Jaime can score 12 points with the numbers 2, 5, and 5?
- Is there a systematic way to record all the possible ways Jaime and Josh can roll 12 points?

SOLVE IT
- What are the possible scores for one roll?
- What are the possible combinations of numbers that equal 12 points?
- One way to set up an organized list is to have three columns labeled Roll 1, Roll 2, and Roll 3. Begin with 1 for Roll 1, 5 for Roll 2, and 6 for Roll 3. Is there another way to arrange these numbers, using 1 for Roll 1?
- Now use 5 for 1. How many ways can you list these same numbers, using 5 for Roll 1?
- Now use 6 for Roll 1, 5 for Roll 2, and 1 for Roll 3. How many different ways can you list these three numbers?
- Finish making your list. How many ways can the players score 12 points?

Combinations of 12
1 + 5 + 6
2 + 5 + 5

Roll 1	Roll 2	Roll 3
1	5	6
1	6	5
5	1	6

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

8 Dinner is over, and it is the twins' turn to clean up and do the dishes. Jaime grabs the dishtowel and waits for Josh to start washing the dishes. Instead of fighting about who has to wash, Josh suggests a game to settle the dispute. He hands Jaime a die whose six faces are marked 1, 2, 3, 4, 5, and 6. To play the game, Josh explains, they take turns rolling the die 3 times in a row. The first one to roll a total of 12 in 3 rolls gets to dry tonight. How many different ways can the twins combine 3 rolls of the die to total 12?

FIND OUT
- What is the question you have to answer? *How many different ways can the twins combine 3 rolls of the die to total 12?*
- How many dice do they have? *1*
- How is the die marked? *1, 2, 3, 4, 5, 6*
- How many rolls does each player get? *3*
- What are the possible scores for one roll? *1, 2, 3, 4, 5, or 6*
- What is the winning total for 3 rolls? *12*

CHOOSE A STRATEGY
- If Jaime gets 2 on the first roll, 5 on the second roll, and 5 on the third roll, is it the same as getting 5 on the first roll, 2 on the second roll, and 5 on the third roll? *No* Is there another way Jaime can score 12 points with the numbers 2, 5, and 5? *Yes, 5-5-2*
- Is there a systematic way to record all the possible ways Jaime and Josh can roll 12 points? *Yes, we can make an organized list.*

SOLVE IT
- What are the possible scores for one roll? *1, 2, 3, 4, 5, or 6*
- What are the possible combinations of numbers that equal 12 points? *1+5+6, 2+5+5, 3+4+5, 2+4+6, 3+3+6, 4+4+4*
- One way to set up an organized list is to have three columns labeled Roll 1, Roll 2, and Roll 3. Begin with 1 for Roll 1, 5 for Roll 2, and 6 for Roll 3. Is there another way to arrange these numbers, using 1 for Roll 1? *Yes, 1-6-5*
- Now use 5 for 1. How many ways can you list these same numbers, using 5 for Roll 1? *5-6-1, 5-1-6*
- Now use 6 for Roll 1, 5 for Roll 2, and 1 for Roll 3. How many different ways can you list these three numbers? *6-5-1, 6-1-5*
- (Have students finish filling in their tables.) How many ways can the players score 12 points? *25*

Solution: 25

Combinations of 12
```
1 + 5 + 6
2 + 5 + 5
3 + 4 + 5
2 + 4 + 6
3 + 3 + 6
4 + 4 + 4
```

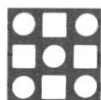| Roll 1	Roll 2	Roll 3	Roll 1	Roll 2	Roll 3
1	5	6	5	3	4
1	6	5	5	4	3
5	1	6	2	4	6
5	6	1	2	6	4
6	1	5	4	2	6
6	5	1	4	6	2
2	5	5	6	4	2
5	5	2	6	2	4
5	2	5	3	3	6
3	4	5	3	6	3
3	5	4	6	3	3
4	3	5	4	4	4
4	5	3			

(continued)

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

EXTEND IT
- What if the winning total for 3 rolls was 8?

PRACTICE
- Similar Practice Problems: 61, 76, 100

9 On July 5, in the area around Center Village, there was great excitement. Six different people reported to the police that they had seen Bigfoot, the large hairy creature sometimes seen but never captured. The next day, twice as many people called the police, sure they had seen the creature. Each day the police received twice as many calls as the day before. After they got a total of more than 300 calls, the police took the phone off the hook! On what day did the police receive their 300th call?

FIND OUT
- What is the question you have to answer?
- How many people first called the police?
- How did the number of calls change from the first day to the second day?
- Did the rate of increase stay the same from day to day?
- When did the police take the phone off the hook?

CHOOSE A STRATEGY
- How did the calls increase each day?
- Can you use this rate of increase to help you solve the problem?
- Is there a systematic way to record the information?

SOLVE IT
- If you set up a table, what are you keeping track of in the top row?
- What are you keeping track of in the second row?
- How many calls did the police get on day 1?
- If the number of calls doubled, how many did they get on day 2?
- How many calls did the police get on day 3?
- Continue filling in your table. On what day did the police receive their 300th call?

Day	1	2	3	4	5	
Number of calls	6	12	24			
Total	6	18				

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of increase. Is your answer reasonable?

9   On July 5, in the area around Center Village, there was great excitement. Six different people reported to the police that they had seen Bigfoot, the large hairy creature sometimes seen but never captured. The next day, twice as many people called the police, sure they had seen the creature. Each day the police received twice as many calls as the day before. After they got a total of more than 300 calls, the police took the phone off the hook! On what day did the police receive their 300th call?

FIND OUT
- What is the question you have to answer? *On what day did the police receive their 300th call?*
- How many people first called the police? *6*
- How did the number of calls change from the first day to the second day? *There were twice as many calls on the second day.*
- Did the rate of increase stay the same from day to day? *Yes*
- When did the police take the phone off the hook? *After they got more than 300 calls*

CHOOSE A STRATEGY
- How did the calls increase each day? *Twice as many*
- Can you use this rate of increase to help you solve the problem? *Yes, we can use this pattern to figure out how many calls the police got each day.*
- Is there a systematic way to record the information? *Yes, we can make a table.*

SOLVE IT
- If you set up a table, what are you keeping track of in the top row? *The number of days*
- What are you keeping track of in the second row? *The number of phone calls*
- How many calls did the police get on day 1? *6*
- If the number of calls doubled, how many did they get on day 2? *12*
- How many calls did the police get on day 3? *24*
- (Have the students continue to fill in their tables, by using the pattern of increase.) On what day did the police receive their 300th call? *6*

Solution: 6

Day	1	2	3	4	5	6	7	8	
Number of calls	6	12	24	48	96	192			
Total	6	18	42	90	186	378			

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of increase. Is your answer reasonable?

EXTEND IT
- What if the number of calls tripled each day?
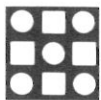
10 On the ancient island of Circulus archaeologists found a series of caves. In the first cave they found a circle with 560 stones. In the second cave they found a circle of stones with 8 fewer stones than in the first cave. Each new cave had a circle with twice as many stones missing from the circle than in the previous cave. For example, in the third cave there were 16 fewer stones in the circle than in the second cave. If the stone circles continued in this way, which cave would have the last stone circle?

FIND OUT
- What is the question you have to answer?
- What did the archaeologists find?
- How many stones were in the circle in the first cave?
- What was the difference in the circles between the first and second caves?
- What did the archaeologists notice about each new cave?
- What was the difference in the circles between the second and third caves?

CHOOSE A STRATEGY
- How did the circles decrease from cave to cave?
- Can you use this rate of decrease to help you solve the problem?
- How can you organize and record the information?

SOLVE IT
- When you set up your table, what are you keeping track of in the first row? In the second row? In the third row?
- What is the pattern of decrease in the stone circles?
- How many stones are in the circle in cave 1?
- How many fewer stones are in the circle in cave 2? How many stones are in the circle?
- How many fewer stones are in cave 3 than in cave 2? How many stones are in the circle?
- Continue filling in your table. What cave will have the last stone circle?

Cave	1	2	3	4	5		
Number fewer		8	16	32			
Total in circle	560	552					

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of decrease. Is your answer reasonable?

10 On the ancient island of Circulus archaeologists found a series of caves. In the first cave they found a circle with 560 stones. In the second cave they found a circle of stones with 8 fewer stones than in the first cave. Each new cave had a circle with twice as many stones missing from the circle than in the previous cave. For example, in the third cave there were 16 fewer stones in the circle than in the second cave. If the stone circles continued in this way, which cave would have the last stone circle?

FIND OUT
- What is the question you have to answer? *Which cave would have the last stone circle?*
- What did the archaeologists find? *Caves with stone circles in them*
- How many stones were in the circle in the first cave? *560*
- What was the difference in the circles between the first and second caves? *There were 8 fewer stones in the circle in the second cave.*
- What did the archaeologists notice about each new cave? *There were twice as many stones missing from the circle than in the previous cave.*
- What was the difference in the circles between the second and third caves? *There were 16 fewer stones in the third cave.*

CHOOSE A STRATEGY
- How did the circles decrease from cave to cave? *By twice as many as the previous circle*
- Can you use this rate of decrease to help you solve the problem? *Yes, this is a pattern of decrease that we can use to find how many stones are in each cave.*
- How can you organize and record the information? *We can use a table.*

SOLVE IT
- When you set up your table, what are you keeping track of in the first row? *The number of the cave* In the second row? *How many fewer stones* In the third row? *The total number of stones in the circle*
- What is the pattern of decrease? *Twice as many*
- How many stones are in the circle in cave 1? *560*
- How many fewer stones are in the circle in cave 2? *8* How many stones are in the circle? *552*
- How many fewer stones are in cave 3 than in cave 2? *16* How many stones are in the circle? *536*
- (Have the students continue to fill in their tables, using the pattern of decrease.) What cave will have the last stone circle? *The 7th cave, with only 56 stones*

Solution: Cave 7

Cave	1	2	3	4	5	6	7	8	
Number fewer		8	16	32	64	128	256		
Total in circle	560	552	536	504	440	312	56		

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of decrease. Is your answer reasonable?

EXTEND IT
- What if the pattern of decrease was twice the number + 2?

PRACTICE
- Similar Practice Problems: 58, 95, 117

11   It is 12:00 and people are lining up for the matinee at the Bijou Cinema Six. In the first five minutes (12:05), 6 people get into line. At the end of the second five minutes (12:10), there are 11 people in line. At the end of the third five minutes (12:15), there are 16 people in line. If the people keep lining up at this rate, what time will it be when there are 81 people in line?

FIND OUT
- What is the question you have to answer?
- What time do people start lining up?
- How many people line up in the first five minutes? In the second five minutes? In the third five minutes?

CHOOSE A STRATEGY
- Would it help to keep track of every five minutes and how many people are in line?
- What strategy can help you figure out how the line is increasing?

SOLVE IT
- When you set up your table, what are you keeping track of in the first row?
- What are you keeping track of in the second row?
- What are you keeping track of in the third row?
- What is the difference between the numbers of people in the first and second five minutes? The second and third five minutes?
- Do you see a pattern of increase?
- Continue to fill in your table. What time will it be when 81 people are in line?

	1	2	3	4	5			
Time	12:05	12:10	12:15					
Total in line	6	11	16					

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

11 It is 12:00 and people are lining up for the matinee at the Bijou Cinema Six. In the first five minutes (12:05), 6 people got into line. At the end of the second five minutes (12:10), there are 11 people in line. At the end of the third five minutes (12:15), there are 16 people in line. If the people keep lining up at this same rate, what time will it be when there are 81 people in line?

FIND OUT
- What is the question you have to answer? *At what time will there be 81 people in line?*
- What time do people start lining up? *12:00*
- How many people line up in the first five minutes? *6* How many are in line after the second five minutes? *11* After the third five minutes? *16*

CHOOSE A STRATEGY
- Would it help to keep track of every five minutes and how many more people are in line? *Yes, we can make a table.*
- What strategy can help you figure out how the line is increasing? *We can look for a pattern of increase in the line.*

SOLVE IT
- When you set up your table, what are you keeping track of in the first row? *The number of five minute periods*
- What are you keeping track of in the second row? *The time*
- What are you keeping track of in the third row? *The people in line*
- What is the difference between the numbers of people in the first and second five minutes? *5* The second and third five minutes? *5 again*
- Do you see a pattern of increase? *Yes, 5*
- (Have the students continue to fill in their tables.) What time will it be when there are 81 people in line? *1:20*

Solution: 1:20

	1	2	3	4	5	6	7	8
Time	12:05	12:10	12:15	12:20	12:25	12:30	12:35	12:40
Total in line	6	11	16	21	26	31	36	41

	9	10	11	12	13	14	15	16
Time	12:45	12:50	12:55	1:00	1:05	1:10	1:15	1:20
Total in line	46	51	56	61	66	71	76	81

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT
- If there are 5 people in line at 12:05; 7 people in line after the second five minutes; 10 people in line after the third five minutes; and 14 people in line after the fourth five minutes, when will there be over 100 people in line?

12 At Bernie's Burgers, Marlowe and Bart are learning to make burgers in the kitchen. Marlowe is making Bernie's Super, with cheese and bacon. He made 4 in the first hour, 8 in the second hour, then 7 in the third hour, 11 in the fourth hour, and 10 in the fifth hour. Bart is making Bernie's Best, with onions and mushrooms. He made 5 in the first hour, 6 in the second hour, 9 in the third hour, 10 in the fourth hour, and 13 in the fifth hour. Marlowe and Bart continued making burgers at the same rates until they made a combined total of 58 burgers in the same hour. Out of the 58 burgers, how many were Bernie's Supers and how many were Bernie's Best?

FIND OUT
- What is the question you have to answer?
- Who is making burgers?
- How many burgers does Marlowe make in hour 1? in hour 2? in hour 3? in hour 4? in hour 5?
- How many burgers does Bart make in hour 1? in hour 2? in hour 3? in hour 4? in hour 5?
- What is the greatest combined total of hamburgers Marlowe and Bart made in one hour?

CHOOSE A STRATEGY
- How can you systematically keep track of all the burgers being made?
- What strategy can help you look at how the number of burgers being made is increasing or decreasing?

SOLVE IT
- When you set up a table, what are you keeping track of in the first row?
- What are you keeping track of in the second row?
- What are you keeping track of in the third row?
- What are you keeping track of in the fourth row?
- Look at the number of burgers that Marlowe is making. What is the difference between hours 1 and 2? between hours 2 and 3? between hours 3 and 4? between hours 4 and 5?
- Do you see a pattern in the way Marlowe is making burgers?
- Look at the number of burgers that Bart is making. What is the difference between hours 1 and 2? between hours 2 and 3? between hours 3 and 4? between hours 4 and 5?
- Do you see a pattern in the way Bart is making burgers?
- When you find the patterns, continue the table. How many hours do they have to make burgers before they make a total of 58 together?
- How many Bernie's Best and how many Bernie's Supers were made?

Hour	1	2	3	4	5		
Marlowe	4	8	7	11	10		
Bart	5	6	9	10	13		
Total	9	14	16	21	23		

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

12 At Bernie's Burgers, Marlowe and Bart are learning to make burgers in the kitchen. Marlowe is making Bernie's Super, with cheese and bacon. He made 4 in the first hour, 8 in the second hour, then 7 in the third hour, 11 in the fourth hour, and 10 in the fifth hour. Bart is making Bernie's Best, with onions and mushrooms. He made 5 in the first hour, 6 in the second hour, 9 in the third hour, 10 in the fourth hour, and 13 in the fifth hour. Marlowe and Bart continued making burgers at the same rates until they made a combined total of 58 burgers in the same hour. Out of the 58 burgers, how many were Bernie's Supers and how many were Bernie's Best?

FIND OUT
- What is the question you have to answer? *When Marlowe and Bart made a combined total of 58 burgers in one hour, how many Bernie's Supers and how many Bernie's Best had been made?*
- Who is making burgers? *Marlowe and Bart*
- How many burgers does Marlowe make in hour 1? *4* in hour 2? *8* in hour 3? *7* in hour 4? *11* in hour 5? *10*
- How many burgers does Bart make in hour 1? *5* in hour 2? *6* in hour 3? *9* in hour 4? *10* in hour 5? *13*
- What is the greatest combined total of hamburgers Marlowe and Bart made in one hour? *58*

CHOOSE A STRATEGY
- How can you systematically keep track of all the burgers being made? *We can make a table.*
- What strategy can help you look at how the number of burgers being made is increasing or decreasing? *We can make a table, then look at the difference between each hour for Marlowe and Bart. We can look for patterns.*

SOLVE IT
- When you set up a table, what are you keeping track of in the first row? *The number of hours*
- What are you keeping track of in the second row? *The number of burgers that Marlowe makes*
- What are you keeping track of in the third row? *The number of burgers that Bart makes*
- What are you keeping track of in the fourth row? *The total number of burgers made*
- Look at the number of burgers that Marlowe is making. What is the difference between hours 1 and 2? *+4* between hours 2 and 3? *−1* between hours 3 and 4? *+4* between hours 4 and 5? *−1*
- Do you see a pattern in the way Marlowe is making burgers? *Yes, first it increases by 4, then decreases by 1, and this continues.*
- Look at the number of burgers that Bart is making. What is the difference between hours 1 and 2? *+1* between hours 2 and 3? *+3* between hours 3 and 4? *+1* between hours 4 and 5? *+3*
- Do you see a pattern in the way Bart is making burgers? *Yes, first he makes 1 more, then 3 more, and this continues.*
- (Have the students look for patterns and use them to fill in the rest of the table.) How many hours do they have to make burgers before they make a total of 58 together? *15 hours*
- How many Bernie's Best and how many Bernie's Super were made in that hour? *Best - 33, Super - 25*

Solution: Best - 33, Super - 25

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Marlowe	4	8	7	11	10	14	13	17	16	20	19	23	22	26	25
Bart	5	6	9	10	13	14	17	18	21	22	25	26	29	30	33
Total	9	14	16	21	23	28	30	35	37	42	44	49	51	56	58

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

PRACTICE
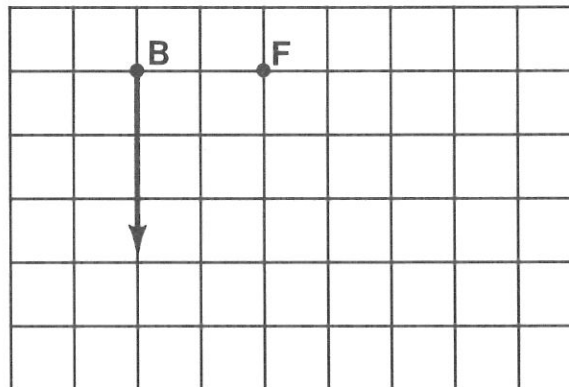- Similar Practice Problems: 50, 79, 105

13 Gino and Mark had found all the things on the list for the treasure hunt and had only a few minutes to get to the finish point. But they were lost! Gino said, "When we were at the bridge, we were 2 blocks west of the finish point. Can you remember where we went after that?" Mark recalled that they had gone south 3 blocks, then they went to their left 5 blocks, left again for 2 blocks, then north for 1 block. What is the quickest route from where they are to the finish point?

FIND OUT • What is the question you have to answer?
  • What are Gino and Mark doing? What happened to them?
  • What do Gino and Mark know about the location of the finish point?
  • What did Mark remember about where they had gone after they left the bridge?

CHOOSE A STRATEGY • Would it help to draw a diagram, to see where Gino and Mark have gone?

SOLVE IT • Begin your map at the bridge. What can you mark on the map to the east of the bridge?
  • Where did they go first from the bridge?
  • What did Mark remember about where they went next?
  • Where did they go after that?
  • Finish mapping all the information you have. Where are they now, when they think they are lost?
  • What is the fastest route to the finish point?

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your map. Is your answer reasonable?

13 Gino and Mark had found all the things on the list for the treasure hunt and had only a few minutes to get to the finish point. But they were lost! Gino said; "When we were at the bridge, we were 2 blocks west of the finish point. Can you remember where we went after that?" Mark recalled that they had gone south 3 blocks, then they went to their left 5 blocks, left again for 2 blocks, then north for 1 block. What is the quickest route from where they are to the finish point?

FIND OUT
- What is the question you have to answer? *What is the quickest route from where they are to the finish point?*
- What are Gino and Mark doing? *They are on a treasure hunt.* What happened to them? *They are lost.*
- What do Gino and Mark know about the location of the finish point? *It is 2 blocks east from the bridge.*
- What did Mark remember about where they had gone after they left the bridge? *South 3 blocks, left 5 blocks, left 2 blocks, north 1 block*

CHOOSE A STRATEGY
- Would it help to draw a diagram, to see where Gino and Mark have gone? *Yes, by making a diagram or map we can figure out where they are, where the finish point is, and look for the shortest route between the two points.*

SOLVE IT
- Begin your map at the bridge. What can you mark on the map to the east of the bridge? *The finish point*
- Where did they go first from the bridge? *South 3 blocks*
- What did Mark remember about where they went next? *To their left 5 blocks*
- Where did they go after that? *Left 2 blocks*
- (Have the students continue to map out all the information they have.) Where are they now, when they think they are lost? *They have just gone north for 1 block.*
- What is the fastest route to the finish point? *West 3 blocks*

Solution: West 3 blocks
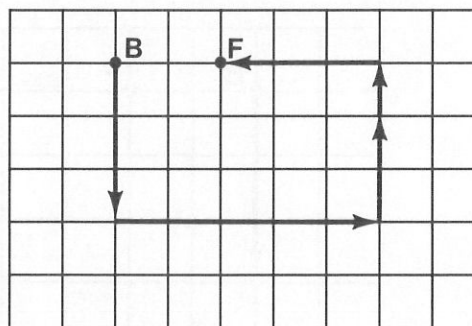
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your map. Is your answer reasonable?

EXTEND IT
- Try writing other directions from the bridge. Can you make it harder to find a quick route to the finish point?

14 Darryl has a summer job with real ups and downs: he delivers packages in the Sears Tower in Chicago. On his first day, he is given a box of sandwiches to deliver during lunchtime. He takes the turkey sandwich to an office 3 floors above the basement. He delivers the ham and cheese 13 floors above the turkey and 7 floors below the egg salad. He delivers the pastrami 9 floors below the ham and cheese and 8 floors below the submarine sandwich. He takes the tuna to an office 12 floors below the submarine sandwich. What is the number of the floor to which Darryl delivered each sandwich?

FIND OUT
- What is the question you have to answer?
- What is Darryl doing?
- What were the sandwiches that Darryl delivered?
- Where does Darryl take the turkey sandwich?
- Where does he deliver the ham and cheese?
- Where does he deliver the pastrami?
- Where does he deliver the tuna?

CHOOSE A STRATEGY
- Would it help to draw a picture of the building, so that you can see where Darryl is going to make his deliveries?

SOLVE IT
- When you make a diagram of the building, what is the first place to mark? What else do you want to label?
- Where does Darryl go to deliver the turkey sandwich? Which floor is that?
- Where does he go to deliver the ham and cheese? Which floor is that on?
- What do you know about where the egg salad goes? Which floor is that?
- Where does he go to deliver the pastrami? Which floor is that on?
- What do you know about where the submarine goes? Which floor is that?
- Where does the tuna go? Which floor?
- Can you see where each sandwich is now?
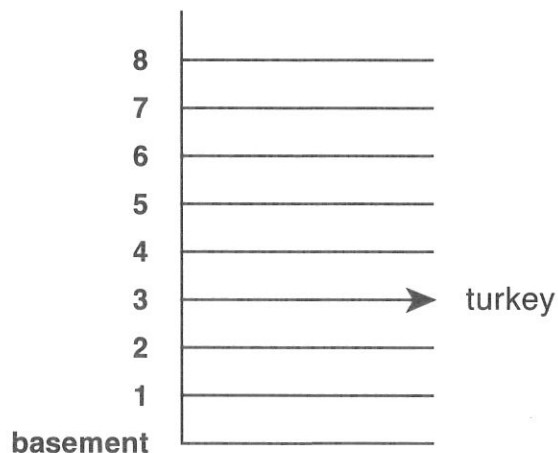
```
8 ─────────────
7 ─────────────
6 ─────────────
5 ─────────────
4 ─────────────
3 ───────────→  turkey
2 ─────────────
1 ─────────────
basement ──────
```

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

14 Darryl has a summer job with real ups and downs: he delivers packages in the Sears Tower in Chicago. On his first day, he is given a box of sandwiches to deliver during lunchtime. He takes the turkey sandwich to an office 3 floors above the basement. He delivers the ham and cheese 13 floors above the turkey and 7 floors below the egg salad. He delivers the pastrami 9 floors below the ham and cheese and 8 floors below the submarine sandwich. He takes the tuna to an office 12 floors below the submarine sandwich. What is the number of the floor to which Darryl delivered each sandwich?

FIND OUT
- What is the question you have to answer? *What is the number of the floor to which Darryl delivered each sandwich?*
- What is Darryl doing? *He is delivering sandwiches to different floors in the Sears building.*
- What were the sandwiches that Darryl delivered? *turkey, ham and cheese, egg salad, pastrami, submarine, tuna*
- Where does Darryl take the turkey sandwich? *3 floors above the basement*
- Where does he deliver the ham and cheese? *13 floors above the turkey and 7 floors below the egg salad*
- Where does he deliver the pastrami? *9 floors above the ham and cheese and 8 floors beneath the submarine*
- Where does he deliver the tuna? *12 floors below the submarine*

CHOOSE A STRATEGY
- Would it help to draw a picture of the building, so that you can see where Darryl is going to make his deliveries? *Yes, a diagram with a lot of numbered floors would help.*

SOLVE IT
- When you make a diagram of the building, what is the first place to mark? *The basement* What else do you want to label? *The numbers of the floors*
- Where does Darryl go to deliver the turkey sandwich? *3 floors above the basement* Which floor is that? *3*
- Where does he go to deliver the ham and cheese? *13 floors above the turkey* Which floor is that on? *16*
- What do you know about where the egg salad goes? *7 floors above the ham and cheese* Which floor is that? *23*
- Where does he go to deliver the pastrami? *9 floors below the ham and cheese* Which floor is that on? *7*
- What do you know about where the submarine goes? *8 floors above the pastrami* Which floor is that? *15*
- Where does the tuna go? *12 floors below the submarine* Which floor? *3*
- Can you see where each sandwich is now? *Yes*

Solution: Turkey - 3, ham and cheese - 16, egg salad - 23, pastrami - 7, submarine - 15, tuna - 3
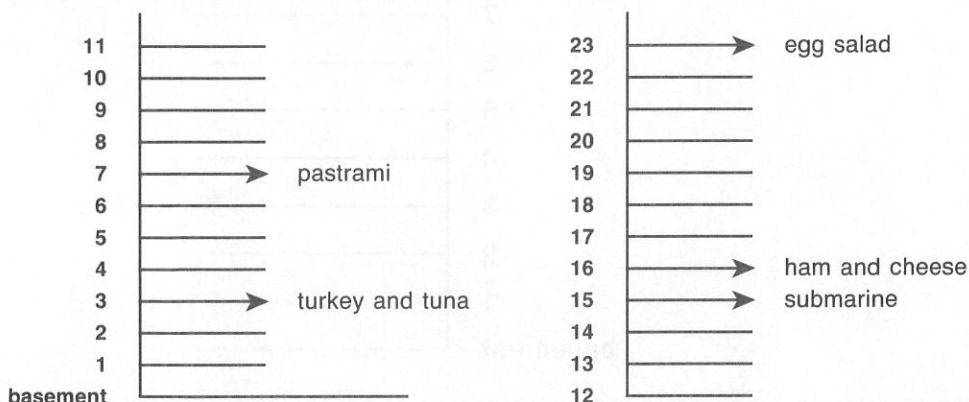
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 65, 78, 97

15   How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday? Joy came by skateboard, Heidi came by bike, and Saul came on the bus. It took Heidi twice as long as Joy to get there. It took Saul 10 minutes more than it took both the girls together. All three skaters together took 64 minutes to get to the rink.

FIND OUT
- What is the question you have to answer?
- What do you know about Heidi's traveling time?
- What do you know about Joy's traveling time?
- What do you know about Saul's traveling time?
- How long did it take all three skaters together to get to the rink?

CHOOSE A STRATEGY
- Will guessing an answer help you to solve this problem?
- How can you use the information from an incorrect guess?

SOLVE IT
- Would you start by making a guess for Heidi, Saul, or Joy? Why?
- What is your guess?
- If you make a guess for one skater, can you figure out the time taken by the other two skaters?
- How can you check your guess?
- How was your guess? If your first guess was wrong, how can you use the information to make your next guess?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

15 How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday? Joy came by skateboard, Heidi came by bike, and Saul came on the bus. It took Heidi twice as long as Joy to get there. It took Saul 10 minutes more than it took both the girls together. All three skaters together took 64 minutes to get to the rink.

FIND OUT
- What is the question you have to answer? *How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday?*
- What do you know about Heidi's traveling time? *Twice as long as Joy*
- What do you know about Joy's traveling time? *One half as much time as Heidi; Saul took 10 more minutes than her time combined with Heidi's*
- What do you know about Saul's traveling time? *10 minutes longer than Joy's and Heidi's combined travel time*
- How long did it take all three skaters together to get to the rink? *64 minutes*

CHOOSE A STRATEGY
- Will guessing an answer help you to solve this problem? *Yes, because we really don't have enough information.*
- How can you use information from an incorrect guess? *We can compare it with the total time and then decide whether to make the next guess higher or lower.*

SOLVE IT
- (Following is an example of one possible guess. The students will probably make several guesses before they get the right answer. Encourage them to look at an incorrect guess and see if the next guess should be higher or lower.)
- Would you start by making a guess for Heidi, Saul, or Joy? *Joy* Why? *Because we know less about her time than the others.*
- What is your guess? *5 for Joy*
- If you make a guess for one skater, can you figure out the time taken by the other two skaters? *Yes, we know that Heidi takes twice as long, so that would make 10 for Heidi. We know that Saul takes 10 minutes longer than Heidi and Joy together, so that makes 25 for Saul.*
- How can you check your guess? *We can add up the three times, which would be 40 minutes.*
- How was your guess? *Not so good* If your first guess was wrong, how can you use the information to make your next guess? *40 is quite a bit below 64, the next guess should be higher.*

  Solution: Joy 9, Heidi 18, Saul 37

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- The next Saturday Saul and Joy took the same amount of time and Heidi took twice as long as Saul and Joy together. All three of them together took 74 minutes to get to the rink. How long did each skater take?

16 This notice was posted: Auditions Next Week for the School Play. After reading a copy of the play, Betty, Hilda, Adam, and Renee have decided which parts to try out for. Hilda wants a part that has twice as many lines as the part that Betty wants. Adam has chosen a part with three more than twice as many lines as the part that Hilda wants. Renee is going to try out for a part with four more lines than Betty's part. If they get the parts they want, together they will have a total of 47 speaking lines. How many lines would each actor have in the play?

FIND OUT
- What is the question you have to answer?
- Who is trying out for the school play?
- What do you know about Hilda's part?
- What do you know about Adam's part?
- What do you know about Renee's part?
- What do you know about Betty's part?
- What is the total number of speaking lines for the four parts that Hilda, Adam, Renee, and Betty want?

CHOOSE A STRATEGY
- Will guessing an answer help you to solve this problem?
- How can information from an incorrect guess help you?

SOLVE IT
- Would you begin with a guess for Hilda, Adam, Renee, or Betty? Why?
- What is your guess?
- If you make a guess for one of the actors, can you figure out the number of lines for the other actors?
- How can you check your guess?
- How was your guess? If your first guess was wrong, how can you use the information to make your next guess?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

16   This notice was posted: Auditions Next Week for the School Play. After reading a copy of the play, Betty, Hilda, Adam, and Renee have decided which parts to try out for. Hilda wants a part that has twice as many lines as the part that Betty wants. Adam has chosen a part with three more than twice as many lines as the part that Hilda wants. Renee is going to try out for a part with four more lines than Betty's part. If they get the parts they want, together they will have a total of 47 speaking lines. How many lines would each actor have in the play?

FIND OUT
- What is the question you have to answer? *How many lines would each actor have?*
- Who is trying out for the play? *Betty, Hilda, Adam, and Renee*
- What do you know about Hilda's part? *It is twice as long as Betty's.*
- What do you know about Adam's part? *It is twice as long as Hilda's plus 3 lines.*
- What do you know about Renee's part? *It is 4 lines longer than Betty's.*
- What do you know about Betty's part? *Not much*
- What is the total number of speaking lines for the four parts? *47*

CHOOSE A STRATEGY
- Will guessing an answer help you to solve this problem? *Yes, because we don't have enough information to do it another way.*
- How can information from an incorrect guess help you? *We can see if it is too high or too low and then make another guess based on that.*

SOLVE IT
- (Following is an example of one possible guess. Students will probably have to make several guesses before getting the right answer. Be sure they use information from an incorrect guess to make their next guess.)
- Would you begin with a guess for Hilda, Adam, Renee, or Betty? *Betty* Why? *If we make a guess about her, then we can figure out the other parts.*
- What is your guess? *8 for Betty*
- If you make a guess for one of the actors, can you figure out the number of lines for the other actors? *Yes, because we know that Hilda's is twice Betty's, so that makes her part 16 lines. We know that Adam is twice Hilda's plus 3, making his 35. Then Renee's is the same as Betty's plus 4, making 12.*
- How can you check your guess? *Add up the lines, which give a total of 71.*
- How was your guess? *Too high* If your first guess was wrong how can you use the information for making your next guess? *The total is above 47, so the next guess should be lower.*

  Solution: Hilda - 10, Betty - 5, Adam - 23, Renee - 9

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- What if Hilda's part is three times as long as Betty's; Adam's part is one half the length of Hilda's; Renee's part is four times as long as Adam's part plus an additional 13 lines. The total number of lines is 82. How long would each person's part be?

PRACTICE
- Similar Practice Problems: 71, 80, 104

17 Kevin, Barbara, and their mother and father went backpacking in Yosemite National Park. On the first and second days, each hiker had a serving of food for breakfast, lunch, and dinner. A large, noisy, brown bear barged into camp the second night, got the food pack down from the tree where they had hung it, and ate one half of the food that was left. The next morning, after they all had breakfast, they found they had 4 food servings left. They decided they had better hike back to their car. How many servings of food did they begin the trip with?

FIND OUT
- What is the question you have to answer?
- How many hikers were eating at each meal?
- What do you know about the number of servings eaten on day 1 and 2?
- How much did the bear eat?
- How many servings did the hikers have after the bear had his share?
- How many servings of food were left at the end of the hike?

CHOOSE A STRATEGY
- Think about the best way to begin solving this problem. The only specific information you have is how much food is left on the last day. Therefore you need to work backwards from the last day to the first day.
- Is there a systematic way to record the information?

SOLVE IT
- If you begin with the last day of the trip, how many servings of food did they have left?
- How much did the hikers eat on the last day? How many servings did the hikers have altogether before breakfast the last day?
- Can you figure out how much the bear ate?
- How many servings of food were left at the end of the second day before the bear came?
- How many servings were eaten on the second day?
- How many servings were eaten on the first day?
- How many servings did they begin with?

<div align="center">

Day 3 - 4 left
        4 eaten

Day 2 -

</div>

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

17 Kevin, Barbara, and their mother and father went backpacking in Yosemite National Park. On the first and second days, each hiker had a serving of food for breakfast, lunch, and dinner. A large, noisy, brown bear barged into camp the second night, got the food pack down from the tree where they had hung it, and ate one half of the food that was left. The next morning, after they all had breakfast, they found they had 4 food servings left. They decided they had better hike back to their car. How many servings of food did they begin the trip with?

FIND OUT
- What is the question you have to answer? *How many servings of food did they begin the trip with?*
- How many hikers were eating at each meal? *4*
- What do you know about the number of servings eaten on day 1 and 2? *4 hikers ate 3 meals each day*
- How much did the bear eat? *One half of what was left*
- How many servings did the hikers have after the bear had his share? *4*
- How many servings of food were left at the end of the hike? *4*

CHOOSE A STRATEGY
- Think about the best way to begin solving this problem. The only specific information you have is how much food is left on the last day. Therefore you need to work backwards from the last day to the first day.
- Is there a systematic way to record the information? *Yes, we can use an organized list.*

SOLVE IT
- If you begin with the last day of the trip, how many servings of food did they have left? *4*
- How much did the hikers eat on the last day? *4* How many servings did the hikers have altogether before breakfast the last day? *8*
- Can you figure out how much the bear ate? *If 8 servings were left after the bear ate his share, then the bear must have eaten 8 servings.*
- How many servings were left at the end of the second day before the bear came? *16*
- How many servings were eaten on the second day? *4 × 3 = 12*
- How many servings were eaten on the first day? *4 × 3 = 12*
- How many servings did they begin with? *16 + 12 + 12 = 40*

    Solution: 40 servings of food

    Day 3 - 4 servings left
        4 servings eaten by hikers

    Day 2 - 8 servings eaten by bear
        12 servings eaten by hikers

    Day 1 - 12 servings eaten by hikers

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

EXTEND IT
- After breakfast on the 4th day, they lose one third of their remaining food while fording a river. They have 6 servings of food left. How many servings did they start with, assuming that each hiker ate a serving at breakfast, lunch, and dinner?

18 Keri's father, Bill, was a baker. Out of his usual morning batch of chocolate chip cookies, Bill burned the first two dozen cookies. He gave half of what was left to Keri to take to school. He wrapped up half of the remaining cookies and gave them to the gas station crew next door. He gave half of what was left to the policeman on the beat. If Bill had only 7 cookies left, how many cookies were in the batch of chocolate chip cookies?

FIND OUT
- What is the question you have to answer?
- What was Bill doing?
- What was the first thing that happened to Bill's cookies?
- How many cookies did Bill give Keri?
- How many cookies did Bill give the gas station crew?
- How many cookies did Bill give to the policeman?
- How many cookies did Bill have left?

CHOOSE A STRATEGY
- You have very little specific information. If you begin with the information you have, then what can you do?
- Is there a way to show your work that will help?

SOLVE IT
- Begin with what Bill had left after he burned and gave away his cookies: 7 cookies. Who is the last person(s) that Bill gave cookies to? How many?
- If you continue to work backwards, who is the next to last person(s) that he gave cookies to? How many?
- Who is the other person(s) that Bill gave cookies to? How many?
- What is the first thing that happened to the cookies?
- Now can you figure out how many cookies were in the first batch?

$\frac{1}{2}$ of what is left	$\frac{1}{2}$ of what is left	
	$\frac{1}{2}$	7

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your picture and work. Is your answer reasonable?

18   Keri's father, Bill, was a baker. Out of his usual morning batch of chocolate chip cookies, Bill burned the first two dozen cookies. He gave half of what was left to Keri to take to school. He wrapped up half of the remaining cookies and gave them to the gas station crew next door. He gave half of what was left to the policeman on the beat. If Bill had only 7 cookies left, how many cookies were in the batch of chocolate chip cookies?

FIND OUT
- What is the question you have to answer? *How many cookies were in the batch of chocolate chip cookies?*
- What was Bill doing? *Baking cookies*
- What was the first thing that happened to Bill's cookies? *He burned two dozen.*
- How many cookies did Bill give Keri? *One half of what was left*
- How many cookies did Bill give the gas station crew? *One half of what was left*
- How many cookies did Bill give to the policeman? *One half of what was left*
- How many cookies did Bill have left? *7*

CHOOSE A STRATEGY
- You have very little specific information. If you begin with the information you have, then what can you do? *Work backwards from the 7 cookies that are left.*
- Is there a way to show your work that will help? *We can make a picture or diagram.*

SOLVE IT
- Begin with what Bill had left after he burned and gave away his cookies: 7 cookies. Who is the last person(s) that Bill gave cookies to? *Policeman* How many? *If he got one half of what was left, then he got 7 cookies.*
- If you continue to work backwards, who is the next to the last person(s) that he gave cookies to? *Gas station crew* How many? *If they got half of what was left, then that would be 14 (7 to policeman + 7 left over).*
- Who is the other person(s) that Bill gave cookies to? *Keri* How many? *One half of what is left, and that would be 28 (14 to crew + 7 to policeman + 7 left over).*
- What is the first thing that happened to the cookies? *Two dozen get burned.*
- Now can you figure out how many cookies were in the first batch? *Yes, add 24 + 28 + 14 + 7 + 7 = 80*

Solution: 80 cookies
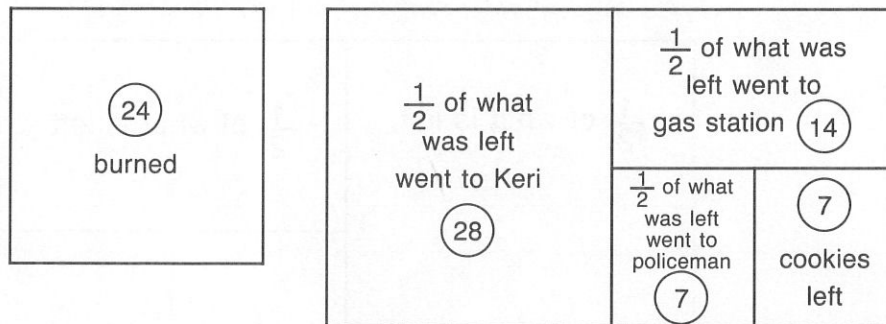
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your picture and work. Is your answer reasonable?

EXTEND IT
- Bill burned a dozen cookies, gave one half of what was left to Keri, then gave one half of what was left to the gas station crew, gave one third of what was left to the drugstore manager, another one third to the policeman, and had 8 cookies left. How many cookies were in Bill's batch?

PRACTICE
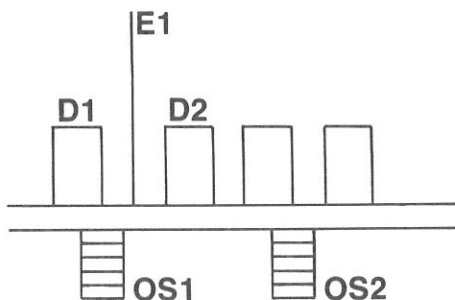- Similar Practice Problems: 70, 90, 118

19 Sandy is delivering pizza to the second floor of the apartment house at 645 Birch Street. There are two outside stairways from the street to the building. Then there are four doors into the building. Once inside there are two elevators and two inside stairways that go to the second floor. How many different ways can Sandy go from the street to the second floor and deliver the pizza?

FIND OUT
- What is the question you have to answer?
- Where is Sandy going?
- How many stairways go from the street to the building?
- How many doors are there into the building?
- How many elevators are there?
- How many inside stairways are there?

CHOOSE A STRATEGY
- It is difficult to "see" this problem in your mind. Is there a good way to lay out the information?

SOLVE IT
- Begin by showing the outside stairways up to the building. How many are there? Then draw the doors into the building. How many are there? How many elevators do you need to show that go to the second floor? How many stairways go to the second floor?
- Begin with one outside stairway. If you can go up one stairway, through 4 doors and up 2 elevators, how many different ways can you go? Now if you go up the same stairway, through 4 doors, and up 2 inside stairways, how many different ways can you go? How many ways are there altogether from one outside stairway?
- Do the same thing with the other outside stairway. How many different ways can you go from this stairway to the second floor?
- How many different ways can Sandy go from the street to the second floor and deliver the pizza?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

19 Sandy is delivering pizza to the second floor of the apartment house at 645 Birch Street. There are two outside stairways from the street to the building. Then there are four doors into the building. Once inside there are two elevators and two inside stairways that go to the second floor. How many different ways can Sandy go from the street to the second floor and deliver the pizza?

FIND OUT
- What is the question you have to answer? *How many different ways can Sandy go from the street to the second floor and deliver the pizza?*
- Where is Sandy going? *From the street to the second floor*
- How many stairways go from the street to the building? *2*
- How many doors are there into the building? *4*
- How many elevators are there? *2*
- How many inside stairways are there? *2*

CHOOSE A STRATEGY
- It is difficult to "see" this problem in your mind. Is there a good way to lay out the information? *We can make a picture or a diagram to show the building and the entrances, elevators, and stairways.*

SOLVE IT
- Begin by showing the outside stairways up to the building. How many are there? *2* Then draw the doors into the building. How many are there? *4* How many elevators do you need to show that go to the second floor? *2* How many stairways go to the second floor? *2*
- Begin with one outside stairway. If you can go up one stairway, through 4 doors and up 2 elevators, how many different ways can you go? *1 × 4 × 2 = 8* Now if you go up the same stairway, through 4 doors and up 2 inside stairways, how many different ways can you go? *1 × 4 × 2 = 8* How many ways are there altogether from one outside stairway? *8 + 8 = 16*
- Do the same thing with the other outside stairway. How many different ways can you go from this stairway to the second floor? *1 × 4 × 4 = 16*
- How many different ways can Sandy go from the street to the second floor and deliver the pizza? *16 + 16 = 32, 2 × 4 × 4 = 32*

Solution: 32
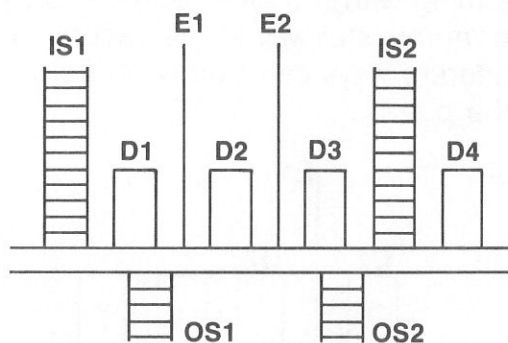
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- Suppose there are 4 outside stairways, 6 doors, 8 elevators, and 2 inside stairways?

20 Being a double agent, Mr. X must be very careful about his daily routine to insure his safety. He never eats at a restaurant more than once, and he must be careful going back and forth to the hotel room where he is presently hiding. There are four street entrances to the hotel: two entrances into the lobby, one entrance in the east wing, and one entrance in the west wing. There are six elevators in the building: two in the lobby and two in each wing. The elevators only go as high as the 15th floor. Each wing also has a stairway which goes to every floor of the hotel. How many different ways can Mr. X get to his room by entering the hotel, taking the elevator to the 15th floor, and walking up to his floor?
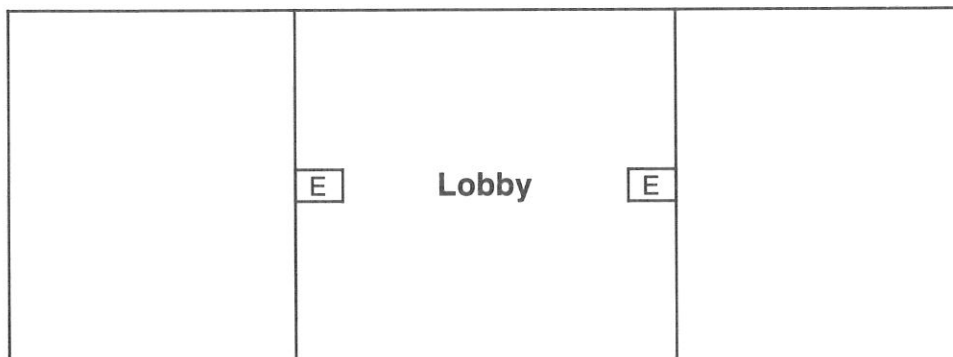
FIND OUT
- What is the question you have to answer?
- Where is Mr. X going?
- How many entrances to the hotel are there?
- How many elevators to the 15th floor are there?
- How many stairways up to Mr. X's floor are there?

CHOOSE A STRATEGY
- Would it be helpful to "see" all this information in the form of a picture, so you can get a better idea of Mr. X's choices?

SOLVE IT
- If you make a diagram of the hotel, how many entrances do you need to show?
- How many elevators need to be included on your diagram?
- How many stairways do you need to mark on the diagram?
- If Mr. X begins outside the hotel, and has a choice of 4 entrances and then 6 elevators to the 15th floor, how many different ways can he get to the 15th floor? Now how many ways can he walk up to his floor from the 15th floor? Then how many ways can he get from the street to his room?

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│              │  [E]    Lobby    [E]   │              │
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LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

20 Being a double agent, Mr. X must be very careful about his daily routine to insure his safety. He never eats at a restaurant more than once, and he must be careful going back and forth to the hotel room where he is presently hiding. There are four street entrances to the hotel: two entrances into the lobby, one entrance in the east wing, and one entrance in the west wing. There are six elevators in the building: two in the lobby and two in each wing. The elevators only go as high as the 15th floor. Each wing also has a stairway which goes to every floor of the hotel. How many different ways can Mr. X get to his room by entering the hotel, taking the elevator to the l5th floor, and walking up to his floor?

FIND OUT
- What is the question you have to answer? *How many different ways can Mr. X get to his room by entering the hotel, taking the elevator to the 15th floor, and walking up to his floor?*
- Where is Mr. X going? *From outside the hotel to his room*
- How many entrances to the hotel are there? *4*
- How many elevators to the 15th floor are there? *6*
- How many stairways up to Mr. X's floor are there? *2*

CHOOSE A STRATEGY
- Would it be helpful to "see" all this information in the form of a picture, so you can get a better idea of Mr. X's choices? *Yes, we can make a picture or diagram of the hotel.*

SOLVE IT
- If you make a diagram of the hotel, how many entrances do you need to show? *4*
- How many elevators need to be included on your diagram? *6*
- How many stairways do you need to mark on the diagram? *2*
- If Mr. X begins outside the hotel, and has a choice of 4 entrances and then 6 elevators to the 15th floor, how many different ways can he get to the 15th floor? *$4 \times 6 = 24$* Now how many ways can he walk up to his floor from the 15th floor? *2* Then how many ways can he get from the street to his room? *$4 \times 6 \times 2 = 48$*

Solution: 48
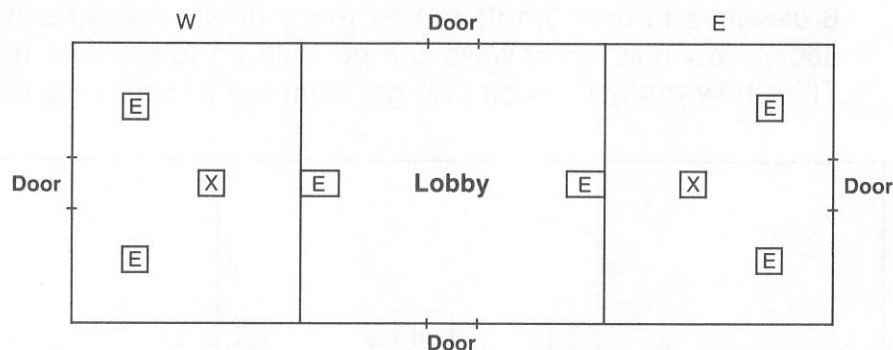
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- There are 3 outside stairways to the hotel, 6 doors into the hotel, 11 elevators to the 15th floor, and 4 stairways. If Mr. X is hiding out again, how many different ways can he get to his floor?

PRACTICE
- Similar Practice Problems: 59, 67, 92

21 Bill, his sister Martha, and Ann are sitting down to learn a new game from Phil Hill. Each player has a partner and the partners are seated across the table from each other. Bill Dill is sitting to the right of Phil's sister. Phil Hill is sitting to the right of Bill's sister's partner. Where is each player sitting at the table?
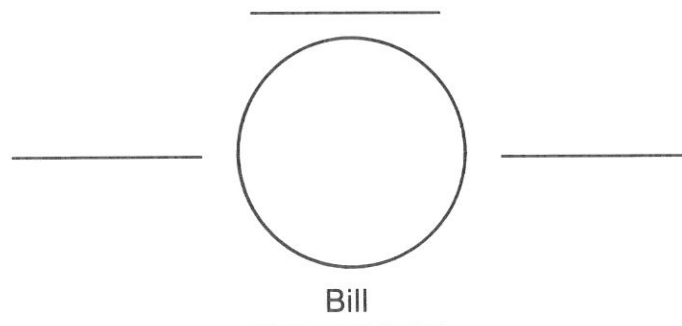
FIND OUT
- What is the question you have to answer?
- Who are the players?
- How are the players sitting at the table?
- What do you know about where Bill Dill is sitting?
- What do you know about where Phil Hill is sitting?

CHOOSE A STRATEGY
- You can use a series of "If this is true, then this must be true," statements to help you solve this problem. What kind of thinking do we call this?
- Is there a way to show your work that can be helpful?

SOLVE IT
- What are you going to put in a diagram?
- Begin with one player. Write the name of Bill Dill on one side of the table. What do you know about where Bill is sitting?
- If Martha is Bill's sister, then who is Phil's sister? Where is she sitting?
- If there are two places left at the table then Phil has to be in one of them. Could Phil be to the right of Martha? Is there only one place where Phil can be sitting?
- Where is each player sitting at the table?

Bill

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your picture and work. Is your answer reasonable?

21 Bill, his sister Martha, and Ann are sitting down to learn a new game from Phil Hill. Each player has a partner and the partners are seated across the table from each other. Bill Dill is sitting to the right of Phil's sister. Phil Hill is sitting to the right of Bill's sister's partner. Where is each player sitting at the table?

FIND OUT
- What is the question you have to answer? *Where is each player sitting at the table?*
- Who are the players? *Bill, his sister Martha, Ann, and Phil*
- How are the players sitting at the table? *As partners, across from each other*
- What do you know about where Bill Dill is sitting? *To the right of Phil's sister*
- What do you know about where Phil Hill is sitting? *To the right of Bill's sister's partner*

CHOOSE A STRATEGY
- You can use a series of "If this is true, then this must be true," statements to help you solve this problem. What kind of thinking do we call this? *Logical reasoning*
- Is there a way to show your work that can be helpful? *We can make a picture or diagram of the table and players.*

SOLVE IT
- What are you going to put in a diagram? *A table and 4 places*
- Begin with one player. Write the name of Bill Dill on one side of the table. What do you know about where Bill is sitting? *We know that Bill is sitting to the right of Phil's sister*
- If Martha is Bill's sister, then who is Phil's sister? *It must be Ann.* Where is she sitting? *To the left of Bill*
- If there are two places left at the table then Phil has to be in one of them. Could Phil be to the right of Martha? *No, because that would put him to the right of Bill's sister, not Bill's sister's partner.* Is there only one place where Phil can be sitting? *Yes, to the right of Bill, making Bill his sister's partner.*
- Where is each player sitting at the table? *Martha across from Bill, Phil on Bill's right and across from Ann*

   Solution: Martha across from Bill, Phil on Bill's right and across from Ann
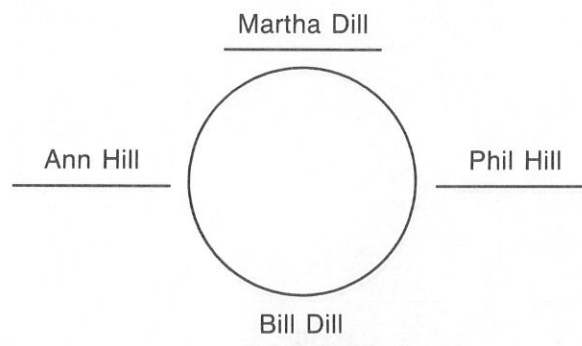
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your picture and work. Is your answer reasonable?

EXTEND IT
- Jim Andrews and his brother Andy are playing cards with their sister Maria and Sue Jackson. Sue is on Jim's right. Maria is to the right of Jim's brother. Where is each player sitting?

22 Donna's father, an archaeologist, is sharing his latest diggings with Donna's class: stone tablets from an ancient culture. The number notation system used on the tablets included squares and circles. Donna holds up one stone tablet that looks like this:
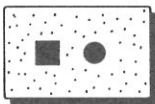
The square and circle on this tablet represent the number 14. The next stone tablet looks like this:
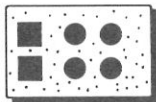
The squares and circles on this tablet represent the number 34. Then Donna holds up a third tablet that looks like this:

What number do the square and circles represent on the third tablet?

FIND OUT
- What is the question you have to answer?
- What are Donna and her father sharing with the class?
- What do the squares and circles on the tablets stand for?
- What number is represented by the square and circle on the first tablet?
- What number is represented by the squares and circles on the second tablet?

CHOOSE A STRATEGY
- What kind of thinking can you use to organize the information in this problem?
- Is there another strategy you can use with this problem?

SOLVE IT
- What do you know about the first tablet?
- Try guessing some numbers for the square and the circle; which numbers will add up to 14?
- If you find two numbers that work for the first tablet, then what is the next step? Do you need to do more guessing and checking?
- If you have values for the square and circle that give you 14 and 34 for the first and second tablets, then can you find a value for the mystery tablet?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?
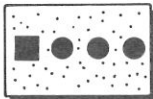
22

Donna's father, an archaeologist, is sharing his latest diggings with Donna's class: stone tablets from an ancient culture. The number notation system used on the tablets included squares and circles. Donna holds up one stone tablet that looks like this:
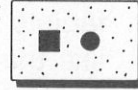
The square and circle on this tablet represent the number 14. The next stone tablet looks like this:
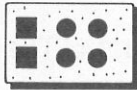
The squares and circles on this tablet represent the number 34. Then Donna holds up a third tablet that looks like this:

What number do the square and circles represent on the third tablet?

FIND OUT
- What is the question you have to answer? *What number do the square and circles represent on the third tablet?*
- What are Donna and her father sharing with the class? *Some ancient stone tablets with squares and circles on them*
- What do the squares and circles on the tablets stand for? *Numbers*
- What number is represented by the square and circle on the first tablet? *14*
- What number is represented by the squares and circles on the second tablet? *34*

CHOOSE A STRATEGY
- What kind of thinking can you use to organize the information in this problem? *Logical thinking*
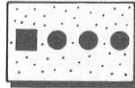- Is there another strategy you can use with this problem? *Yes, we can use "guess and check" to come up with possible number combinations for the squares and circles.*

SOLVE IT
- What do you know about the first tablet? *It has one square and one circle on it, and together they represent the number 14.*
- Try guessing some numbers for the square and the circle. Which numbers will add up to 14? *9 + 5, 10 + 4, 11 + 3, 12 + 2*
- If you find two numbers that work for the first tablet, then what is the next step? *We need to try the same numbers for the squares and circles on the second tablet, to see if they represent 34. If we try 9 for the square and 5 for the circle, that total is 38. If we try 8 for the square and 6 for the circle, this total is 40. Do you need to do more guessing and checking? Yes*
- If you have values for the square and circle that give you 14 and 34 for the first and second tablets, then can you find a value for the mystery tablet? *Yes, with 11 and 3 giving the right answer for tablet 1 and 2, we get 20 for the third tablet.*

Solution: 20

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- If the square and circle represent 26 on the first tablet, and the squares and circles represent 74 on the second tablet, what do the square and circles represent on the third tablet?

PRACTICE
- Similar Practice Problems: 83, 93, 119

23 Travis works at the Fantasy in Flight Factory. He checks all the kites made in the factory before they are packaged. One day Travis discovered that for every 30 kites that passed inspection there were 7 kites that didn't pass: 4 kites without tails and 3 kites with the wrong colors. Of the 296 kites Travis examined, how many didn't have tails and how many had the wrong colors?

FIND OUT
- What is the question you have to answer?
- What is Travis doing?
- How many kites pass inspection?
- How many kites do not pass inspection? How many of these don't have tails? How many have the wrong colors?
- How many kites did Travis look at?

CHOOSE A STRATEGY
- For 30 kites that passed inspection, 7 did not pass. For 60 kites that passed, how many wouldn't pass? How many would be missing tails? How many would have the wrong colors?
- How can you keep track of the kites in a systematic way?

SOLVE IT
- When you set up your table, what are you keeping track of in the first row?
- What are you keeping track of in the second row?
- What are you keeping track of in the third row?
- What are you keeping track of in the fourth row?
- How many kites don't pass inspection for 90 kites that do? How many are missing a tail? How many have the wrong colors?
- Continue to fill in the table until you have a total of 296 kites.
- When you have a total of 296 kites, how many don't pass inspection because they are missing a tail? How many have the wrong colors?

Pass	30	60	90		
No tail	4	8			
Wrong color	3	6			
Total	37	74			

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

23

Travis works at the Fantasy in Flight Factory. He checks all the kites made in the factory before they are packaged. One day Travis discovered that for every 30 kites that passed inspection there were 7 kites that didn't pass: 4 kites without tails and 3 kites with the wrong colors. Of the 296 kites Travis examined, how many didn't have tails and how many had the wrong colors?

FIND OUT
- What is the question you have to answer? *Of the 296 kites Travis examined, how many didn't have tails and how many had the wrong colors?*
- What is Travis doing? *Examining kites at a kite factory*
- For every 30 kites that pass inspection, how many don't pass? *7*
- Out of the kites that don't pass inspection, how many of these don't have tails? *4* How many have the wrong colors? *3*
- How many kites did Travis look at? *296*

CHOOSE A STRATEGY
- For 30 kites that passed inspection, 7 did not pass. For 60 kites that passed, how many wouldn't pass? *14* How many would be missing tails? *8* How many would have the wrong colors? *6*
- How can you keep track of the kites in a systematic way? *We can set up a table.*

SOLVE IT
- When you set up your table, what are you keeping track of in the first row? *The number that passed inspection*
- What are you keeping track of in the second row? *The number without tails*
- What are you keeping track of in the third row? *The number with the wrong colors*
- What are you keeping track of in the fourth row? *The total number of kites examined*
- For 90 kites that pass inspection, how many are missing a tail? *12* How many have the wrong colors? *9*
- (Have students continue to fill in the table until they have a total of 296 kites.)
- When you have a total of 296 kites, how many don't pass inspection because they are missing a tail? *32* How many have the wrong colors? *24*

Solution: 32 without tails, 24 with the wrong colors

Pass	30	60	90	120	150	180	210	240
No tail	4	8	12	16	20	24	28	32
Wrong color	3	6	9	12	15	18	21	24
Total	37	74	111	148	185	222	259	296

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT
- At the pet store, Travis' friend Vince found that out of every 10 cats 6 were yellow and 2 were grey. Out of 120 cats, how many would be yellow and how many would be grey?

24 The Rialto Theater is celebrating its 11th anniversary. In honor of the occasion, they are giving away free passes! They have hidden a gold, silver, purple, or green star under every seat. Every person who sits in a seat with a gold star gets a free pass to the next show. For every 2 gold stars they hid 18 silver, 16 purple, and 12 green stars. If there are 384 seats in the Rialto Theater, how many people won free passes?

FIND OUT
- What is the question you have to answer?
- What are they doing at the Rialto Theater for their 11th anniversary? How does someone get a free pass?
- What kind of stars did they put under the seats?
- For every 2 gold stars how many silver were there? purple? green?
- How many seats are there in the Rialto?

CHOOSE A STRATEGY
- You have to keep track of gold stars, silver stars, purple stars, and green stars. What else do you need to keep track of?
- How can you systematically record all the information you need?

SOLVE IT
- When you make a table, what are you keeping track of in the first row?
- What are you keeping track of in the second row? third row? fourth row? fifth row?
- Why do you need to keep track of the total number of stars?
- After filling in the first column, with 2 gold stars in the first row, how many seats have stars on them?
- After filling in the column with 4 gold stars in the first row, how many seats have stars on them?
- Continue to fill in the table until you have a total of 384 stars which represent 384 seats in the theater.
- When you have stars for 384 seats, how many gold stars are hidden? How many people won free passes?

Gold	2	4		
Silver	18	36		
Purple	16	32		
Green	12			
Total	48			

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

24   The Rialto Theater is celebrating its 11th anniversary. In honor of the occasion, they are giving away free passes! They have hidden a gold, silver, purple, or green star under every seat. Every person who sits in a seat with a gold star gets a free pass to the next show. For every 2 gold stars they hid 18 silver, 16 purple, and 12 green stars. If there are 384 seats in the Rialto Theater, how many people won free passes?

FIND OUT
- What is the question you have to answer? *If there are 384 seats in the Rialto Theater, how many people won free passes?*
- What are they doing at the Rialto Theater for their 11th anniversary? *Giving away free passes* How does someone get a free pass? *Anyone who sits in a seat with a gold star gets a free pass.*
- What kind of stars did they put under the seats? *Gold, silver, purple, green*
- For every 2 gold stars how many silver were there? *18* purple? *16* green? *12*
- How many seats are there in the Rialto? *384*

CHOOSE A STRATEGY
- You have to keep track of gold stars, silver stars, purple stars, and green stars. What else do you need to keep track of? *The total number of stars for every 2 gold stars*
- How can you systematically record all the information you need? *We can make a table.*

SOLVE IT
- When you make a table, what are you keeping track of in the first row? *The number of gold stars*
- What are you keeping track of in the second row? *The number of silver stars* third row? *Purple stars* fourth row? *Green stars* fifth row? *Total number of stars*
- Why do you need to keep track of the total number of stars? *To watch for a total of 384, which is the total number of seats in the theater.*
- After filling in the first column, with 2 gold stars in the first row, how many seats have stars on them? *48*
- After filling in the column with 4 gold stars in the first row, how many seats have stars on them now? *96*
- (Have the students continue to fill in the table until they have a total of 384 stars which represent 384 seats in the theater.)
- When you have stars for 384 seats, how many gold stars are hidden? *16* How many people won free passes? *16*

Solution: 16

Gold	2	4	6	8	10	12	14	16	
Silver	18	36	54	72	90	108	126	144	
Purple	16	32	48	64	80	96	112	128	
Green	12	24	36	48	60	72	84	96	
Total	48	96	144	192	240	288	336	384	

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

EXTEND IT
- What if there were 432 seats at the Rialto, and for every 3 gold stars they hid 19 silver stars, 15 purple stars, and 11 green stars. How many people would win free passes?

PRACTICE
- Similar Practice Problems: 53, 69, 115

25 It is Family Day at Camp Turtlecreek. One of the most popular activities is the canoe relay race. Teams are organized for the race and each team must have two adults and three campers. In order to win the race, you must canoe all your team members from one shore across a shallow river to the other shore. They have to follow these rules for who can be in the canoe on a crossing: One adult; one camper; one adult and one camper; or two campers. What is the fewest trips you would have to make to win the relay?

FIND OUT
- What is the question you have to answer?
- What is one of the most popular activities on Family Day?
- How are the teams organized for the race?
- How does a team win the race?
- What are the rules for each river crossing?

CHOOSE A STRATEGY
- Is there a good way to keep track of all these river crossings and "see" how a team is progressing?

SOLVE IT
- What do you want to keep track of in your diagram?
- Begin with two campers crossing the river. What is the next step? How many team members have you gotten across?
- If one camper returns on crossing 2, what are your choices for crossing 3?
- What is the fewest number of crossings you would have to make to win the relay?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

25 It is Family Day at Camp Turtlecreek. One of the most popular activities is the canoe relay race. Teams are organized for the race and each team must have two adults and three campers. In order to win the race, you must canoe all your team members from one shore across a shallow river to the other shore. They have to follow these rules for who can be in the canoe on a crossing: One adult; one camper; one adult and one camper; or two campers. What is the fewest trips you would have to make to win the relay?
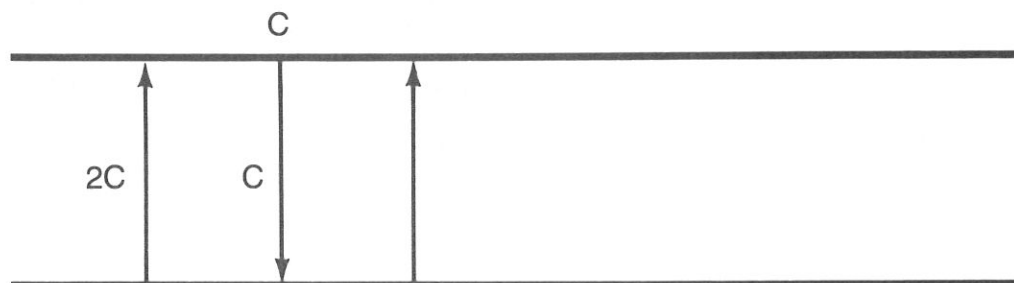
FIND OUT
- What is the question you have to answer? *What is the fewest trips you would have to make to win the relay?*
- What is one of the most popular activities on Family Day? *A team canoe race across the river*
- How are the teams organized for the race? *2 adults and 3 campers*
- How does a team win the race? *By getting all the team members across the river in a canoe*
- What are the rules for each river crossing? *In one canoe there can be: 1 adult; 1 camper; 1 adult and 1 camper; or 2 campers.*

CHOOSE A STRATEGY
- Is there a good way to keep track of all these river crossings and "see" how a team is progressing? *Yes, we can make a diagram and mark each crossing and also write down the team members as they get across the river.*

SOLVE IT
- (Following is an example of one way to do this. Encourage students to experiment with other ways.)
- What do you want to keep track of in your diagram? *Each crossing; who is in the canoe, who has gotten across the river*
- Begin with two campers crossing the river. What is the next step? *One camper stays there, and then one camper takes the canoe back across for crossing 2.* How many team members have you gotten across? *1*
- If one camper returns on crossing 2, what are your choices for crossing 3? *You have 2 campers and 2 adults left, so you could have 1 adult, 2 campers, or 1 adult and 1 camper, or 1 camper.*
- (Have students continue to mark crossings, and show which team members have gotten across the river, until everyone is across.)
- What is the fewest trips you would have to make to win the relay? *7*

Solution: 7 crossings
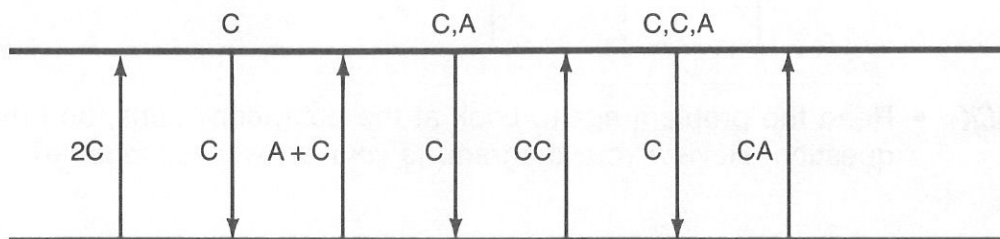
	C			C,A		C,C,A	
2C	C	A + C	C	CC	C	CA	

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

EXTEND IT
- What if you may never have 2 adults alone together or 2 campers alone together, in the boat or on either shore of the river?

26   Jessica is hiking in the mountains with her llama, her dog, and her cat. She comes to a deep river, where she must help each animal across, one at a time. She has a problem: she can't leave the llama alone with the dog, and she can't leave the cat alone with the dog. What is the fewest trips Jessica can make to get the llama, the dog, and the cat to the other side of the river?

FIND OUT
- What is the question you have to answer?
- What does Jessica have to do?
- What are the conditions for crossing the river?
- What are the special conditions for the llama and the dog?
- What are the special conditions for the cat and the dog?

CHOOSE A STRATEGY
- What is the best way to keep track of all the river crossings?
- Is there another strategy that might be useful here?

SOLVE IT
- What do you need to keep track of on your diagram?
- Who would you pick to make the first crossing? Why?
- If Jessica returns alone on crossing 2, who would be a good choice for crossing 3? Why?
- What should Jessica do on crossing 4? Why?
- If it is crossing 5 and the llama is on the other side, should Jessica take the cat or the dog? What happens if she takes the cat? What happens if she takes the dog?
- What is the fewest trips Jessica can take to get everyone across?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your picture and work. Is your answer reasonable?

T•51

26 Jessica is hiking in the mountains with her llama, her dog, and her cat. She comes to a deep river, where she must help each animal across, one at a time. She has a problem: she can't leave the llama alone with the dog, and she can't leave the cat alone with the dog. What is the fewest trips Jessica can take to get the llama, the dog, and the cat to the other side of the river?
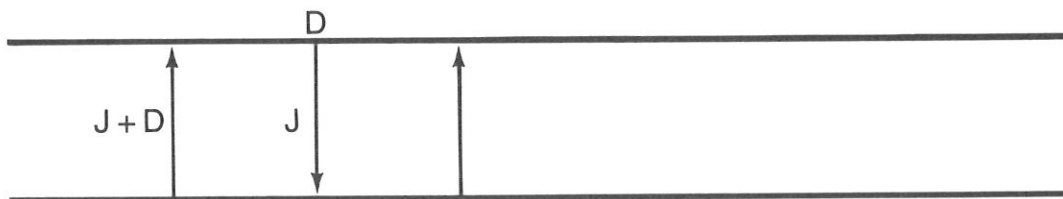
FIND OUT
- What is the question you have to answer? *What is the fewest trips Jessica can take to get the llama, the dog, and the cat to the other side of the river?*
- What does Jessica have to do? *Take the animals across a river*
- What are the conditions for crossing the river? *She can only take one animal at a time.*
- What are the special conditions for the llama and the dog? *They can't be alone together.*
- What are the special conditions for the cat and the dog? *They can't be alone together.*

CHOOSE A STRATEGY
- What is the best way to keep track of all the river crossings? *We can make a picture or diagram.*
- Is there another strategy that might be useful here? *Because of all the conditions we should use logical reasoning.*

SOLVE IT
- (Following is one example of how to do this problem. Encourage students to experiment and see if there is another way to do this.)
- What do you need to keep track of on your diagram? *Each crossing, and who is on which side of the river*
- Who would you pick to make the first crossing? *Jessica and the dog* Why? *Because the only two she can leave alone are the llama and the cat.*
- If Jessica returns alone on crossing 2, who would be a good choice for crossing 3? *Jessica and the cat* Why? *It doesn't really matter too much because the other animal is left alone.*
- What should Jessica do on crossing 4? *Come back with the dog* Why? *Because she can't leave the cat and dog together alone and she should be looking ahead, and trying to get the llama and the cat together on the same side first.*
- (Have students continue making crossings, always making sure that the conditions are met each time.)
- What is the fewest trips Jessica can take to get everyone across? 7

Solution: 7
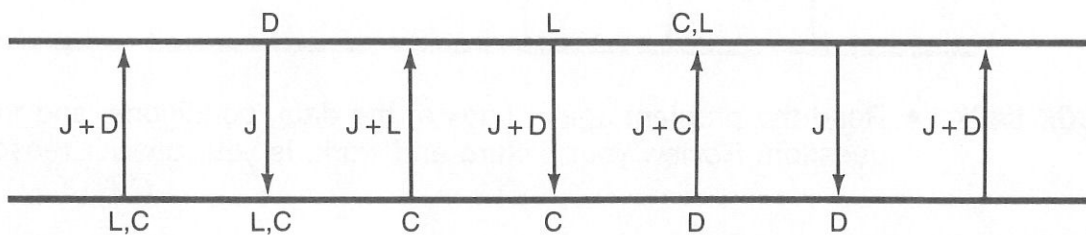
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your picture and work. Is your answer reasonable?

EXTEND IT
- What if there are 3 dogs and 3 cats and there can never be more of one animal than another on either side of the river? Jessica can take as many as 3 animals across at a time.

PRACTICE
- Similar Practice Problems: 52, 62, 109

27 Noelani was saving Luscious Lemon soda coupons, which came in the caps of the the Luscious Lemon soda bottles. There were two kinds of coupons: 5-point coupons and 2-point coupons. So far, she had collected 23 coupons, worth 76 points. How many 5-point and 2-point coupons did Noelani have?

FIND OUT
- What is the question you have to answer?
- What is Noelani saving?
- What are the two different kinds of coupons?
- How many coupons does Noelani have?
- How many points are her coupons worth?

CHOOSE A STRATEGY
- How can you systematically lay out the information for this problem?

SOLVE IT
- When you set up a list, you will need two columns. What do you want to keep track of in each column?
- What do you want to keep track of in each row of your list?
- Fill in several rows of your list. Do you have enough coupons that you can combine numbers from each column to make 23?
- Fill in more rows of your list. What are some combinations of numbers that add up to 23?
- When you find two numbers that make 23, what does the combined value of those coupons need to be? What is the value of nine 2-point coupons and fourteen 5-point coupons?
- Continue to fill in your list and test combinations of numbers that make 23. How many 2-point coupons and how many 5-point coupons does Noelani have?

2 point		5 point	
1	2	1	5
2	4	2	10
3	6	3	15

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

27 Noelani was saving Luscious Lemon soda coupons, which came in the caps of the the Luscious Lemon soda bottles. There were two kinds of coupons: 5-point coupons and 2-point coupons. So far, she had collected 23 coupons, worth 76 points. How many 5-point and 2-point coupons did Noelani have?

FIND OUT
- What is the question you have to answer? *How many 5-point and 2-point coupons did Noelani have?*
- What is Noelani saving? *Coupons from Luscious Lemon soda tops*
- What are the two different kinds of coupons? *5-point and 2-point*
- How many coupons does Noelani have? *23*
- How many points are her coupons worth? *76*

CHOOSE A STRATEGY
- How can you systematically lay out the information for this problem? *We can make an organized list.*

SOLVE IT
- When you set up a list, you will need two columns. What do you want to keep track of in each column? *5-point coupons and 2-point coupons*
- What do you want to keep track of in each row of your list? *The number of coupons and how many points that number of coupons is worth*
- Fill in several rows of your list. Do you have enough coupons that you can combine numbers from each column to make 23? *No*
- Fill in more rows of your list. What are some combinations of numbers that add up to 23? *14 + 9, 16 + 7*
- When you find two numbers that make 23, what does the combined value of those coupons need to be? *76* What is the value of nine 2-point coupons and fourteen 5-point coupons? *88*
- Continue to fill in your list and test combinations of numbers that make 23. How many 2-point coupons and how many 5-point coupons does Noelani have? *Thirteen 2-point coupons and ten 5-point coupons*

Solution: thirteen 2-point coupons and ten 5-point coupons

2 point		5 point	
1	2	1	5
2	4	2	10
3	6	3	15
4	8	4	20
5	10	5	25
6	12	6	30
7	14	7	35
8	16	8	40
9	18	9	45
10	20	10	50
11	22	11	55
12	24		
13	26		

26 + 50

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

EXTEND IT
- Noelani is trying to collect 100 points. If she has 76, what are the possible combinations of coupons she could get to give her 100 points?

28 Steven is leaning over the video game, concentrating on shooting down as many spaceships as he can within the time limit. Vulcan spaceships are worth 20 points each, and Android spaceships are worth 25 points. If he shoots down 21 spaceships for a total of 465 points, how many Vulcan spaceships and how many Android spaceships did Steven shoot down?

FIND OUT
- What is the question you have to answer?
- What is Steven doing?
- How many different kinds of spaceships are there?
- How much are Vulcan spaceships worth?
- How much are Android spaceships worth?
- How many spaceships does Steven shoot down?
- How many points are the spaceships worth?

CHOOSE A STRATEGY
- How can you systematically lay out the information for this problem?

SOLVE IT
- How many columns do you need in your list? What are you going to put at the top of each column?
- What are you keeping track of in each row of your list?
- Fill in several rows of your list. Do you have enough spaceships that you can combine numbers from each column to make 21?
- Fill in more rows of your list. What are some combinations of numbers that add up to 21?
- When you find two numbers that make 21, what does the combined value of these spaceships need to be? What is the value of eleven Vulcan spaceships and ten Android spaceships?
- Continue to fill in your list and test combinations of numbers that make 21. How many Vulcan and Android spaceships did Steven shoot down?

Vulcan		Android	
1	20	1	25
2	40	2	50

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

28 Steven is leaning over the video game, concentrating on shooting down as many spaceships as he can within the time limit. Vulcan spaceships are worth 20 points each, and Android spaceships are worth 25 points. If he shoots down 21 spaceships for a total of 465 points, how many Vulcan spaceships and how many Android spaceships did Steven shoot down?

FIND OUT
- What is the question you have to answer? *How many Vulcan spaceships and how many Android spaceships did Steven shoot down?*
- What is Steven doing? *Playing a video game and shooting down spaceships*
- How many different kinds of spaceships are there? *2*
- How much are Vulcan spaceships worth? *20*
- How much are Android spaceships worth? *25*
- How many spaceships does Steven shoot down? *21*
- How many points are the spaceships worth? *465*

CHOOSE A STRATEGY
- How can you systematically lay out the information for this problem? *We can use an organized list.*

SOLVE IT
- How many columns do you need in your list? *2* What are you going to put at the top of each column? *Android and Vulcan*
- What are you keeping track of in each row of your list? *The number of spaceships and how many points that number of ships is worth*
- Fill in several rows of your list. Do you have enough spaceships that you can combine numbers from each column to make 21? *No*
- Fill in more rows of your list. What are some combinations of numbers that add up to 21? *10 + 11, 13 + 8*
- When you find two numbers that make 21, what does the combined value of these spaceships need to be? *465* What is the value of eleven Vulcan spaceships and ten Android spaceships? *470*
- Continue to fill in your list and test combinations of numbers that make 21. How many Vulcan and Android spaceships did Steven shoot down? *12 Vulcan spaceships and 9 Android spaceships*

Solution: 12 Vulcan, 9 Android

Vulcan ships		Android ships	
#	pts	#	pts
1	20	1	25
2	40	2	50
3	60	3	75
4	80	4	100
5	100	5	125
6	120	6	150
7	140	7	175
8	160	8	200
9	180	9	225
10	200		
11	220		
12	240		

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your list. Is your answer reasonable?

EXTEND IT
- Steven's goal is to score 1000 points. Right now he has 465 points. How many more Vulcan spaceships and Android spaceships does Steven have to shoot down to reach his goal?

PRACTICE
- Similar Practice Problems: 55, 82, 107

29 At Marvelous Mark's T-Shirts, you can win a free T-shirt if you can solve the pebble puzzle. Next to the jar at the entrance is this information:

There are 825 pebbles in the jar.
There are 375 more brown pebbles than white pebbles.
Guess the number of brown and white pebbles in the jar and win a free T-shirt!

What is the right answer to the pebble puzzle?

FIND OUT
- What is the question you have to answer?
- What is going on at Marvelous Mark's T-Shirts?
- How can you win a free T-shirt?
- How many pebbles are there in the jar?
- What do you know about the brown pebbles?

CHOOSE A STRATEGY
- Will guessing the answer help you to solve this problem?
- How can you use the information from an incorrect guess?

SOLVE IT
- What is the total number of pebbles in the jar?
- How many more brown pebbles than white pebbles are there?
- Make a guess, keeping in mind the total number of pebbles and how many more brown than white there are. What is your guess?
- How can you check your guess? How did you do?
- If your guess was wrong, how can you use the information to make your next guess?
- Keep making guesses and checking them until you get the right answer. How many brown and how many white pebbles are in the jar?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your guess. Is your answer reasonable?

29   At Marvelous Mark's T-Shirts, you can win a free T-shirt if you can solve the pebble puzzle. Next to the jar at the entrance is this information:

There are 825 pebbles in the jar.
There are 375 more brown pebbles than white pebbles.
Guess the number of brown and white pebbles in the jar and win a free T-shirt!

What is the right answer to the pebble puzzle?

FIND OUT
- What is the question you have to answer? *How many brown and white pebbles are in the jar?*
- What is going on at Marvelous Mark's T-Shirts? *They are giving away free T-shirts.*
- How can you win a free T-shirt? *By solving the pebble puzzle.*
- How many pebbles are there in the jar? *825*
- What do you know about the brown pebbles? *There are 375 more brown pebbles than white pebbles.*

CHOOSE A STRATEGY
- Will guessing the answer help you to solve this problem? *Yes, because it's not really clear how to solve this problem in another way.*
- How can you use the information from an incorrect guess? *It can help us decide whether the next guess should be higher or lower.*

SOLVE IT
- What is the total number of pebbles in the jar? *825*
- How many more brown pebbles than white pebbles are there? *375*
- Make a guess, keeping in mind the total number of pebbles and how many more brown than white there are. What is your guess for the number of brown pebbles? *500* White pebbles? *325*
- How can you check your guess? *Subtract the white from the brown.* How did you do? *There are 175 more brown than white.*
- If your guess was wrong, how can you use the information to make your next guess? *We need to make the number higher for brown and lower for white.*
- Keep making guesses and checking them until you get the right answer. How many brown and how many white pebbles are in the jar? *600 brown, 225 white*

Solution: 600 brown, 225 white

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your guess. Is your answer reasonable?

EXTEND IT
- There are 1190 pebbles in the jar today, and there are 310 more brown than white pebbles. How many brown and white pebbles are in the jar?

30 At a recent convention of millipedes and centipedes held at the Hotel Fleeflea, the desk clerk registered 293 guests. He noticed that each millipede had 16 legs and that each centipede had 14 legs. Seth, the shoeshine spider, reported that there were 4,408 legs in total at the convention. He had to shine all their shoes! Can you help the desk clerk at the Hotel Fleeflea figure out how many of the registered guests were millipedes, and how many were centipedes?

FIND OUT
- What is the question you have to answer?
- What was happening at the Hotel Fleeflea?
- Who was attending the convention?
- What did the desk clerk notice about the millipedes and centipedes?
- What did Seth report to the desk clerk about the total number of legs at the convention?

CHOOSE A STRATEGY
- Will guessing the answer help you to solve this problem?
- How can you use the information from an incorrect guess?

SOLVE IT
- What was the total number of centipedes and millipedes?
- How many legs did each centipede have?
- How many legs did each millipede have?
- Make a guess, and remember that the total number can't be more than 293. How many centipedes? How many millipedes?
- How can you check your guess? How did you do?
- If your guess was wrong, how can you use this information to make your next guess?
- Keep making guesses and checking them until you find the right combination. How many millipedes were there? How many centipedes?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your guess. Is your answer reasonable?

30 At a recent convention of millipedes and centipedes held at the Hotel Fleeflea, the desk clerk registered 293 guests. He noticed that each millipede had 16 legs and that each centipede had 14 legs. Seth, the shoeshine spider, reported that there were 4,408 legs in total at the convention. He had to shine all their shoes! Can you help the desk clerk at the Hotel Fleeflea figure out how many of the registered guests were millipedes, and how many were centipedes?

FIND OUT
- What is the question you have to answer? *How many millipedes and how many centipedes were registered at the hotel?*
- What was happening at the Hotel Fleeflea? *They were having a convention.*
- Who was attending the convention? *Millipedes and centipedes*
- What did the desk clerk notice about the millipedes and centipedes? *The millipedes had 16 legs and the centipedes had 14 legs.*
- What did Seth report to the desk clerk about the total number of legs at the convention? *There were 4,408 legs altogether.*

CHOOSE A STRATEGY
- Will guessing the answer help you to solve this problem? *Yes, because we really don't know how to figure this out.*
- How can you use the information from an incorrect guess? *We can figure out whether the numbers are too high or too low.*

SOLVE IT
- What was the total number of centipedes and millipedes? *293*
- How many legs did each centipede have? *14*
- How many legs did each millipede have? *16*
- Make a guess, and remember that the total number can't be more than 293. How many centipedes? *110* How many millipedes? *183*
- How can you check your guess? *Multiply each number times the right number of legs and then add these two numbers together.* How did you do? *110 × 14 = 1540, 183 × 16 = 2928, 1540 + 2928 = 4468*
- If your guess was wrong, how can you use this information to make your next guess? *The answer for the guess was too high, so we probably want to reduce the number of millipedes and increase the number of centipedes.*
- Keep making guesses and checking them until you find the right combination. How many millipedes were there? *153* How many centipedes? *140*

  Solution: 153 millipedes, 140 centipedes

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your guess. Is your answer reasonable?

EXTEND IT
- At the next millipede and centipede convention there were 355 registered guests and Seth reported a total of 5340 legs. How many centipedes and how many millipedes attended the convention?

PRACTICE
- Similar Practice Problems: 66, 106, 111

31 The Chen family has one grandmother and one grandfather, five mothers and five fathers. Each mother has three daughters and each daughter has two brothers. The grandmother has only one daughter-in-law. What is the fewest members of the Chen family there could be?

FIND OUT
- What is the question you have to answer?
- Who are the oldest members of the Chen family?
- How many mothers are in the family? How many fathers?
- How many daughters does each mother have?
- How many brothers does each daughter have?
- How many daughters-in-law does the grandmother have?

CHOOSE A STRATEGY
- Would it be helpful to try and "see" this problem by making a picture or diagram?
- Is there another strategy that would be helpful to use with this problem?

SOLVE IT
- Draw a family tree diagram. Who goes at the top of the family tree?
- How many children do the grandmother and the grandfather have? How did you come to this conclusion? Where are you going to put these children on the family tree?
- How many of these children are married? How many daughters-in-law does the grandmother have?
- What clue do you have to the number of grandchildren there are? How many girls? How many boys? Where are you going to put the grandchildren on the family tree?
- What is the fewest members of the Chen family there could be?

              Grandmother   Grandfather

daughter   daughter   daughter

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

31 The Chen family has one grandmother and one grandfather, five mothers and five fathers. Each mother has three daughters and each daughter has two brothers. The grandmother has only one daughter-in-law. What is the fewest members of the Chen family there could be?
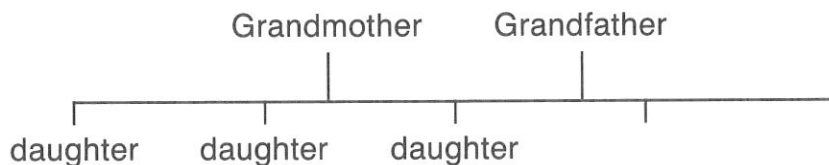
FIND OUT
- What is the question you have to answer? *What is the fewest members of the Chen family there could be?*
- Who are the oldest members of the Chen family? *The grandmother and the grandfather*
- How many mothers are in the family? *5* How many fathers? *5*
- How many daughters does each mother have? *3*
- How many brothers does each daughter have? *2*
- How many daughters-in-law does the grandmother have? *1*

CHOOSE A STRATEGY
- Would it be helpful to try and "see" this problem by making a picture or diagram? *Yes, we can make a diagram.*
- Is there another strategy that would be helpful to use with this problem? *Yes, we can use logical reasoning.*

SOLVE IT
- Draw a family tree diagram. Who goes at the top of the family tree? *The grandmother and grandfather*
- How many children do the grandmother and the grandfather have? *5* How did you come to this conclusion? *The problem states that each mother has 3 daughters and each daughter has 2 brothers, 3 + 2 = 5. Where are you going to put these children on the family tree? In the second row down from the grandmother and grandfather*
- How many of these children are married? *4, because there are 5 mothers, so if the grandmother is 1 mother there are 4 more.* How many daughters-in-law does the grandmother have? *One, therefore one son is not married.*
- What clue do you have to the number of grandchildren there are? *We know that there are 5 mothers that have children and each mother has 5 children. The grandmother makes 1 mother, leaving 4, and 4 × 5 = 20* How many girls? *12* How many boys? *8* Where are you going to put the grandchildren on the family tree? On the next level down from the grandmother and grandfather's children*
- What is the fewest members of the Chen family there could be? *31*

Solution: 31
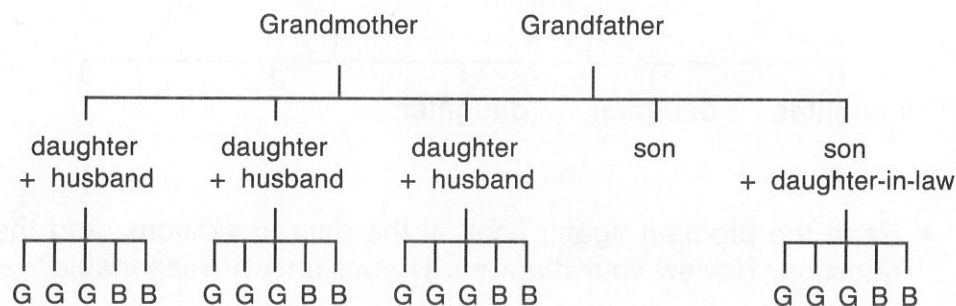
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your diagram. Is your answer reasonable?

EXTEND IT
- There are 6 mothers and 6 fathers. Half the mothers have sons. One mother has 2 sons and 2 mothers have 3 sons. Half the mothers have daughters. One mother has 5, one has 1, and the other has 3. There are 8 grandsons. Can you draw a family tree and figure out the fewest members of the family there could be?

32 Gonzalo sits down on the step in front of his house with his clipboard and pencil, and begins counting the pickup trucks that go past. Gonzalo has to make a graph for his sixth-grade math class. Out of the 18 pickup trucks that he counts, 17 have camper shells, 8 have CB antennas, and 7 have both. When Gonzalo sits down to draw the graph, how many pickups belong in the group that has ONLY camper shells, and how many belong in the group that has ONLY CB antennas?

FIND OUT
- What is the question you have to answer?
- What is Gonzalo doing?
- How many pickup trucks does Gonzalo count altogether?
- How many pickups with camper shells does Gonzalo count?
- How many pickups with CB antennas does Gonzalo count?
- How many pickups with both a camper shell and a CB antenna does Gonzalo count?

CHOOSE A STRATEGY
- In this case where you are trying to separate groups by attributes, you can use a special type of diagram called a Venn diagram. Draw intersecting circles and in the intersection put the number that represents shared group members.
- What other strategy can help you organize your thinking about this problem?

SOLVE IT
- Set up 2 intersecting circles. What are you going to label circle 1? circle 2?
- What number goes in the middle, or intersection?
- How many camper shells did Gonzalo count? How many pickups had both camper shells and CB antennas? How can you find the number that represents pickups with only camper shells?
- How many pickups with CB antennas did Gonzalo count? How many pickups had both CB antennas and camper shells? How can you find the number of pickups that had only CB antennas?
- How many pickups had only a camper shell? How many pickups had only a CB antenna?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

32 Gonzalo sits down on the step in front of his house with his clipboard and pencil, and begins counting the pickup trucks that go past. Gonzalo has to make a graph for his sixth-grade math class. Out of the 18 pickup trucks that he counts, 17 have camper shells, 8 have CB antennas, and 7 have both. When Gonzalo sits down to draw the graph, how many pickups belong in the group that has ONLY camper shells, and how many belong in the group that has ONLY CB antennas?
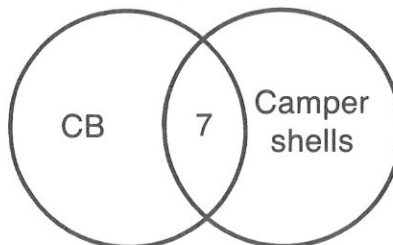
FIND OUT
- What is the question you have to answer? *How many pickups belong in the group that has only camper shells and how many pickups belong in the group that has only CB antennas?*
- What is Gonzalo doing? *Counting pickup trucks and marking whether they have camper shells or CB antennas, or both*
- How many pickup trucks does Gonzalo count altogether? *18*
- How many pickups with camper shells does Gonzalo count? *17*
- How many pickups with CB antennas does Gonzalo count? *8*
- How many pickups with both a camper shell and a CB antenna does Gonzalo count? *7*

CHOOSE A STRATEGY
- In this case where you are trying to separate groups by attributes, you can use a special type of diagram called a Venn diagram. Draw intersecting circles and in the intersection put the number that represents shared group members.
- What other strategy can help you organize your thinking about this problem? *We can use logical reasoning.*

SOLVE IT
- Set up 2 intersecting circles. What are you going to label circle 1? *Camper shells* circle 2? *CB*
- What number goes in the middle, or intersection? *7*
- How many camper shells did Gonzalo count? *17* How many pickups had both camper shells and CB antennas? *7* How can you find the number that represents pickups with only camper shells? *Subtract the number for both from the total number, 17 − 7 = 10.*
- How many pickups with CB antennas did Gonzalo count? *8* How many pickups had both CB antennas and camper shells? *7* How can you find the number of pickups that had only CB antennas? *Subtract the number for both from the total number with CB antennas, 8 − 7 = 1.*
- How many pickups had only a camper shell? *10* How many pickups had only a CB antenna? *1*

Solution: Camper shells 10, CB antennas 1
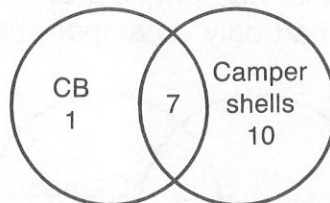
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- On another day Ramon counted a total of 39 pickup trucks. He counted 37 pickups with camper shells and 13 with CB antennas. He counted 11 pickups that had both. How many pickup trucks had only camper shells and how many had only CB antennas?

PRACTICE
- Similar Practice Problems: 81, 96, 117
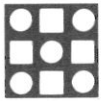
33 In the country of Tran, the towns were set up in an interesting way. The first town had only one house: • , then the houses in the second town looked like this: •
                                                                            • •

and the third town looked like this:
       •
      • •
     • • •

and the fourth town looked like this:
        •
       • •
      • • •
     • • • •

Each town was formed in a triangle that increased each time in the same way. How many houses would be in town 9?

FIND OUT
- What is the question you have to answer?
- How many houses are in town 1?
- How many houses are in town 2?
- How many houses are in town 3?
- How many houses are in town 4?

CHOOSE A STRATEGY
- If you look at each town and compare it to the next town, what do you discover?
- How can you organize the information in the problem?

SOLVE IT
- Can you draw the next town? The houses in each town are in what geometric shape?
- Is there a faster way to find out about town 9 than drawing each town?
- If you set up a table, what do you want to keep track of in the first row?
- What do you want to keep track of in the second row?
- Look for a pattern of change. What is the difference between towns 1 and 2? towns 2 and 3? towns 3 and 4?
- What is the pattern of change?
- Continue filling in the table until you get to town 9. How many houses are in town 9?

| Town	1	2	3	4	5	
Houses	1	3	6			

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

33 In the country of Tran, the towns were set up in an interesting way. The first town had only one house: • , then the houses in the second town looked like this: ••

and the third town looked like this: •••

and the fourth town looked like this: ••••

Each town was formed in a triangle that increased each time in the same way. How many houses would be in town 9?

FIND OUT
- What is the question you have to answer? *How many houses would be in town 9?*
- How many houses are in town 1? *1*
- How many houses are in town 2? *3*
- How many houses are in town 3? *6*
- How many houses are in town 4? *10*

CHOOSE A STRATEGY
- If you look at each town and compare it to the next town, what do you discover? *A pattern*
- How can you organize the information in the problem? *We can make a table.*

SOLVE IT
- Can you draw the next town? *Yes* The houses in each town are in what geometric shape? *A triangle*
- Is there a faster way to find out about town 9 than drawing each town? *Yes, if we make a table and find a pattern, then we can fill in the table.*
- If you set up a table, what do you want to keep track of in the first row? *The number of the town*
- What do you want to keep track of in the second row? *The number of houses in that town*
- Look for a pattern of change. What is the difference between towns 1 and 2? *2* towns 2 and 3? *3* towns 3 and 4? *4*
- What is the pattern of change? *The difference increases by 1 each time.*
- Continue filling in the table until you get to town 9. How many houses are in town 9? *45*

Solution: 45

Town	1	2	3	4	5	6	7	8	9
Houses	1	3	6	10	15	21	28	36	45

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

34 A large crowd gathered in the cold and watched preparations for the winter snow festival. Some kids from the high school built a pyramid out of huge snowballs. The pyramid had a square base. The top of the pyramid looked like this: ◯

The top two layers looked like this: ⬤

The top three layers looked like this: ⬤

and the top four layers looked like this: ⬤

If the students built a pyramid with 10 layers and each layer increased in the same way, how many snowballs did they use?

FIND OUT
- What is the question you have to answer?
- What were the high school kids doing for the winter festival?
- How many snowballs were in the top layer of the pyramid?
- How many snowballs were in the top two layers?
- How many snowballs were in the top three layers?
- How many snowballs were in the top four layers?
- How many layers did the pyramid have?

CHOOSE A STRATEGY
- If you look at each layer of the pyramid and compare it to the next layer, what do you discover?
- How can you organize the information in the problem?

SOLVE IT
- Can you draw the top five layers of the pyramid? Is there an easier way?
- If you make a table, what are you keeping track of in the first row?
- What are you keeping track of in the second row of the table?
- What is the difference between layers 1 and 2? layers 2 and 3? layers 3 and 4?
- If you look for a pattern of change, what do you see?
- Continue the table, until you reach 10 layers. How many snowballs did the students use?

Layer	1	2	3	4	5	
Snowballs	1	5	14			

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

34

A large crowd gathered in the cold and watched preparations for the winter snow festival. Some kids from the high school built a pyramid out of huge snowballs. The pyramid had a square base. The top of the pyramid looked like this: ○

The top two layers looked like this:

The top three layers looked like this:

and the top four layers looked like this:

If the students built a pyramid with 10 layers and each layer increased in the same way, how many snowballs did they use?

FIND OUT
- What is the question you have to answer? *How many snowballs did the students use?*
- What were the high school kids doing for the winter festival? *Building a pyramid out of snowballs*
- How many snowballs were in the top layer of the pyramid? *1*
- How many snowballs were in the top two layers? *5*
- How many snowballs were in the top three layers? *14*
- How many snowballs were in the top four layers? *30*
- How many layers did the pyramid have? *10*

CHOOSE A STRATEGY
- If you look at each layer of the pyramid and compare it to the next layer, what do you discover? *There is a pattern.*
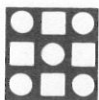- How can you organize the information in the problem? *We can make a table.*

SOLVE IT
- Can you draw the top five layers of the pyramid? *Yes* Is there an easier way? *Yes, we can make a table.*
- If you make a table, what are you keeping track of in the first row? *The number of the layers*
- What are you keeping track of in the second row of the table? *How many snowballs in that number of layers*
- What is the difference between layers 1 and 2? *4* layers 2 and 3? *9* layers 3 and 4? *16*
- If you look for a pattern of change, what do you see? *Each increase is a square of a number, beginning with 2 and then increasing by 1. After the square of 2 is the square of 3, then the square of 4, etc.*
- Continue the table until you reach 10 layers. How many snowballs did the students use? *385*

Solution: 385

Layer	1	2	3	4	5	6	7	8	9	10
Snowballs	1	5	14	30	55	91	140	204	285	385

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 68, 87, 112

35 Mario, Jake, Erica, and Erica's sister Charlene all enjoy sports. Jake likes to play tennis and always carries a racket. Mario and Erica's sister can't swim. Neither Jake nor Mario have a bike. Can you match each person with these items: a bike, a skateboard, a tennis racket, and a surfboard?

FIND OUT
- What is the question you have to answer?
- Who enjoys sports?
- What do you know about Jake?
- What do you know about Mario and Erica's sister?
- What do you know about Jake and Mario?
- What sports equipment do you have to match up with Mario, Jake, Erica, and Charlene?

CHOOSE A STRATEGY
- The information in this problem is given in a set of clues. If you use a series of "If. . .then" statements to solve this problem, what kind of thinking will you be using?
- Is there another strategy you can use to record the information?

SOLVE IT
- When you set up a table, how many columns do you need? How many rows?
- How are you going to label the columns? The rows?
- What do you know about Jake?
- In which box of the table can you put a Y for yes?
- If you match Jake with one of the items, then you can put an N in the other boxes in Jake's column. Where can you put more Ns?
- What do you know about Mario and Erica's sister? Where can you put an N in each column for them?
- Now you should be able to draw a logical conclusion about who belongs to the surfboard. Who has the surfboard?
- Continue to fill in the table until all the boxes have a Y or N. Who belongs to the bike? skateboard? tennis racket? surfboard?

	Mario	Jake	Erica	Charlene
Bike				
Racket		Y		
Skate-board				
Surf-board				

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and logical reasoning. Is your answer reasonable?

35 Mario, Jake, Erica, and Erica's sister Charlene all enjoy sports. Jake likes to play tennis and always carries a racket. Mario and Erica's sister can't swim. Neither Jake nor Mario have a bike. Can you match each person with these items: a bike, a skateboard, a tennis racket, and a surfboard?

FIND OUT
- What is the question you have to answer? *Can you match each person with these items: a bike, a skateboard, a tennis racket, and a surfboard?*
- Who enjoys sports? *Mario, Jake, Erica, and her sister Charlene*
- What do you know about Jake? *He always carries a racket.*
- What do you know about Mario and Erica's sister? *They can't swim.*
- What do you know about Jake and Mario? *They don't have bikes.*
- What sports equipment do you have to match up with Mario, Jake, Erica, and Charlene? *A bike, skateboard, tennis racket, surfboard*

CHOOSE A STRATEGY
- The information in this problem is given in a set of clues. If you use a series of "If...then" statements to solve this problem, what kind of thinking will you be using? *Logical reasoning*
- Is there another strategy you can use to record the information? *Yes, we can make a table.*

SOLVE IT
- When you set up a table, how many columns do you need? *4* How many rows? *4*
- How are you going to label the columns? *Mario, Jake, Erica, Charlene* The rows? *racket, bike, skateboard, surfboard* (Students could reverse the columns and the rows.)
- What do you know about Jake? *He carries a racket everywhere.*
- In which box of the table can you put a Y for yes? *Tennis racket*
- If you match Jake with one of the items, then you can put an N in the boxes for the other items in Jake's column. Where can you put more Ns? *For everyone else in the tennis racket row.*
- What do you know about Mario and Erica's sister? *They can't swim.* Where can you put an N in each column for them? *Next to surfboard*
- Now you should be able to draw a logical conclusion about who belongs to the surfboard. Who has the surfboard? *If Jake, Mario, and Charlene have an N for the surfboard, then it must belong to Erica.*
- Continue to fill in the table until all the boxes have a Y or N. Who belongs to the bike? *Charlene* skateboard? *Mario* tennis racket? *Jake* surfboard? *Erica*

Solution: bike - Charlene, skateboard - Mario
tennis racket - Jake, surfboard - Erica

	Mario	Jake	Erica	Charlene
Bike	N	N	N	Y
Racket	N	Y	N	N
Skate-board	Y	N	N	N
Surf-board	N	N	Y	N

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and logical reasoning. Is your answer reasonable?

EXTEND IT
- Try this: Charlene loves to take bike rides; Jake hates the water and isn't good at games that use balls; and Mario doesn't like to get wet. Who belongs to the skateboard, tennis racket, surfboard, and bike?

36 It is 7:00 P.M. on Back-to-School Night. Mrs. Anderson is welcoming the parents in her first class: Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo. She sees five of her students: Peter, Mary, Jack, Sam, and Jill. Mrs. Anderson notices that Mr. Green's daughter did not inherit his freckles; Mrs. Brown has big dimples when she smiles; Mary and Sam both have freckles and dimples; Mr. Black's son looks just like him; and Sam's father and Peter's father were unable to attend. Can you match up each student with a parent?

FIND OUT
- What is the question you have to answer?
- What is Mrs. Anderson doing?
- Who are the parents and students that came to Back-to-School Night?
- What do you know about Mr. Green? Mrs. Brown? Mary and Sam? Mr. Black? Sam and Peter?

CHOOSE A STRATEGY
- What kind of thinking can you use to organize the information in this problem?
- Is there another strategy you can use to record the information?

SOLVE IT
- When you set up your table, how many columns do you need? How many rows?
- How are you going to label the columns? the rows?
- What do you know about Mr. Green?
- What do you know about Mary and Sam?
- If Mary has freckles and Mr. Green's daughter does not, then what can you conclude about the only other girl?
- If you can put a Y in one of the boxes, then where can you put Ns?
- What do you know about Mr. Black?
- What do you know about Sam and Peter?
- If Sam's and Peter's fathers didn't come, but Mr. Black has a son, then what can you conclude about the only other boy?
- What do you know about Mrs. Brown?
- Continue to use "If. . .then" statements and fill in the boxes in the table, until you have marked each box with a Y or N.
- Who belongs to Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo?

	Mary	Sam	Peter	Jack	Jill
Mr. Black					
Mr. Green					
Mr. White					
Mrs. Brown					
Mrs. Rojo					

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and logical reasoning. Is your answer reasonable?

36 It is 7:00 P.M. on Back-to-School Night. Mrs. Anderson is welcoming the parents in her first class: Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo. She sees five of her students: Peter, Mary, Jack, Sam, and Jill. Mrs. Anderson notices that Mr. Green's daughter did not inherit his freckles; Mrs. Brown has big dimples when she smiles; Mary and Sam both have freckles and dimples; Mr. Black's son looks just like him; and Sam's father and Peter's father were unable to attend. Can you match up each student with a parent?

FIND OUT
- What is the question you have to answer? *Can you match up each student with a parent?*
- What is Mrs. Anderson doing? *Welcoming parents and students in her classroom for Back-to-School Night.*
- Who are the parents and students that came to Back-to-School Night? *Students: Peter, Mary, Jack, Sam, Jill Parents: Mr. Black, Mr. Green, Mr. White, Mrs. Brown, Mrs. Rojo*
- What do you know about Mr. Green? *His daughter didn't inherit his freckles.*
  What do you know about Mrs. Brown? *She has big dimples when she smiles.*
  What do you know about Mary and Sam? *They both have freckles and dimples.*
  What do you know about Mr. Black? *He has a son that looks just like him.*
  What do you know about Sam and Peter? *Their fathers didn't come.*

CHOOSE A STRATEGY
- What kind of thinking can you use to organize the information in this problem? *Logical reasoning*
- Is there another strategy you can use to record the information? *We can make a table.*

SOLVE IT
- When you set up your table, how many columns do you need? *5* How many rows? *5*
- How are you going to label the columns? *Mary, Sam, Peter, Jack, Jill* the rows? *Mr. Black, Mr. Green, Mr. White, Mrs. Brown, Mrs. Rojo* (The columns and rows could easily be reversed.)
- What do you know about Mr. Green? *His daughter did not inherit his freckles.*
- What do you know about Mary and Sam? *They both have freckles and dimples.*
- If Mary has freckles and Mr. Green's daughter does not then what can you conclude about the only other girl? *Jill must be Mr. Green's daughter.*
- Where can you put a Y and where can you put Ns? *You can put a Y for Jill in the row for Mr. Green, then put Ns in the rest of Jill's column and in the rest of the row for Mr. Green.*
- What do you know about Mr. Black? *He has a son that looks just like him.*
- What do you know about Sam and Peter? *Their fathers didn't come.*
- If Sam's and Peter's fathers didn't come, but Mr. Black has a son, then what can you conclude about the only other boy? *Jack must be Mr. Black's son.*
- What do you know about Mrs. Brown? *She has dimples.*
- Continue to use "If...then" statements and fill in the boxes in the table, until you have marked each box with a Y or N.
- Who belongs to Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo? *Mr. Black - Jack, Mr. Green - Jill, Mr. White - Mary, Mrs. Brown - Sam, Mrs. Rojo - Peter*

  Solution: Mr. Black - Jack, Mr. Green - Jill, Mr. White - Mary,
          Mrs. Brown - Sam, Mrs. Rojo - Peter

	Mary	Sam	Peter	Jack	Jill
Mr. Black	N	N	N	Y	N
Mr. Green	N	N	N	N	Y
Mr. White	Y	N	N	N	N
Mrs. Brown	N	Y	N	N	N
Mrs. Rojo	N	N	Y	N	N

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your table and logical reasoning. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 57, 72, 85

37 Armando has a card trick for Andrew: "I have 10 cards, numbered from 1 to 10. I have arranged the cards in a stack in a special way. The first card facing up is a 1, and I'm putting it on the table. The second card I'm putting at the bottom of the stack. The third card, which is a 2, I'm putting on the table next to 1. Then the fourth card goes to the bottom of the stack. I'll continue putting one card on the table and the next card to the bottom of the stack until I put card 10 on the table. How did I first arrange the cards in the stack?"

FIND OUT
- What is the question you have to answer?
- What is Armando doing?
- How many cards does Armando have? What are they?
- How does Armando lay out the cards?
- What is the first card in the stack? Where does Armando put it?
- Where does Armando put the second card?
- What is the third card? Where does he put it?
- Where does he put card 10?

CHOOSE A STRATEGY
- Would it help to move around pieces of paper that represent the cards?
- Is there another strategy that can help organize your thinking about this problem?

SOLVE IT
- Do you know the number of the first card in the stack? second? third? fourth? fifth?
- Try laying out the cards like this diagram, where each X means you leave a space for a card that goes under the stack:

  1 X 2 X 3 X 4 X 5 X

  Be sure to do this before going to the next questions.
- When you leave a space after 5, how many cards are accounted for? Where will the 1, 2, 3, 4, and 5 go? Where are the cards that the Xs stand for?
- Can you go back and fill in the first space now?
- Do you know what goes in the space after 2, or does this card go under again?
- What goes in the space after 3?
- What do you know about the card after 4?
- What goes in the space after 5?
- Fill in all the spaces. What order do you have the cards in?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your arrangement of cards. Is your answer reasonable?

37 Armando has a card trick for Andrew: "I have 10 cards, numbered from 1 to 10. I have arranged the cards in a stack in a special way. The first card facing up is a 1, and I'm putting it on the table. The second card I'm putting at the bottom of the stack. The third card, which is a 2, I'm putting on the table next to 1. Then the fourth card goes to the bottom of the stack. I'll continue putting one card on the table and the next card to the bottom of the stack until I put card 10 on the table. How did I first arrange the cards in the stack?"

FIND OUT
- What is the question you have to answer? *How did Armando arrange the cards in the stack?*
- What is Armando doing? *Showing Andrew a card trick*
- How many cards does Armando have? *10* What are they? *They are numbered from 1 to 10.*
- How does Armando lay out the cards? *First one face up on the table, then one to the bottom of the stack, the next face up, etc.*
- What is the first card in the stack? *1* Where does Armando put it? *Face up on the table*
- Where does Armando put the second card? *At the bottom of the stack*
- What is the third card? *2* Where does he put it? *Face up on the table next to the 1*
- Where does he put the fourth card? *At the bottom of the stack*
- Where does he put card 10? *Face up on the table*

CHOOSE A STRATEGY
- Would it help to move around pieces of paper that represent the cards? *Yes, because it will take a lot of experimenting to figure this out.*
- Is there another strategy that can help organize your thinking about this problem? *We can use logical reasoning.*

SOLVE IT
- Do you know the number of the first card in the stack? *Yes, 1* second? *No* third? *Yes, 2* fourth? *No* fifth? *It must be 3*
- Try laying out the cards like this diagram, where each X means you leave a space for a card that goes under the stack:

  1 X 2 X 3 X 4 X 5 X

  Be sure to do this before going to the next questions.
- When you leave a space after 5, how many cards are accounted for? *10* Where will the 1, 2, 3, 4, and 5 go? *Face up on the table* Where are the cards that the Xs stand for? *Still in the stack*
- Can you go back and fill in the first space now? *This should be a 6.*
- Do you know what goes in the space after 2, or does this card go under again? *The next card after the one put on the table always goes to the bottom of the stack, if we continue the pattern.*
- What goes in the space after 3? *The next number is 7.*
- What do you know about the card after 4? *It must go under.*
- What goes in the space after 5? *This should be 8.*
- Fill in all the spaces. What order do you have the cards in? *1-6-2-10-3-7-4-9-5-8*

  Solution: 1-6-2-10-3-7-4-9-5-8

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your arrangement of cards. Is your answer reasonable?

EXTEND IT
- Try the same trick, but use 16 cards, numbered 1 to 16.

38 A case of Seth's Sizzling Soda holds 24 bottles. Matt and Marnie are going to fill a case with 12 empty bottles to return to the store. The case has 6 columns across and 4 rows down. Matt challenges Marnie: "Can you find at least two different ways to place the 12 bottles in the case so that each row and each column has an odd number of bottles?"

FIND OUT
- What is the question you have to answer?
- What are Matt and Marnie doing?
- How many bottles are they returning to the store?
- What is the size of the case?
- What is Matt's challenge to Marnie?

CHOOSE A STRATEGY
- Would it help to have objects to represent the 12 bottles that you can arrange and then rearrange in the case?
- What other strategy do you need to use for this problem?

SOLVE IT
- If you use pieces of paper to represent the bottles, how many do you need?
- When you make a diagram, how many columns does it need? How many rows?
- What are the conditions for arranging the bottles?
- If you start by filling in row 1, what are some possible numbers of bottles you could put in this row? What are possible numbers of bottles for any row in the case? How many bottles are you putting in row 1?
- How many bottles are you going to put in row 2?
- Continue to fill in the rows. What do you need to keep track of for each row? What do you need to check on for each column? What does the total number of bottles need to be? How did you do?
- Keep moving the bottles around until you have the right arrangement. Now find another way to arrange the bottles with the same conditions.

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

38 A case of Seth's Sizzling Soda holds 24 bottles. Matt and Marnie are going to fill a case with 12 empty bottles to return to the store. The case has 6 columns across and 4 rows down. Matt challenges Marnie: "Can you find at least two different ways to place the 12 bottles in the case so that each row and each column has an odd number of bottles?"

FIND OUT
- What is the question you have to answer? *Can you find two different ways to place the 12 bottles in the case so that each row and each column has an odd number of bottles?*
- What are Matt and Marnie doing? *Taking bottles back to the store*
- How many bottles are they returning to the store? *12*
- What is the size of the case? *6 columns across and 4 rows down*
- What is Matt's challenge to Marnie? *Find two different ways to place the bottles in the case so that each row and each column has an odd number of bottles.*

CHOOSE A STRATEGY
- Would it help to have objects to represent the 12 bottles that you can arrange and then rearrange in the case? *Yes, then we can keep changing the arrangement easily until we find one, record it, then look for a different one.*
- What other strategy do you need to use for this problem? *We need to have diagrams of the case, so we can mark the arrangement when we find it.*

SOLVE IT
- If you use pieces of paper to represent the bottles, how many do you need? *12*
- When you make a diagram, how many columns does it need? *6* How many rows? *4*
- What are the conditions for arranging the bottles? *Each column and each row needs an odd number of bottles.*
- (Following is just an example for row 1 and 2, there are other ways to do this. Encourage the students to experiment. Are there more than two ways to do this?)
- If you start by filling in row 1, what are some possible numbers of bottles you could put in this row? *1, 3, 5, 7* What are possible numbers of bottles for any row in the case? *1, 3, 5, probably not 7 because each row needs at least 1 bottle in it.* How many bottles are you putting in row 1? *3*
- How many bottles are you going to put in row 2? *1*
- Continue to fill in the rows. What do you need to keep track of for each row? *The total, to make sure it's an odd number.* What do you need to check on for each column? *The total, to make sure it's an odd number.* What does the total number of bottles need to be? *12* How did you do?
- Keep moving the bottles around until you have the right arrangement. Now find another way to arrange the bottles with the same conditions.

Solution:

X	X	X			
X		X		X	
	X				
X	X	X	X		X

	X		X		X
X	X	X		X	X
	X		X		X
		X			

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 54, 77, 101

39 Delia, Tracy, and Bella are going cross-country skiing with a school group. They are carrying packs and each pack can weigh up to and including 10 pounds. The packs are weighed two at a time. Delia and Tracy weigh their packs together and the total is 24 pounds. When Delia and Bella weigh their packs, the total is 20 pounds. Tracy's and Bella's packs together weigh 18 pounds. Which skiers have packs that are too heavy, and by how much?
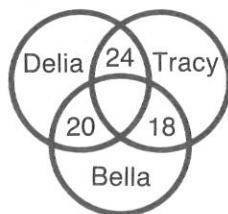
FIND OUT
- What is the question you have to answer?
- What are Delia, Tracy, and Bella doing?
- How many pounds are allowed per pack?
- How are the packs being weighed?
- What is the combined weight of Tracy's and Delia's packs?
- What is the combined weight of Delia's and Bella's packs?
- What is the combined weight of Tracy's and Bella's packs?

CHOOSE A STRATEGY
- You can use a series of "If...then" statements to solve this problem. What kind of thinking do we call this?
- Is there a special kind of diagram that could help you organize the information?
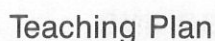
SOLVE IT
- When you make your Venn diagram, how many intersecting circles do you need? What are you going to label each circle?
- How are the packs weighed? Where do you put these numbers in the circles? Where does the number go for Tracy's and Delia's packs? the number for Delia's and Bella's packs? the number for Tracy's and Bella's packs?
- There are two numbers in Delia's circle. What other number goes in the circle and where does it go?
- If you estimate a number for Delia's pack, then you have to add it to what number to make 24? At the same time you have to add it to what number to make 20? If you estimate a number for Delia's pack, then you have to estimate what other numbers?
- How do you go about checking your estimates? If your estimates are incorrect, how can you use this information?
- Now make estimates, based on the totals in the intersections. Check to see that the numbers in each two circles add up to the total in the intersection between those two circles.
- Which packs are over the limit, and by how much?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?
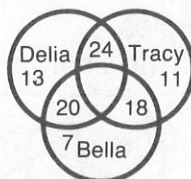
39 Delia, Tracy, and Bella are going cross-country skiing with a school group. They are carrying packs and each pack can weigh up to and including 10 pounds. The packs are weighed two at a time. Delia and Tracy weigh their packs together and the total is 24 pounds. When Delia and Bella weigh their packs, the total is 20 pounds. Tracy's and Bella's packs together weigh 18 pounds. Which skiers have packs that are too heavy, and by how much?

FIND OUT
- What is the question you have to answer? *Which skiers have packs that are too heavy, and by how much?*
- What are Delia, Tracy, and Bella doing and what is each one carrying? *They are going on a cross-country skiing trip and they are each carrying a pack.*
- How many pounds are allowed per pack? *10 pounds*
- How are the packs being weighed? *Two at a time*
- What is the combined weight of Tracy's and Delia's packs? *24 pounds*
- What is the combined weight of Delia's and Bella's packs? *20 pounds*
- What is the combined weight of Tracy's and Bella's packs? *18 pounds*

CHOOSE A STRATEGY
- You can use a series of "If...then" statements to solve this problem. What kind of thinking do we call this? *Logical reasoning*
- Is there a special kind of diagram that could help you organize the information? *We can make a Venn diagram.*

SOLVE IT
- When you make your Venn diagram, how many intersecting circles do you need? *3* What are you going to label each circle? *Tracy, Delia, or Bella*
- How are the packs weighed? *Two at a time* Where do you put these numbers in the circles? *In the intersections between each two circles* Where does the number go for Tracy's and Delia's packs? *In the intersection between Tracy and Delia* the number for Delia's and Bella's packs? *In the intersection between Delia and Bella* the number for Tracy's and Bella's packs? *The intersection between Tracy and Bella*
- There are two numbers in Delia's circle, what other number goes in the circle and where does it go? *The number for Delia's pack, in the space outside the intersections*
- If you estimate a number for Delia's pack, then you have to add it to what number to make 24? *The amount of Tracy's pack* At the same time you have to add it to what number to make 20? *The weight of Bella's pack* If you estimate a number for Delia's pack, then you have to estimate what other numbers? *We have to make an estimate for the weight of Tracy's and Bella's packs too.*
- How do you go about checking your estimates? *Add each two numbers outside the intersection and see if they total the number inside the intersection.* If your estimates are incorrect, how can you use this information? *We can make the estimates higher or lower, depending on how far off we are.*
- (Have the students make estimates, based on the totals in the intersections. Have them check to see that the numbers in each two circles add up to the total in the intersection between those two circles.)
- Which packs are over the limit, and by how much? *Delia is over by 3, Tracy by 1.*

Solution: Delia = 3, Tracy = 1

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- John, Harry, and Barry are going on the trip. John's and Harry's packs together weigh 28 pounds; Harry's and Barry's packs together weigh 24 pounds; John's and Barry's packs together weigh 16 pounds. How much does each pack weigh?

40 Albert is squirming excitedly in his bleacher seat, awaiting the results of the men's 50-meter freestyle. His older brother, David, was one of the swimmers, along with David's friends Mick and Rick. Mick's and Rick's combined times totaled :93.00 seconds. Rick's and David's times totaled :93.22, and David's and Mick's times totaled :92.74. Who had the best time, and by how much?
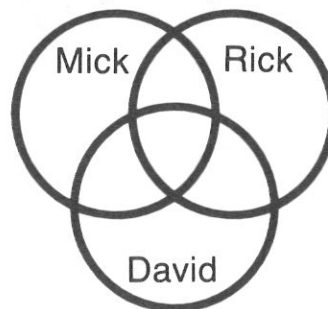
FIND OUT
- What is the question you have to answer?
- What is Albert doing?
- Who are the swimmers?
- How are the swimmers' times reported?
- What is the combined time for Mick and Rick?
- What is the combined time for Rick and David?
- What is the combined time for David and Mick?

CHOOSE A STRATEGY
- What kind of thinking can help you organize the information in this problem?
- Is there another strategy you can use with this problem?

SOLVE IT
- When you make your Venn diagram, how many intersecting circles do you need? How are you going to label each circle?
- How are the times reported? Where do you put these numbers in the circles? Where does the number go for Mick's and Rick's times? Rick's and David's times? David's and Mick's times?
- There are two numbers in Mick's circle. What other number goes in the circle and where does it go?
- If you estimate a number for Mick's time, then you have to add it together with what number to make :93.00? At the same time, you have to add it to what number to make :92.74? If you estimate a number for Mick's time, then you have to estimate what other numbers?
- How do you go about checking your estimates? If your estimates are incorrect, how can you use this information?
- Now make estimates, based on the totals in the intersections. Check to see that the numbers in each two circles add up to the total in the intersection between those two circles.
- Who had the fastest time, and by how much?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

40 Albert is squirming excitedly in his bleacher seat, awaiting the results of the men's 50-meter freestyle. His older brother, David, was one of the swimmers, along with David's friends Mick and Rick. Mick's and Rick's combined times totaled :93.00 seconds. Rick's and David's times totaled :93.22, and David's and Mick's times totaled :92.74. Who had the best time, and by how much?

FIND OUT
- What is the question you have to answer? *Who had the best time and by how much?*
- What is Albert doing? *Watching a 50-meter freestyle race*
- Who are the swimmers? *David, Rick, and Mick*
- How are the swimmers' times reported? *Two combined*
- What is the combined time for Mick and Rick? *:93.00*
- What is the combined time for Rick and David? *:93.22*
- What is the combined time for David and Mick? *:92.74*

CHOOSE A STRATEGY
- What kind of thinking can help you organize the information in this problem? *Logical reasoning*
- Is there another strategy you can use with this problem? *We can make a Venn diagram.*

SOLVE IT
- When you make your Venn diagram, how many intersecting circles do you need? *3* How are you going to label each circle? *Mick, Rick, or David*
- How are the times reported? *Two combined* Where do you put these numbers in the circles? *In the intersections* Where does the number go for Mick's and Rick's times? *In the intersection between Mick and Rick* Rick's and David's times? *In the intersection between Rick and David* David's and Mick's times? *In the intersection between David and Mick*
- There are two numbers in Mick's circle. What other number goes in the circle and where does it go? *The number for Mick's time which goes outside the intersections*
- If you estimate a number for Mick's time, then you have to add it together with what number to make :93.00? *The number for Rick* At the same time, you have to add it to what number to make :92.74? *The number for David* If you estimate a number for Mick's time, then you have to estimate what other numbers? *The time for Rick and David*
- How do you go about checking your estimates? *Add the numbers for two circles and see if the total is the same as the number in the intersection.* If your estimates are incorrect, how can you use this information? *Adjust the estimates up or down*
- (Have the students make estimates, based on the totals in the intersections. Then have them check to see that the numbers in each two circles add up to the total in the intersection between those two circles.)
- Who had the fastest time, and by how much? *Mick had the fastest time by .22 seconds.*

Solution: Mick = :46.26, faster by .22 seconds
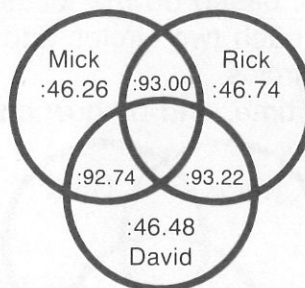
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE
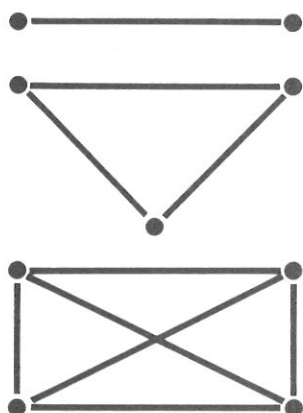- Similar Practice Problems: 99, 108, 114

41 A group of 13 friends were planning a trip. On the night before they left they made a lot of phone calls. Each friend talked to every other friend at least once. What is the fewest phone calls that could have been made?

FIND OUT
- What is the question you have to answer?
- Who was planning a trip?
- What did the friends do the night before the trip?
- Who did each friend talk with?

CHOOSE A STRATEGY
- Would it help to solve this same problem with a smaller number of friends, then apply what you've learned to a larger number?
- Are there other strategies that you can use with the first one?

SOLVE IT
- When you set up a table, what are you going to keep track of?
- If you begin with 2 friends and draw a diagram, how many calls will be made?
- If you have 3 friends, how many calls will be made?
- If you have 4 friends, how many calls will be made?
- How many calls will be made for 5 friends?
- Do you see a pattern in the way the number of calls increases?
- What is the fewest phone calls that could have been made?

Number of friends	Calls
2	1
3	3
4	6

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

41   A group of 13 friends were planning a trip. On the night before they left they made a lot of phone calls. Each friend talked to every other friend at least once. What is the fewest phone calls that could have been made?

FIND OUT
- What is the question you have to answer? *What is the fewest phone calls that could have been made?*
- Who was planning a trip? *A group of 13 friends*
- What did the friends do the night before the trip? *They called each other on the phone.*
- Who did each friend talk with? *With every other friend at least once*

CHOOSE A STRATEGY
- Would it help to solve this same problem with a smaller number of friends, then apply what you've learned to a larger number? *Yes, it would help to make the problem simpler.*
- Are there other strategies that you can use with the first one? *We can make diagrams and keep track of the number of calls in a table. We can use the table to look for a pattern.*

SOLVE IT
- When you set up a table, what are you going to keep track of? *The number of friends and the number of calls the friends make*
- If you begin with 2 friends and draw a diagram, how many calls will be made? *1*
- If you have 3 friends, how many calls will be made? *3*
- If you have 4 friends, how many calls will be made? *6*
- How many calls will be made for 5 friends? *10*
- Do you see a pattern in the way the number of calls increases? *Yes, each time the difference between the number of calls increases by one.*
- What is the fewest phone calls that could have been made? *78*

Solution: 78

Number of friends	Calls
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66
13	78

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- What if 20 friends each talk to each other friend twice. How many calls are made?

42 The annual Chinese New Year's parade is ready to begin. The parade marshal has stationed the 15 judges along the parade route at 10-yard intervals, to judge the students' papier-mache masks. At what place should the judges meet for ballot counting, so that together they travel the fewest possible yards?
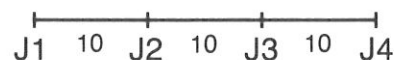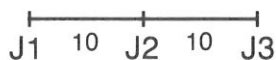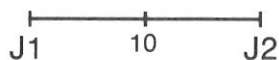
FIND OUT
- What is the question you have to answer?
- What is about to begin?
- Who is stationed along the parade route? How many are there?
- How far apart are the judges stationed?
- What are the conditions for the judges' meeting place?

CHOOSE A STRATEGY
- Would it help to solve this problem first for 2 and 3 judges?
- How can you organize the information in this problem?

SOLVE IT
- If you make a diagram for 2 judges, what labels do you need to put on your diagram?
- What are the possible meeting places for 2 judges?
- Which is the best meeting place for 2 judges?
- If you make a diagram for 3 judges, what labels do you need?
- What are the possible meeting points for 3 judges?
- What is the best place for them to meet, so that together they travel the fewest possible yards?
- Make a diagram for 4 judges. Then try making a diagram for 15 judges. What labels do you need?
- What are the possible meeting points?
- Where is the best place for the judges to meet, so that together they travel the fewest possible yards?

J1   10   J2   J1 10 J2 10 J3   J1 10 J2 10 J3 10 J4

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

The Problem Solver 6   T•83   © 1987 Creative Publications

42 The annual Chinese New Year's Parade is ready to begin. The parade marshal has stationed the 15 judges along the parade route at 10-yard intervals, to judge the students' papier-mache masks. Where should the judges meet to count ballots, so that together they travel the fewest possible yards?

FIND OUT
- What is the question you have to answer? *Where should the judges meet to count ballots, so that together they travel the fewest possible yards?*
- What is about to begin? *The Chinese New Year's Parade*
- Who is stationed along the parade route? *Judges* How many are there? *15*
- How far apart are the judges stationed? *At 10-yard intervals*
- What are the conditions for the judges' meeting place? *It should be where the judges can get to by traveling the shortest possible distance.*

CHOOSE A STRATEGY
- Would it help to solve this problem first for 2 and 3 judges? *Yes, it would help to make the problem simpler.*
- How can you organize the information in this problem? *We can make diagrams of the judges on the parade route.*

SOLVE IT
- If you make a diagram for 2 judges, what labels do you need to put on your diagram? *J1 and J2, and 10 between the 2*
- What are the possible meeting places for 2 judges? *At J1, J2, or in between*
- Which is the best meeting place for 2 judges? *At any location, because the total distance traveled would be 10 yards.*
- If you make a diagram for 3 judges, what labels do you need? *J1, J2, J3, and 10 between each 2 judges*
- What are the possible meeting points for 3 judges? *J1, J2, or J3*
- What is the best place for them to meet, so that together they travel the fewest possible yards? *At J2, then shortest total distance would be 20 yards*
- Make a diagram for 4 judges. Then try making a diagram for 15 judges. What labels do you need? *J1 - J15, and 10 for the 14 spaces in between*
- What are the possible meeting points? *J1 - J15*
- Where is the best place for the judges to meet, so that together they travel the fewest possible yards? *At J8, then total distance traveled would be 560 yards*
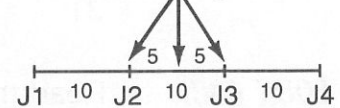
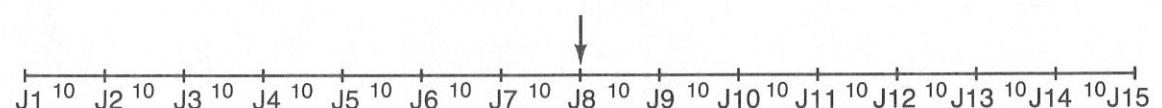Solution: Judge 8

Total = 10 yards
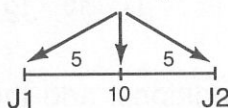
Total = 20 yards
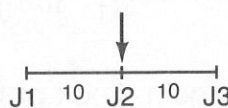
Total = 40 yards

Total = 560 yards

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- What if there are 25 judges placed at 15-yard intervals. Where is the best place for the judges to meet?

PRACTICE
- Similar Practice Problems: 73, 86, 120

43　Sara's softball team can be divided into four groups: $\frac{1}{2}$ the players are strong hitters, $\frac{1}{4}$ are good pitchers, $\frac{1}{8}$ like to play the outfield, and 2 players are catchers. If each player is in only one group, how many players are on the team and how many players are in each group?
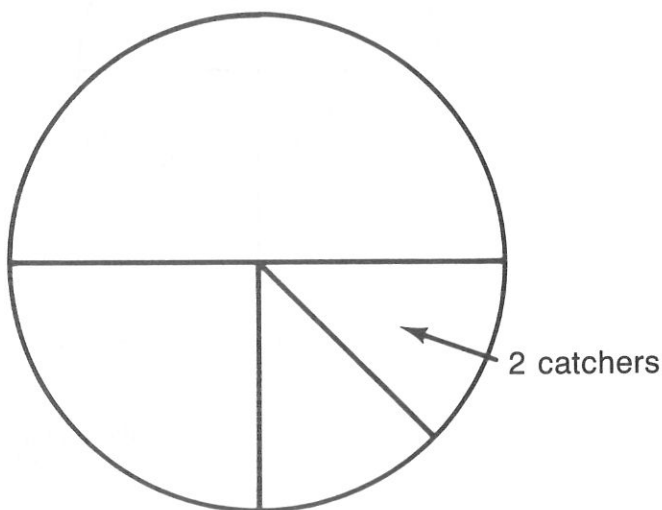

FIND OUT
- What is the question you have to answer?
- How many groups can you divide Sara's team into? Are there any conditions given about the groups?
- What do you know about the number of players on the team who are hitters?
- What do you know about the number of players on the team who are pitchers?
- What do you know about the number of players on the team who like the outfield?
- How many players on the team are catchers?

CHOOSE A STRATEGY
- To solve this problem you need to begin with the specific information you have, the number of catchers. How can you organize the rest of the information?
- Is there a good way to lay out the information?

SOLVE IT
- What kind of diagram would you make?
- How can you divide up the diagram? How can you label each part?
- Where can you begin filling in the diagram?
- If you work backwards, what can you fill in next?
- Continue to work backwards and fill in each part of the diagram. How many players are on Sara's team? How many players are in each group?

2 catchers

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

43 Sara's softball team can be divided into four groups: ½ the players are strong hitters, ¼ are good pitchers, ⅛ like to play the outfield, and 2 players are catchers. If each player is in only one group, how many players are on the team and how many players are in each group?

FIND OUT
- What is the question you have to answer? *How many players are on the team and how many players are in each group?*
- How many groups can you divide Sara's team into? *4* Are there any conditions given about the groups? *Each player is in only one group.*
- What do you know about the number of players on the team who are hitters? *½*
- What do you know about the number of players on the team who are pitchers? *¼*
- What do you know about the number of players on the team who like the outfield? *⅛*
- How many players on the team are catchers? *2*

CHOOSE A STRATEGY
- To solve this problem you need to begin with the specific information you have, the number of catchers. How can you organize the rest of the information? *We can work backwards from the 2 catchers.*
- Is there a good way to lay out the information? *We can make a circle graph.*

SOLVE IT
- What kind of diagram would you make? *A circle graph*
- How can you divide up the diagram? *½, ¼, ⅛, ⅛* How can you label each part? *½ = hitters, ¼ = pitchers, ⅛ = outfield, ⅛ (2) = catchers*
- Where can you begin filling in the diagram? *We can fill in the part for catchers, 2.*
- If you work backwards, what can you fill in next? *The other ⅛ must also be 2, so there is ⅛ that likes the outfield.*
- Continue to work backwards and fill in each part of the diagram. How many players are on Sara's team? *16* How many players are in each group? *Hitters = 8, pitchers = 4, outfield = 2, catchers = 2*
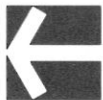
Solution: Hitters - 8, pitchers - 4, outfield - 2, catchers - 2
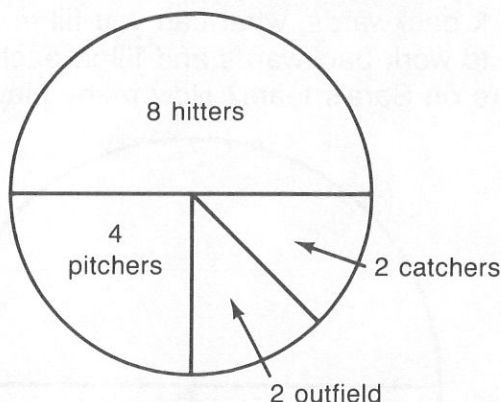
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- Sara's brother Terry is on a baseball team. Of the team, ⅔ are hitters, ⅙ are pitchers, and 3 are catchers. How many players are on the team and how many are in each group?

44   The National Space Board was taking applications for places in the experimental space station city being built near the moon. Half of the applications came from the United States, $\frac{1}{4}$ came from Canada, $\frac{1}{8}$ came from Russia, $\frac{1}{16}$ came from England, $\frac{1}{32}$ came from India, and 18 applications came from Mexico. How many applications in total have been received for the space city, and how many came from each country?
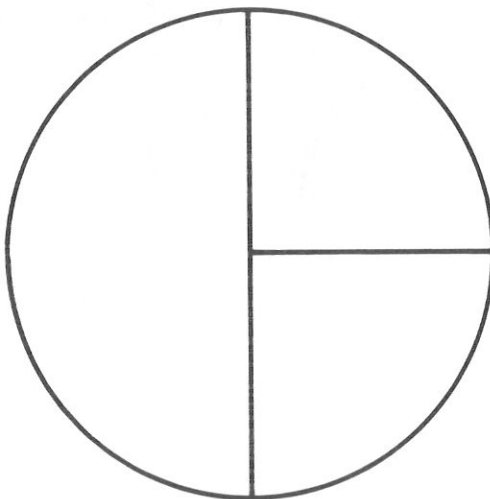
FIND OUT
- What is the question you have to answer?
- What is the National Space Board receiving?
- What do you know about the number of applications from the U.S.?
- What do you know about the number of applications from Canada?
- What do you know about the number of applications from Russia?
- What do you know about the number of applications from England?
- What do you know about the number of applications from India?
- How many applications came from Mexico?

CHOOSE A STRATEGY
- To solve this problem you need to begin with the specific information you have, the number of applications from Mexico. How can you organize the rest of the information?
- Is there a good way to lay out the information?

SOLVE IT
- What kind of diagram would you make?
- How can you divide up the diagram? How can you label each part?
- Where can you begin filling in the diagram?
- If you work backwards, what can you fill in next?
- Continue to work backwards and fill in each part of the diagram.
- How many applications did the National Space Board receive and how many came from each country?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

44 The National Space Board was taking applications for places in the experimental space station city being built near the moon. Half of the applications came from the United States, ¼ came from Canada, ⅛ came from Russia, ¹⁄₁₆ came from England, ¹⁄₃₂ came from India, and 18 applications came from Mexico. How many applications in total have been received for the space city, and how many came from each country?

FIND OUT
- What is the question you have to answer? *How many applications were received for the space city and how many came from each country?*
- What is the National Space Board receiving? *Applications for a future space city*
- What do you know about the number of applications from the U.S.? *½*
- What do you know about the number of applications from Canada? *¼*
- What do you know about the number of applications from Russia? *⅛*
- What do you know about the number of applications from England? *¹⁄₁₆*
- What do you know about the number of applications from India? *¹⁄₃₂*
- How many applications came from Mexico? *18*

CHOOSE A STRATEGY
- To solve this problem you need to begin with the specific information you have, the number of applications from Mexico. How can you organize the rest of the information? *We can work backwards.*
- Is there a good way to lay out the information? *We can use a circle graph.*

SOLVE IT
- What kind of diagram would you make? *A circle graph*
- How can you divide up the diagram? *½, ¼, ⅛, ¹⁄₁₆, ¹⁄₃₂, ¹⁄₃₂* How can you label each part? *½ = U.S., ¼ = Canada, ⅛ = Russia, ¹⁄₁₆ = England, ¹⁄₃₂ = India, ¹⁄₃₂ = Mexico (18)*
- Where can you begin filling in the diagram? *Mexico, 18*
- If you work backwards, what can you fill in next? *The other ¹⁄₃₂, India, must also have 18 applications.*
- Continue to work backwards and fill in each part of the diagram.
- How many applications did the National Space Board receive and how many came from each country? *576 applications, 288 = U.S., 144 = Canada, 72 = Russia, 36 = England, 18 = India, 18 = Mexico*

Solution: Total = 576, 288 - U.S., 144 - Canada, 72 - Russia, 36 - England, 18 - India, 18 - Mexico
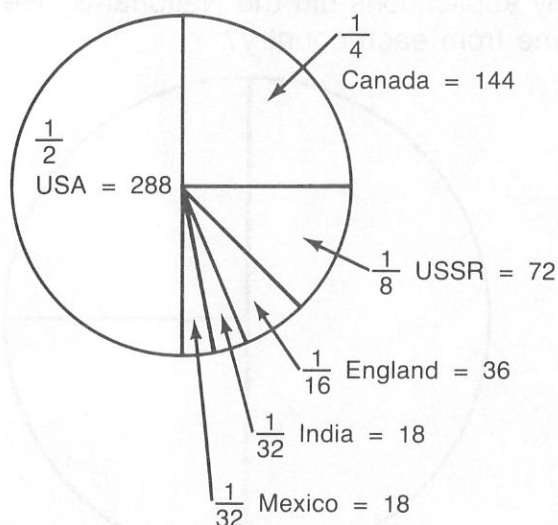
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 75, 94, 103

45 Your sock drawer has 25 electric yellow socks, 30 blue striped socks, 17 orange socks, 13 magnetic magenta socks, 33 pale purple socks, 30 royal red socks, 11 gruesome green socks, 14 midnight black socks, and 23 bruin brown socks! If you reach into the drawer in the dark, how many socks do you need to pull out to be sure you have a matching pair?

FIND OUT
- What is the question you have to answer?
- How many yellow socks are in the drawer? blue? orange? magenta? purple? red? green? black? brown?
- How many different colors are there?

CHOOSE A STRATEGY
- Would it be easier to solve this problem with fewer different colors?
- Is there another strategy you can use along with the first one?

SOLVE IT
- Begin with 3 different colors and 2 socks of each color: yellow, red, and green.
- If you pull out 1 yellow, what are the possibilities for the next sock you pull out? What are your chances that it is yellow?
- If you pull out a green sock, you have 1 yellow and 1 green. What are the possibilities for the next sock you pull out? What are your chances that it is yellow or green?
- If you pulled out a red sock, then you have 1 yellow, 1 red, and 1 green. What are the possibilities for the next sock? What are your chances of pulling out either red, green, or yellow?
- Now consider the problem you started with. How many socks do you need to pull out to be sure you have a matching pair?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

45 Your sock drawer has 25 electric yellow socks, 30 blue striped socks, 17 orange socks, 13 magnetic magenta socks, 33 pale purple socks, 30 royal red socks, 11 gruesome green socks, 14 midnight black socks, and 23 bruin brown socks! If you reach into the drawer in the dark, how many socks do you need to pull out to be sure you have a matching pair?

FIND OUT
- What is the question you have to answer? *If you reach into the drawer in the dark, how many socks do you need to pull out to be sure you have a matching pair?*
- How many yellow socks are in the drawer? *25* blue? *30* orange? *17* magenta? *13* purple? *33* red? *30* green? *11* black? *14* brown? *23*
- How many different colors are there? *9*

CHOOSE A STRATEGY
- Would it be easier to solve this problem with fewer different colors? *Yes, it would make the problem simpler.*
- Is there another strategy you can use along with the first one? *We can use logical reasoning.*

SOLVE IT
- Begin with 3 different colors and 2 socks of each color: yellow, red, and green.
- If you pull out 1 yellow, what are the possibilities for the next sock you pull out? *It can be yellow, red, or green.* What are your chances that it is yellow and that you have a matched pair? *1 out of 3*
- If you pull out a green sock, you have 1 yellow and 1 green. What are the possibilities for the next sock you pull out? *Again, it can be yellow, red, or green.* What are your chances that it is yellow or green and that you have a matched pair? *2 out of 3*
- If you pulled out a red sock, then you have 1 yellow, 1 red, and 1 green. What are the possibilities for the next sock? *Still yellow, green, or red* What are your chances of pulling out either red, green, or yellow and having a matched pair? *3 out of 3, or 100%*
- With 3 different colors, and more than 1 of each color, how many socks do you need to pull out to be sure you have a pair? *4*
- Now consider the problem you started with. How many socks do you need to pull out to be sure you have a matching pair? *You would need to pull out 10 socks, or 1 more than the number of colors. It really doesn't matter how many colors there are, or how many of each color as long as there are at least 2 of each color.*

  Solution: 10

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

46 Maggie opened her locker and found a strange purple paper, folded into the shape of a bird. She opened up the bird and read these lines: "This is being written at noon on Wednesday. The next secret club meeting will be held in the usual place in exactly 1003 hours. Be there." What time and day of the week will Maggie's club have their next meeting?

FIND OUT
- What is the question you have to answer?
- What did Maggie do?
- What was the note about?
- When was the note written?
- How many hours later is the next meeting?

CHOOSE A STRATEGY
- Would it be easier to try to solve the problem using a fewer number of hours?

SOLVE IT
- If you substitute 50 for 1003, how could you divide up 50 hours?
- How many days later would that be? Would the meeting be at noon or at another hour?
- How many hours are in a week?
- If you go back to 1003 hours, how can you divide that first?
- Are there hours left over? How can you divide them up?
- At what hour and on what day will the next meeting be?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

46 Maggie opened her locker and found a strange purple paper, folded into the shape of a bird. She opened up the bird and read these lines: "This is being written at noon on Wednesday. The next secret club meeting will be held in the usual place in exactly 1003 hours. Be there." What time and day of the week will Maggie's club have their next meeting?

FIND OUT
- What is the question you have to answer? *What time and day of the week will Maggie's club have their next meeting?*
- What did Maggie do? *She opened her locker and found a paper folded into a bird.*
- What was the note about? *It told when the next meeting of her club would be.*
- When was the note written? *Noon on Wednesday*
- How many hours later is the next meeting? *1003 hours later*

CHOOSE A STRATEGY
- Would it be easier to try to solve the problem using a fewer number of hours? *Yes, it would help to make the problem simpler.*

SOLVE IT
- If you substitute 50 for 1003, how could you divide up 50 hours? *Divide by 24 to divide the hours into days.*
- How many days later would that be? *2* Would the meeting be at noon or at another hour? *It would be 2 hours after noon, 2:00, because dividing by 24 leaves a remainder of 2.*
- How many hours are in a week? *168*
- If you go back to 1003 hours, how can you divide that first? *By weeks*
- Are there hours left over? *Yes* How many? *163* How can you divide them up? *By days* How many? *6* Are there hours left over? *Yes* How many? *19*
- At what hour and on what day will the next meeting be? *At 7 A.M. on a Wednesday*

   Solution: 7 A.M., Wednesday

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

EXTEND IT
- What if the note said the next meeting would be 2450 hours later, what hour and what day of the week would this be?

PRACTICE
- Similar Practice Problems: 84, 98, 110

47 Juanita presented a problem to Karl. "If you can solve this," said Juanita, "I'll buy you the ice-cream cone of your choice! Here's the problem: Show how one half of five is four." Karl got his ice-cream cone. What was his answer?

FIND OUT
- What is the question you have to answer?
- What does Juanita present to Karl?
- What is the problem?

CHOOSE A STRATEGY
- When the strategies you know about don't apply to a problem, and you don't know where to begin...relax and open up your mind to any and all possibilities. Explore anything you think of; reasonable or unreasonable. We call this "brainstorming."

SOLVE IT
- What is your first reaction to this problem?
- If you set up a visual picture of this problem, what do you see?
- What different kind of visual pictures can you think of that represent five?
- Write down all the things you can think of. Try playing with all the pictures, by dividing them into half in different ways. Is there some way to do this and come up with four?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

47 Juanita presented a problem to Karl. "If you can solve this," said Juanita, "I'll buy you the ice-cream cone of your choice! Here's the problem: Show how one half of five is four." Karl got his ice-cream cone. What was his answer?

FIND OUT
- What is the question you have to answer? *Show how one half of five is four. What was Karl's answer to this problem?*
- What does Juanita present to Karl? *A problem*
- What is the problem? *Show how one half of five is four.*

CHOOSE A STRATEGY
- When the strategies you know about don't apply to a problem, and you don't know where to begin. . . .relax and open up your mind to any and all possibilities. Explore anything you think of; reasonable or unreasonable. We call this "brainstorming."

SOLVE IT
- (Following is an example of some possible responses to these questions. Students will have different visual pictures and many different approaches to this problem. Encourage a variety of ideas.)
- What is your first reaction to this problem? *That it's impossible and crazy!*
- If you set up a visual picture of this problem, what do you see? *5 objects and that one half of them is $2\frac{1}{2}$*
- What different kind of visual pictures can you think of that represent five? *Five, 5, V and. . . . .*
- Write down all the things you can think of. Try playing with all the pictures, by dividing them into half in different ways. Is there some way to do this and come up with four? *Yes*

Solution: F ⎸ I ⎸ V ⎸ E

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

48
Ruben ignored everyone at the table, and even his dinner when it arrived. He was leaning intently over the geometric shapes he was creating with toothpicks. Ruben was trying to make 6 squares with 12 toothpicks. Can you help him?

FIND OUT
- What is the question you have to answer?
- What is Ruben trying to make?
- How many toothpicks does Ruben have?

CHOOSE A STRATEGY
- When the strategies you know about don't apply to the problem, what can you do?

SOLVE IT
- What is your first reaction to Ruben's project?
- What is your visual picture of the 6 squares? How many toothpicks would you need for them?
- Do the squares have to be flat on the table?
- Can a toothpick be in more than one square?
- How can Ruben build 6 squares with 12 toothpicks?

LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

48   Ruben ignored everyone at the table and even his dinner when it arrived. He was leaning intently over the geometric shapes he was creating with toothpicks. Ruben was trying to make 6 squares with 12 toothpicks. Can you help him?

FIND OUT
- What is the question you have to answer? *How can you make 6 squares with 12 toothpicks?*
- What is Ruben trying to make? *6 squares*
- How many toothpicks does Ruben have? *12*

CHOOSE A STRATEGY
- When the strategies you know about don't apply to the problem, what can you do? *Brainstorm*

SOLVE IT
- (Following is one example of the types of responses you will get, students will have different visual images and different approaches to solving this problem. Encourage all types of responses.)
- What is your first reaction to Ruben's project? *It can't be done!*
- What is your visual picture of the 6 squares? *6 squares on the table* How many toothpicks would you need for them? *24*
- Do the squares have to be flat on the table? *No*
- Can a toothpick be in more than one square? *Why not*
- How can Ruben build 6 squares with 12 toothpicks? *We can make a 3-dimensional box that has 6 squares.*

Solution:
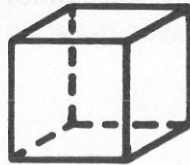
LOOK BACK
- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

PRACTICE
- Similar Practice Problems: 74, 88, 102

49 Spread the word! It is Grand Opening Day at the Jeanery! Every 5th customer will receive a coupon for a free pair of jeans, and every 7th customer will receive a coupon for a free sweater. If 400 customers come in on opening day, how many will leave with both a pair of jeans and a sweater?

- -

50 Lupe and Elena were training for a mile swim. Each week they tried to get better times. The first week the best time for each of them was 70 minutes. Then Lupe had a best time of 66 minutes the second week, 67 the third week, 63 the fourth week, and 64 the fifth week. Elena had a best time of 69 minutes the second week, 67 the third week, 66 the fourth week, and 64 the fifth week. If they continued at this same rate, who would have the best time after 12 weeks of training?
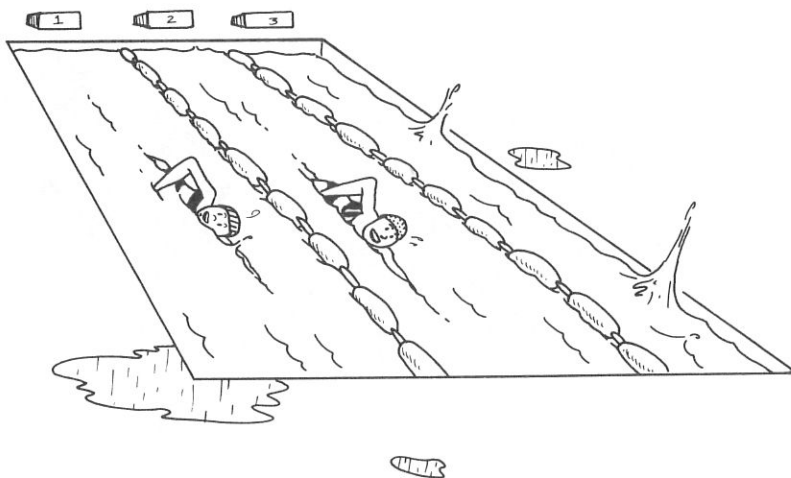
51 Heidi has a summer job at the Perfect Pie Shop. Benjamin, who runs the shop, told Heidi to arrange six freshly baked pies in the front window in the following way:

- pumpkin to the right of the chocolate
- chocolate across from the strawberry
- peach to the right of strawberry
- peach and apple not in the same row with the lemon

How did Heidi arrange the pies?

52 Space Station Alpha is having mechanical problems so the inhabitants (humans, robots, and man-eating plants) have to be moved to another station with a two-person shuttle. Each plant is large enough to require a seat of its own on the shuttle, but a plant and a human cannot travel together for obvious reasons. At no point, at either station, can the man-eating plants outnumber the combination of humans and robots. If there are three plants, three humans, and two robots on Space Station Alpha, what is the fewest trips it would take to empty the station?
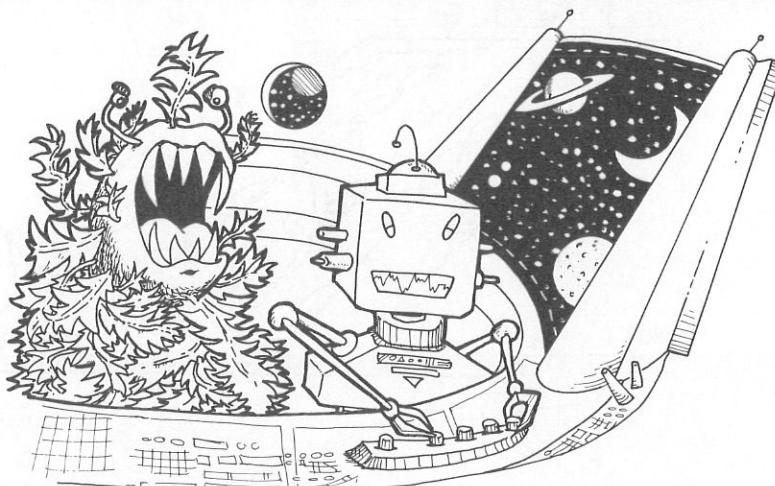
53 The Square Table Pizza Palace sells cheese, plain, sausage, and mushroom pizzas. Marty found one Saturday night that for every 6 orders of cheese pizza, there were 10 orders of plain pizza, 7 orders of sausage pizza, and 4 orders for mushroom pizza. If 30 cheese pizzas were ordered that night, how many pizzas did they sell altogether?

54 Carla is passing out red, white, and blue construction paper for an art project. There are ten people in her group. Carla, who likes to show off, tells everyone to watch closely as she hands out the paper. The first person gets a red paper. Carla puts the second paper at the bottom of the stack. She gives the second person a white paper. Carla puts the fourth paper at the bottom of the stack and gives the third person a blue paper. Carla continues this pattern until there is no more paper. How did she have to stack the colored paper so that she could pass it out this way?

55 Delia went to mail a package to her Aunt Maude in Minneapolis. It cost $2.75 to mail the package, but all she had was stamps worth 15 cents and 20 cents. She had to lick 16 stamps, using the 15-cent and 20-cent stamps, to total $2.75. Now her mouth feels like the Sahara Desert at high noon. How many 15- and 20-cent stamps did Delia have to lick to send Aunt Maude's package to Minneapolis?

- -

56 Nancy, Mason, Ramos, and Angelica have part-time jobs at Dr. Forbe's Veterinary Hospital and Kennels. Nancy gives the dogs baths every 4 days; Mason cleans out the cages in the kennels every 6 days; Ramos feeds the animals in section B every 2 days; and Angelica helps the receptionist every 3 days. How many times in eight weeks will all four helpers be at Dr. Forbe's Hospital on the same day?

57 Stan, Ann, Fran, and Dan are discussing their favorite ways to travel. Fran cannot swim. Stan likes to sleep in the back seat while his mother or father drives. Ann is afraid of heights and water. Which way to travel did each person choose: boat, train, plane, or car?

58 Willy the Wizard has a special hobby, growing huge vegetables. His favorite is zucchini, which he likes to measure. He has one that started out at 6 inches, then grew by 9 inches to be 15 inches on the second day. Each day the zucchini grew by the same amount as the day before plus another 3 inches. On what day would the giant zucchini be longer than 11 feet?
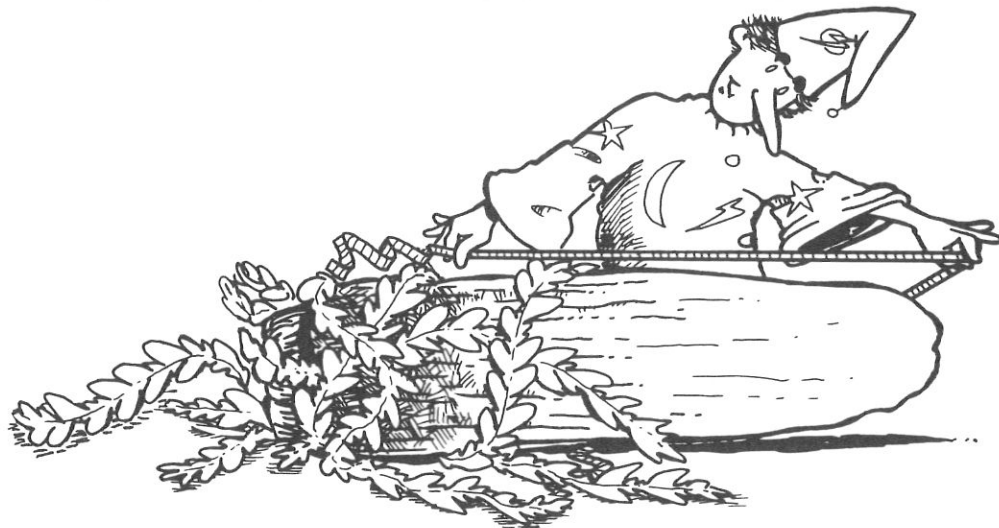
59 Helga is trying to decide which way to go to visit her friend Marisa. There are 6 different streets that lead to her friend's apartment building. Then there are 3 stairways up from the street, where Helga can go into the building through 2 different doors. Once inside she can choose from 2 inside stairways or 3 elevators to get to the third floor where Marisa's apartment is. How many different ways can Helga go to visit her friend?

- -

60 Sidney remembered that next Monday is his favorite Uncle Wally's birthday. Sidney addressed a birthday card to Uncle Wally in Australia, and now he needs 32 cents to send it. At the post office there is a machine that dispenses stamps in denominations of 10 cents, 7 cents, 5 cents, and 2 cents. How many different combinations of stamps totaling 32 cents could Sidney put on the card?

61 Allen is in the Boy Scouts. He has three weeks left in which to complete the hours required for a merit badge in community service. Because of his paper route and homework, he can't put in more than 5 hours in any one week for the merit badge. How many different ways (0 + 5 + 5 is different from 5 + 0 + 5) can Allen work a total of 10 hours over the three-week period?

62 Three old friends — a snake, a mouse, and a cat — were traveling through the woods one day. The friends were looking for a place to eat because they were very hungry. Suddenly they came upon a very wide and deep ravine, at the bottom of which was a raging river. They were puzzling over how to make their way across, when a kindly golden eagle nearby overheard them. He offered to carry them across one at a time for a small fee. Now, even though they were good friends, the snake couldn't be left alone with the mouse nor could the mouse be left alone with the cat, when they were this hungry. What was the fewest trips the eagle had to make in order to take the friends across the ravine safely?

63 Daphne stares wide-eyed at the display of stickers before her in the stationery store. She has $1.50 in babysitting money to spend on stickers. The unicorn sticker costs 80 cents, the rainbow sticker costs 70 cents, the teddy bear sticker is 40 cents, the whale sticker is 25 cents, and the ice-cream soda sticker is 15 cents. How many different combinations of stickers could Daphne buy with her $1.50?

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64 The Bickerton family is leaving for Yosemite Park in their van. Father Bickerton is driving, with Mother Bickerton up front next to him. On the two bench-style seats in the back of the van are the rest of the family: Grandmother, Brad, his two sisters Bernice and Betty, the baby, and Bow-Wow the dog. Grandmother is next to the window, because she gets carsick. She also likes to sit next to Bow-Wow. The baby is to the right of Brad and not far from Mother. About halfway there, Brad complains that Betty is kicking his seat! Where is each Bickerton sitting in the van?

65 Three scientists are observing Ricky and Riley, laboratory rats, to see how long each rat takes to find a piece of cake in a maze. The maze is a large box with open pathways running north-south and east-west in a grid pattern. Riley takes the direct route by traveling 3 rows north, turning right and traveling 2 columns east, proceeding north for 4 rows and turning right for 1 column. He is 1 row south of the cake when the bell goes off. Ricky, on the other hand, fumbles his way along. He enters at the same gate as Riley and advances 12 rows north. He then becomes confused and does an about-face, and moves 8 rows south along the same path. Then he moves east 1 column and north 1 row. Finally he makes a right turn and moves 4 columns to the east. How far away from the cake is Ricky if the bell goes off at this point?
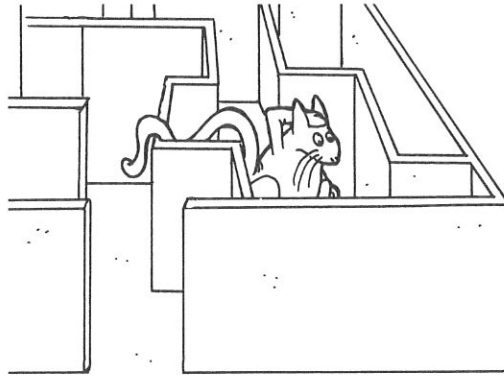
66 The Laurelwood PTA is presenting its 5th annual Talent Show and Kate is collecting tickets at the door. The gym is filled with 215 students and adults. Students tickets are $.50 and adult tickets are $2.00. If Kate collected $250.00, how many adults and how many students purchased tickets?

67 Anita and Terry ride the subway on Sundays to visit their grandparents. There are two entrances to the subway from the street, north and south. Inside each entrance is an escalator, an elevator, and a stairway leading to the first level. On the first level there are eight gates. At each end of the first level (north and south) there is an escalator, an elevator, and a stairway. These lead directly to the trains. How many different ways can Anita and Terry go from street level to the trains?

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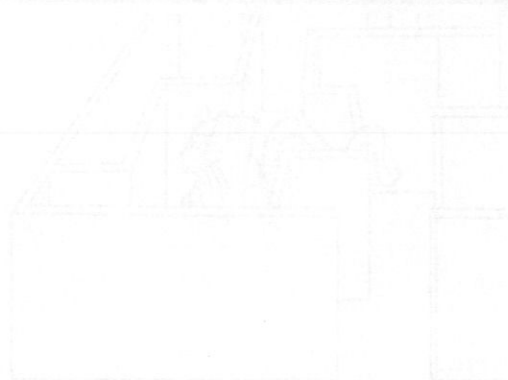
68 Judy, Shirley, and Martin are building a scale model of an Aztec temple as part of a social studies display. They need to start collecting empty laundry detergent boxes from their families and friends. If they design the top level of the temple to

be a 1-by-2 box rectangle:

and the level directly underneath to be a 2-by-3 box rectangle:
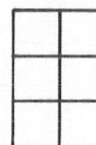
and the level directly underneath that to be a 3-by- 4 box rectangle:

then how many boxes will they need to collect for their project if the temple is 12 levels high?

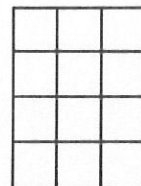**69** Alvin is working as an apprentice to his father, a house-painter, for the summer. Today they begin a contract to paint a chain of Cleo's Ice-Cream Cottages. To get just the right shade of green they have to mix 5 cans of teal blue with 3 cans of lemon yellow and 2 cans of eggshell white. If it takes 60 cans of paint to cover the cottages, how many cans of each color will Alvin need to use?

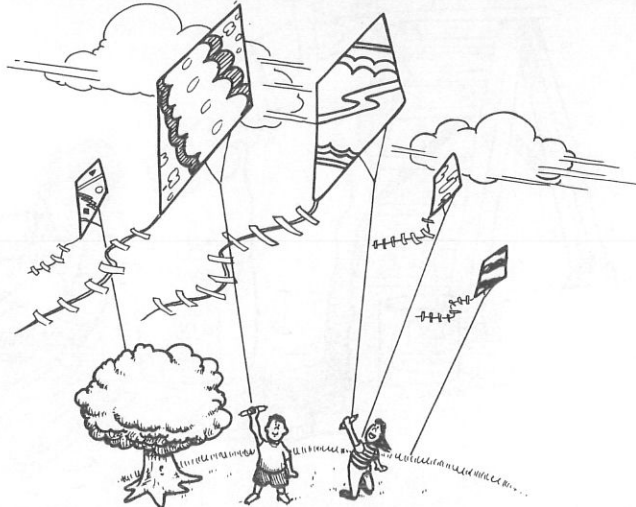**70** Saul is helping his brother Jason do carpentry and general repair jobs. Jason gave Saul some money for supplies. Saul bought some tools for $20.80, and then spent one half of what was left for wood. He spent one third of what was left after that on paint and another third on buckets and brushes, and he returned with $13.20. How much money did Jason give Saul?

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71 The final event of the end-of-the-year picnic is a kite-flying contest. Henry, T.J., Monique, Sukia, and Maliki are participating in the contest to see whose kite can soar the highest. Henry, for all his frantic running around, cannot catch the wind, so his kite barely makes it off the ground. Maliki's kite goes ten times as high as Henry's. Sukia makes his kite go twice as high as Maliki's. Monique's kite flies four yards lower than Sukia's, and T.J.'s kite flies as high as Maliki's and Henry's kites together. If the kites together fly 182 yards off the ground, how high is each kite?

72 Paul, Winnie, John, and Tish are discussing what they want to be when they grow up. Paul is allergic to cat and dog fur; John is good at fixing things; and neither Paul nor Winnie can carry a tune. Which one do you think will pick each of these occupations: a mechanic, a veterinarian, a rock star, and an engineer?

73 It is the last event in the sixth-grade math marathon. Kelly studies the final problem. She has been given an unlimited supply of identical wooden cubes, and directions to build a 30-step staircase with them. If a 3-step staircase looks like this:
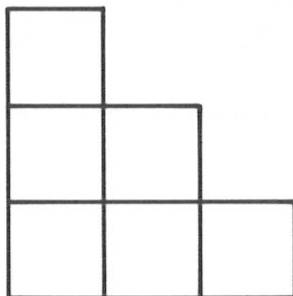
how many cubes will Kelly need to complete the staircase?

74 Two mothers and two daughters divided $21 in dollar bills evenly among themselves. Each received an equal number of dollar bills. How could this be?

75   It is the first school band concert of the year and Brett is responsible for setting up the chairs on the stage. His instructions are to set up the chairs in sections. One half the chairs will go in the brass section, one fourth in the wind section, one eighth in the percussion section, and 5 chairs are needed for the strings. How many chairs will Brett need to set up, and how many will he put in each section?

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76   In your dream you are competing in the semi-finals of the archery contest. You know you must make a total of 50 points to be in the finals. The bulls-eye is worth 50, and is therefore the smallest circle; the next circle just outside that is worth 30; the circle just outside that is worth 20, and the last and largest circle is worth 10. Anything outside the target is worth 0 points. You have 3 arrows in your quiver — 3 shots to make 50 points. Every eye is on you. How many ways can you score 50 points in three attempts? (Remember, 0 + 0 + 50 is different from 0 + 50 + 0.)
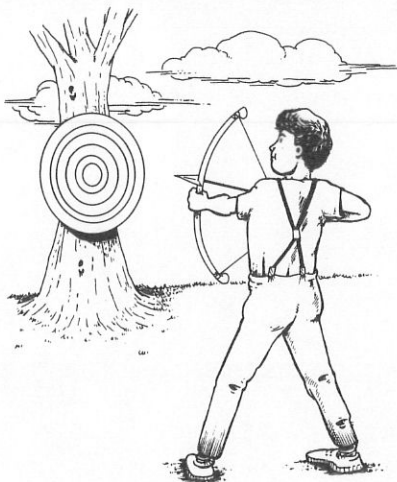
77 Andrew quickly solved the card trick and asked Armando for another. "Okay, Andrew," said Armando. "Listen carefully. I have ten cards: 4 queens, 4 kings, and 2 jacks. Arrange the cards in a stack so that when they are all turned face up on the table the kings and queens alternate with each other, and there is a jack at each end. The cards must be turned up in the following pattern: first card face up on the table (jack), second card to the bottom of the stack, third card face up on the table next to the first card, fourth card to the bottom of the deck, and so forth until your hand is empty." How did Andrew arrange the cards in the stack?

78 Arthur and Angela Ant live inside a wall at the art museum. They come out to admire the paintings after the museum has closed. They find their way around by following the grout pathways in the square-tiled floor. Angela particularly likes Van Gogh, while Arthur likes the Monet paintings. The Monet collection is located just 10 tiles east of their front door. After admiring Monet, they travel 4 tiles north, turn left for 4 tiles, right for 2 tiles, left for 3, right for 1, and west for 3, which brings them directly in front of a Van Gogh. However, they become antsy and must return home quickly. How many tiles are they from their front door, if they take the most direct route?
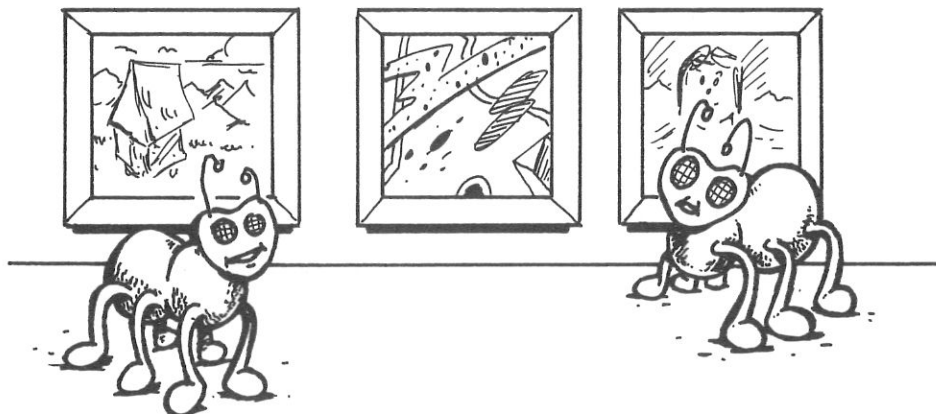
79 Two biologists are searching the woods for purple trumpetworms, giant man-eating moths, and spear-headed spiders. They are successful, and on the first day they count 3 moths, 4 worms, and 17 spiders. On the second day they count 9 moths, 12 worms, and 15 spiders. On day three they find 5 moths, 6 worms, and 14 spiders. They count 11 moths, 18 worms, and 12 spiders on day four; and 7 moths, 12 worms, and 11 spiders on day five. If this pattern continues, on what day will they count 0 spiders, and how many worms and moths will they count?

80 It is finals week at Valley Middle School. The members of the public speaking class are presenting their final speeches. Jeremy's speech is 3 minutes over the minimum time set by the teacher. Sara's speech is twice as long as Jeremy's. Paul's speech is exactly the minimum length of time. The class period is 42 minutes long. If all three speakers together take exactly one-half the class period, what is the minimum time set for the speeches?

81 Poor Diane sprained her ankle just before the family reunion at the beach. Now she has to sit on the sidelines and watch her cousins play volleyball. She notices that of the 12 cousins playing volleyball, 6 are wearing red shorts and white tennis shoes; 7 are wearing white tennis shoes and baseball caps; and 5 cousins are wearing all three. How many of Diane's cousins are wearing baseball caps but no red shorts or white tennis shoes? How many are wearing red shorts but no baseball caps or white tennis shoes? How many are wearing tennis shoes but no baseball caps or red shorts?

82 Mary MacDonald makes mugs in Miami. Mary makes two sizes of mugs: a small one that she sells for $2.50, and a large one that she sells for $5.75. Yesterday Mary made $56.00. Before she opened her shop in the morning, she had 200 mugs in the inventory; at the end of the day she had 188. How many mugs of each price did Mary sell?

83 The Marshall family is sitting down to dinner. Mom and Dad are at opposite heads of the table, and are delighted to have all the children home tonight. In order of oldest to youngest, the children are: Tybony, her brothers Darrell and Melvin, and her sister Crystal. Crystal's oldest brother is seated to the right of Tybony's younger sister. Tybony's youngest brother is seated to the right of Crystal's father's oldest daughter. Where is each child sitting at the table?

- -

84 Carla is excited because her big brother Joe has just come back from six weeks of fishing for salmon in Alaska. Joe earned money in bonuses, and has promised Carla $10 if she can figure out how much he made. Each time Joe caught $500 worth of salmon, he got a bonus. The first time he received a $10 bonus. The second time, he got a $30 bonus. The third time he received $50, and the fourth time he received $70. During the time Joe spent fishing, he caught $500 worth of salmon on 21 days. If the bonuses continued at the same rate, how much bonus money did Joe make?

85   Brian, Paulette, Priscilla, Benny, and Kenny brought their pets to Mrs. Lindy's science class. Paulette's and Kenny's pets are soft and furry. Brian's pet provides him with breakfast three days a week. Benny is afraid his pet will be scooped up and eaten by Paulette's pet. Can you match each of the following pets with its owner: a goldfish, a rabbit, a chicken, a raccoon, and a parrot?

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86   Evelyn is reading about Windemere Castle in Scotland. Many years ago, when prisoners were held in the various cells of the dungeon area, they began to dig passages connecting each cell to each of the other cells in the dungeon. If there were 20 cells in all, what is the fewest passages that had to be tunneled out over the years?
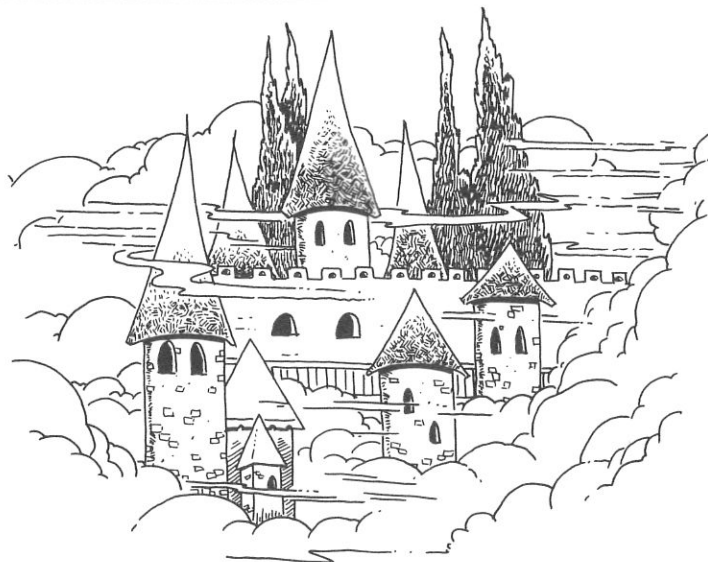
87 Cubelets are cuddly little creatures that live in the underground caverns on the planet Crate. They reproduce only when they are exposed to direct sunlight. When thus exposed, one cubelet remains 1 cubelet, 2 cubelets become 8 cubelets, 3 become 27, and 4 become 64. How many cubelets will 5 become?

88 Two friends were going on an 18,000-mile automobile trip. After they had the car tuned up, the garage mechanic told them that each tire was good for only 12,000 miles. What is the minimum number of spare tires they needed for the trip?

89 Windmill Springs School is having its annual Science Fair. The science instructor, Mr. E. Raser, asks you to set up eight projects for the fair. The projects are to be placed on two rectangular tables that face each other with an aisle in between. Mr. Raser gives you these directions:

- Set up the constellation display directly across from the map of the planets, at one end of the tables.

- Set up the rocket model next to the the soil test kit and the weather graph.

- Set up the rock collection and the water conservation poster on the same table.

- Set up the energy project at the other end of the table from the constellation display.

- Put the water conservation poster across from the rocket model.

How do you set up the display?

90 Mimi sat up next to her grandfather on the big harvesting machine. Mimi and her grandfather harvested 15 acres the first day and half of what was left on the second day. On the third day they harvested 12 acres and half of what was left on the fourth day. On the fifth day there remained four acres to be harvested. How many acres will Mimi and her grandfather have harvested when they finish the job?

91 The Gadfly Gazette is published every day, rain or shine. Marion helps her sister Janet to get the paper delivered. Marion is on a schedule that includes folding, delivering, and collecting. Every 6th day she goes collecting, every 3rd day she delivers the paper, and every 4th day she folds the papers. If she helped Janet for 12 weeks, how many times did Marion do all three jobs in the same day?

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92 Joe and Jim have tickets to see Harvey Pond and the Frogs in concert. The stadium where the concert is being held has six entrances from the street, four stairways to the first balcony and three stairways to the second balcony. Joe and Jim have tickets for the second balcony. How many different paths can they take to get to their seats in the second balcony?

93 Alexander enjoys playing tennis with his father, Adam; his father's brother, Charles; and Charles' son, Christopher. Chris' cousin's father and Chris' uncle's brother are on opposite teams. Charles' brother's son is not on the same team as the son's father's brother. Who are the teammates in the family tennis games?

- -

94 Walt and his dad are at the circus. Walt was busy counting all the animals that performed. He figures out that one half were horses, one fourth were big cats, one eighth were elephants, one sixteenth were camels, and 8 were monkeys. How many animals did Walt see altogether and how many animals were in each group?
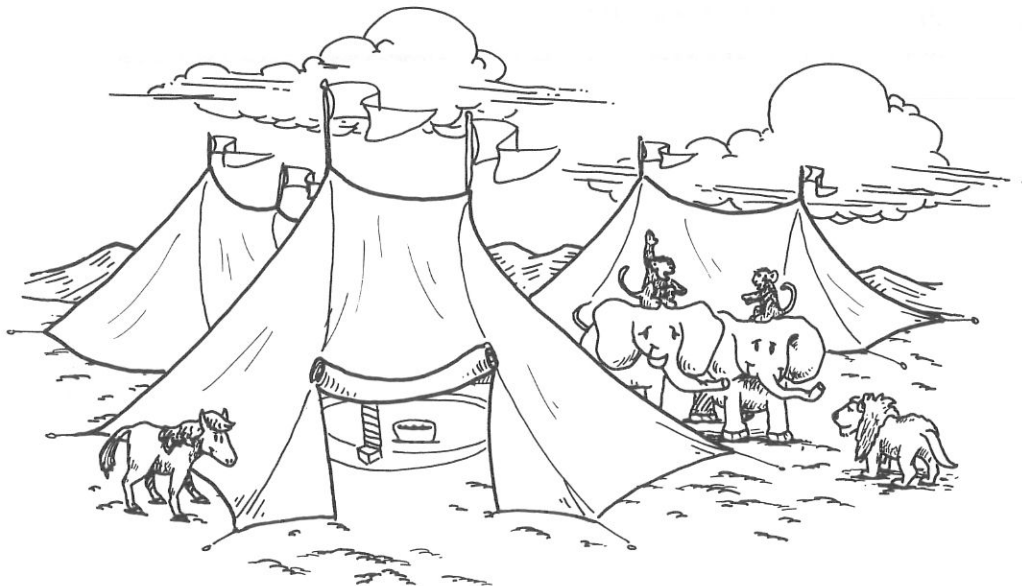
95 Luis and Lupe's class is saving cans to earn money for the orchestra's trip to New York. The first week they had 4 cans, the second week they collected the same amount as the first week plus 8 more cans, or a total of 12 cans. The third week they collected the same amount as the previous week plus 8 cans, for a total of 20 cans. If they continued to collect cans at this rate, how many weeks would it take them to collect over 200 cans?

96 The school newspaper is interviewing sixth-grade students to see what sports they follow regularly on television. Of the 70 students interviewed; 40 enjoyed basketball; 40 enjoyed baseball; 40 enjoyed football; 20 enjoyed basketball and football; 22 enjoyed baseball and basketball; 27 enjoyed football and baseball; and 12 enjoyed all three sports. How many students out of the 70 interviewed didn't follow any one of the three sports?

97 Jason has borrowed a friend's sailboat and has stayed out too late. The fog has settled in, and he is lost and hungry. He radios for help and the harbormaster's voice comes across the wire, giving him directions to the harbor. He asks Jason if he remembers seeing a lighthouse and Jason remembers seeing one several miles back. The harbormaster tells him to return to the lighthouse and go directly north for six miles, then turn east for two miles because of a sandbar. Next he should go north for three more miles, then go west for one mile. The harbor entrance is one mile due west of that. How many miles is the harbor entrance from the lighthouse?
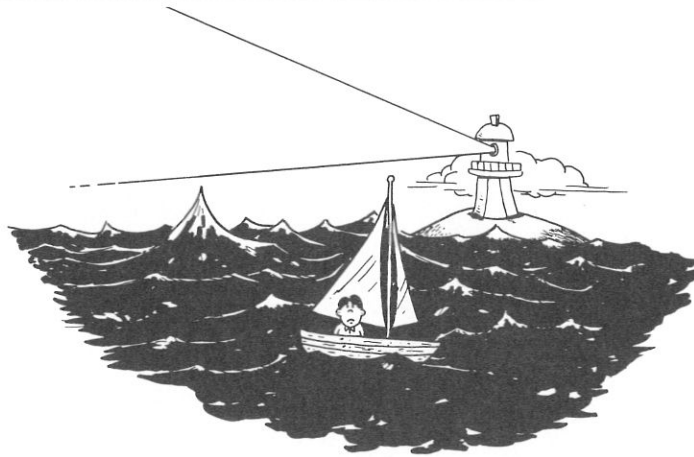
98 Mathilda inherited her grandmother's quilt, that she proudly keeps on her bed. She always thought there were 81 squares in the quilt, because there were squares in 9 columns across and in 9 rows down. One day she realized that there were actually many more squares than 81 on the quilt. How many squares were there altogether?

99 Program cards are being handed out in 5th period P.E. As he passes out the cards to the members of his class, Arthur notices that 8 students are on the basketball team as well as on the leadership council; 7 students are on the track team as well as on the leadership council; and 5 students are on the track team as well as the basketball team. How many more students are in the leadership class than are on the track team?

- -

100 Directly across from the Big Beast Booth at the county fair is the Bowl-A-Ball Booth. Mary Louise is ready to bowl her ball down the miniature alley and into one of the five circles labeled 0, 2, 4, 6, and 8. Mary Louise has three attempts to score 8 points. How many different ways can she score 8 points in three attempts?

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101 Sean and Erin designed a card trick to stump their friends. They arranged cards in a stack so that when they are dealt out the cards are in this order: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, and queen. Sean and Erin deal them out in the following manner: first card face up on the table, next two cards put at the bottom of the stack, the next card face up on the table, next two cards put at the bottom of the stack, and so on. How did Sean and Erin arrange the cards in the stack?

102 "Did you get the job, did you get the job?" asks Brian. His brother Jim just applied for a job parking cars at the largest hotel in the city. Jim hands Brian the written test he had to take in order to qualify for the job. "Yes," he says, "I got the job. But I had to answer this question first: How can you park twenty cars in these ten stalls without doubling up in any of the stalls?" Brian studied the question and finally came up with the solution. Can you?

103 Jennifer received a very special gift on her twelfth birthday: her father's entire collection of baseball cards. "Wow," shouted Jennifer, adding her Dad's collection to her own. "One third of my cards are Yankees, one third are Giants, one sixth are Phillies, one twelfth are Tigers, and 30 cards are Padres!" How many cards from each team did Jennifer have, and how many cards did she have altogether?

- -

104 Zeno, Orb, Yurko, and Sam are friends who live on neighboring space stations of the planet Krayon. They commute to school every day by space shuttle. Orb's space station is one half as far from Krayon as Zeno's space station. Yurko travels as far as the total distance traveled by Zeno and Orb, while Sam travels three times the distance that Zeno travels. How many space miles does each friend travel to school, if the friends together travel a total of 888 space miles?
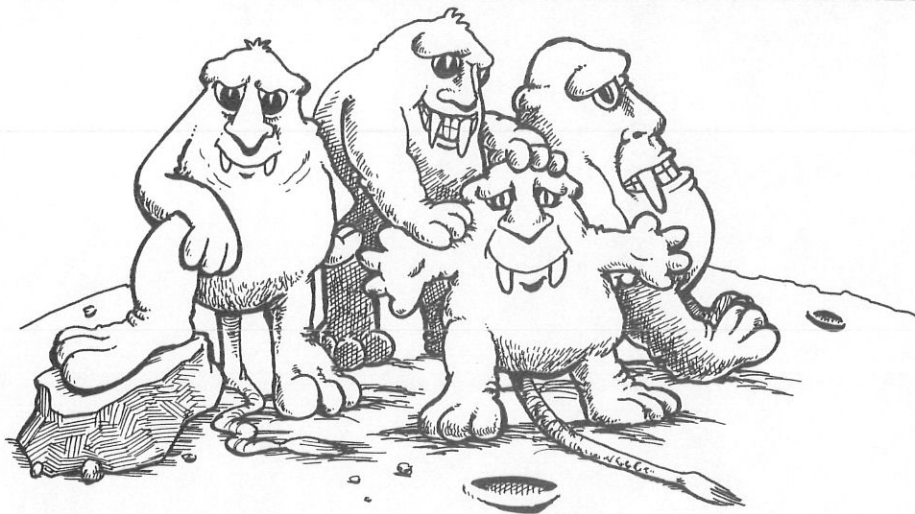
105 At the Cookie Castle, Marty, Jim, and Dan are trying to increase their production of almond puffs, triple-chocolate squares, and lemon mounds. In the first hour Marty makes 12 puffs, Jim makes 11 squares, and Dan makes 7 mounds. In the second hour they make 13 puffs, 22 squares, and 8 mounds. During the third hour they turn out 15 puffs, 12 squares, and 11 mounds. The fourth hour they make 18 puffs, 24 squares, and 16 mounds. The fifth hour they make 22 puffs, 14 squares, and 23 mounds. If they continued at these rates, how many hours would it take before they could make a combined total of more than 15 dozen cookies in one hour?

106 Ryan was working at the summer recreation program. His first job each morning was to lock up the bicycle cage after all the little kids arrived. He entertained himself by counting the number of wheels in the cage each day. Today, Ryan counted a total of 31 vehicles and a total of 84 wheels. If some of the vehicles were Big Wheels, some were bicycles, and some were wagons, how many of each type of vehicle were in the cage?

107 Irene is keeping track of money from yearbook sales at her middle school. She is counting the paper money and placing it in piles by denomination. She has a stack of one dollar bills, a stack of five-dollar bills, and a stack of ten-dollar bills. If she has $88.00 total in paper money and 20 bills altogether, how many bills of each denomination does she have?

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108 It is Saturday afternoon, and Cleve is spending it at his favorite place in the whole world: the pet shop. He is fascinated by the guppies, the puppies, and the frogs. He notices that the total number of guppies and frogs is 51. The total number of frogs and puppies is 17, and the total number of puppies and guppies is 58. How many guppies, how many puppies, and how many frogs were in the pet store?
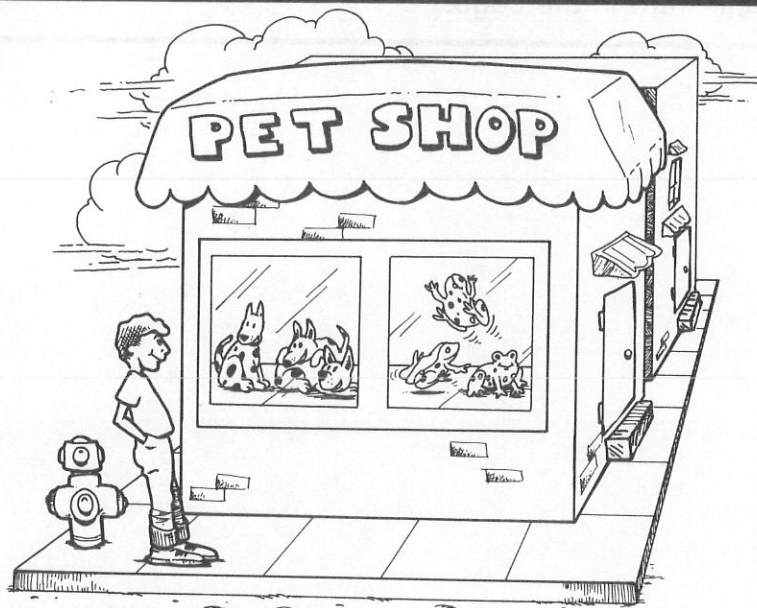
109 A strange virus has broken out aboard the SeaLab I: the six-member crew has been under water for so long now that some of the crew are beginning to take on the characteristics of fish. Two members showed up at breakfast with gills just below their ears, showing that they had just "caught" the virus. The crew must return at once to prevent further outbreak. Their only way out is with a two-person submarine. It is important that neither of the crew members with the virus ride with any member that doesn't have the virus. What is the fewest trips that must be made in the submarine, so that no one else begins to look fishy?

110 Tony's father is a driver for Brank's, an armored truck company that takes money from businesses to the bank. His last stop of the day is at the largest hotel in the city, where he collects a large sum of money. A map of part of the city showing the location of the hotel and the bank looks like this:
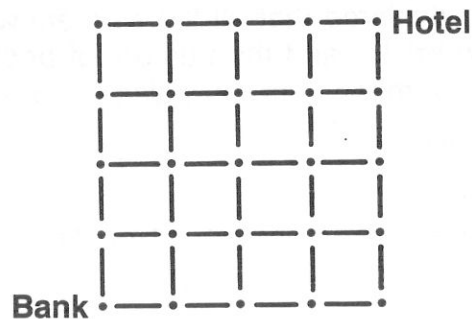
Hotel

Bank

If Tony's father must take a different route every day to prevent a robbery attempt, how many different paths DOWN and to the LEFT can he take from the hotel to the bank?

111 Giselle, Sara, and Sue-Mee were team leaders in a summer vacation read-a-thon. The teams read as many books as possible from June until late August. When the team leaders met in early September to compare their totals, Giselle's team was the winner. Sara sadly reported that her team had read three more than one half the number of books that Giselle's team had read. Sue-Mee, who had a very social group, reported that they had read three less than half the number of books that Sara's team had read. The grand total of books read was 96. How many books did each team read?

- -

112 There was once a very vain Egyptian king who built his entire kingdom around his palace. He declared that all the roads in the kingdom would begin at his front door, with mileage markers placed along the way on each road to show the king how far he was from his palace when he traveled around his kingdom. The king loved pyramids, so he declared that all the markers would be miniature pyramids composed of separate bricks, and the number of bricks in each pyramid would represent the number of miles from the palace. The first marker looked like this

and represented one mile:

The second marker looked like this and represented 5 miles:
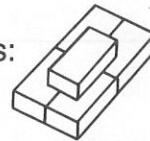
The third marker looked like this and represented 14 miles:
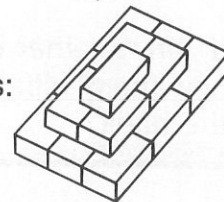
How many miles from the palace would the king be when he arrived at the fifteenth marker?

Name

113 It is Louis' first day at summer camp, and he is selecting snacks to bring on the hike. The camp store has a limited inventory: first aid supplies, beverages, and some snacks. The snacks are in 1-ounce cellophane packages: trail mix for 12 cents, nuts for 10 cents, dried pineapple for 8 cents, and dried apricots for 4 cents. Louis has 46 cents to spend on snacks. How many different ways can he combine snacks to total 46 cents?

- -

Name

114 Darrin and Sharon are riding the merry-go-round with their son, Aaron. Sharon and Aaron together are 32 years old; the sum of Sharon's and Darren's ages is 54; and Darrin's age added to Aaron's equals 34 years. How many years difference is there between father and son?

115 The Fabulous Fudge Emporium serves fudge made from an old secret recipe. Marianne works at the shop after school. She doesn't know what all the ingredients are, but she knows that in 3 days they use 15 pounds of dark chocolate, 30 pounds of sugar and 21 pounds of butter. When Marianne orders chocolate, sugar, and butter for 3 weeks, how many pounds does she order of dark chocolate, sugar, and butter?

- -

116 The McGuire family choir is singing gleefully before the fire. The choir has one grandmother and one grandfather; seven mothers and seven fathers; each father has two sons; and each son has four daughters. What is the fewest singers there could be in the McGuire choir?

117 Claudia worked at the rare coin exhibit in the museum. She noticed something very mysterious going on in one case with coins from the island of Crotia. There had been 460 coins in the case. Then one day she noticed that 2 coins were missing. On the second day 6 more coins were missing, and on the third day 18 more coins were missing. Every day there were three times as many coins missing as had disappeared the day before. If the coins continued to disappear at this rate, on what day would there be fewer than one half of the original number of Crotian coins left?

---

118 It's the weekend for the annual Edison School Car Wash. The homeroom representatives are signing up volunteer car washers. The first 6 homerooms have 7 volunteers each. Half of the remaining volunteers come from the next 6 homerooms. The next 4 representatives bring in the names of 36 volunteers. The next 2 representatives bring in one half of the remaining volunteers, and the last homeroom brings in the names of 8 volunteers. How many students volunteer to be car washers?
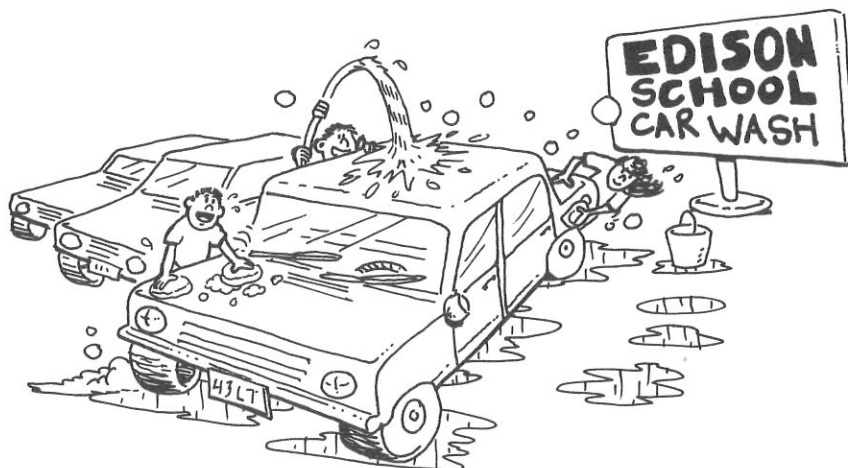
119 Peter studied the partner assignment list on the gym wall. It was a rainy day, and rainy days meant square dancing in P.E. His square included the names T.J., Carl, Christopher, Jenny, Becky, Blanca, and Marisol. When they got started, T.J. and his partner were to the left of Blanca. Across from T.J. was Peter, who was to the right of Christopher. T.J.'s brother's partner, Jenny, was across the square from Blanca. Becky was not on Blanca's right. Can you name the four pairs of partners?

120 It is the year 2029, and the Cosmos communication Network is unveiling its newly-completed multi-million dollar video communication system, linking all 23 earth satellites to each other and to earth. If each connection cost $1,000,000.00, what will be the total cost of the network?

1 4

2 6

3 24

4 19

5

truck	jeep	van
sportscar	2-door	4-door

or

sportscar	2-door	4-door
truck	jeep	van

6 (Other solutions are possible.)

4	2	3
2	2	2
3	2	4

3	1	5
3	3	1
3	3	3

7 15

8 25

9 6

10 7

11 1:20

12 Best—33, Super—25

13 west 3 blocks

14 turkey—3, ham and cheese—16, egg salad—23, pastrami—7, submarine—15, tuna—3

15 Joy—9, Heidi—18, Saul—37

16 Hilda—10, Betty—5, Adam—23, Renee—9

17 40 servings of food

18 80 cookies

19 32

20 48

21 Martha across from Bill, Phil on Bill's right and across from Ann

22 20

23 32 without tails, 24 with the wrong colors

24 16

25 7 crossings

26 7

27 thirteen 2-point coupons, ten 5-point coupons

28 12 Vulcan, 9 Android

29 600 brown, 225 white

30 153 millipedes, 140 centipedes

31 31

32 camper shells—10, CB antennas—1

33 45

34 385

35 bike—Charlene, skateboard—Mario, tennis racket—Jake, surfboard—Erica

36 Mr. Black—Jack, Mr. Green—Jill, Mr. White—Mary, Mrs. Brown—Sam, Mrs. Rojo—Peter

37 1—6—2—10—3—7—4—9—5—8

38

39 Delia—3, Tracy—1

40 Mick—:46.26, faster by .22 seconds

41 78

42 Judge 8

43 hitters—8, pitchers—4, outfield—2, catchers—2

44 total—576; 288—U.S., 144—Canada, 72—Russia, 36—England, 18—India, 18—Mexico

45 10

46 7 A.M., Wednesday

47 F I V E

48

49 11

day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
jeans					X					X					X					X					X					X					X
sweater						O						O						O						O						O					O

50 Lupe, with 51 minutes

	1	2	3	4	5	6	7	8	9	10	11	12
Lupe	70	66	67	63	64	60	61	57	58	54	55	51
Elena	70	69	67	66	64	63	61	60	58	57	55	54

51
lemon   chocolate   pumpkin   or   chocolate   pumpkin   lemon
apple   strawberry   peach   strawberry   peach   apple
                    or
peach   strawberry   apple
lemon   chocolate   pumpkin

52 13

53 total—135

cheese	6	12	18	24	30
plain	10	20	30	40	50
sausage	7	14	21	28	35
mushroom	4	8	12	16	20
				total	135

54 R—B—W—R—B—R—R—B—W—W

55 nine 15¢ stamps, seven 20¢ stamps

15¢ Stamps		20¢ Stamps	
number	cost	number	cost
1	$ .15	1	$ .20
2	.30	2	.40
3	.45	3	.60
4	.60	4	.80
5	.75	5	1.00
6	.90	6	1.20
7	1.05	7	1.40
8	1.20	8	1.60
9	1.35	9	1.80

56 4

day	1	2	3	4	5	6	7	8	9	10	11	12
Nancy				X				X				X
Mason						X						X
Ramos		X		X		X		X		X		X
Angelica			X			X			X			X

1 time every 12 days

57 Dan—boat, Ann—train, Fran—plane, Stan—car

	Dan	Ann	Fran	Stan
boat	Y	N	N	N
train	N	Y	N	N
plane	N	N	Y	N
car	N	N	N	Y

58 9th day

day	1	2	3	4	5	6	7	8	9
grows	6	9	12	15	18	21	24	27	30
total inches	6	15	27	42	60	81	105	132	162

59 180

6 streets → 3 stairways → 2 doors → 3 elevators → to 3rd floor
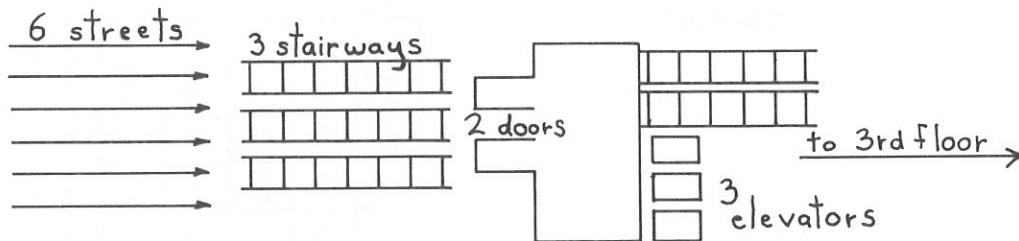
60 21

	10¢	7¢	5¢	2¢
1	3	0	0	1
2	2	1	1	0
3	2	0	2	1
4	2	0	0	6
5	1	2	0	4
6	1	1	3	0
7	1	1	1	5
8	1	0	4	1
9	1	0	2	6
10	1	0	0	11
11	0	4	0	2

	10¢	7¢	5¢	2¢
12	0	3	1	3
13	0	2	2	4
14	0	2	0	9
15	0	1	5	0
16	0	1	3	5
17	0	1	1	10
18	0	0	6	1
19	0	0	4	6
20	0	0	2	11
21	0	0	0	16

61 21 Combinations of 10
0+5+5		
1+4+5		
2+4+4		
3+3+4		
2+3+5		

1st week	2nd	3rd
0	5	5
5	5	0
5	0	5
1	4	5
4	5	1
5	1	4
5	4	1
1	5	4
4	1	5
2	4	4
4	4	2

1st	2nd	3rd
4	2	4
3	3	4
3	4	3
4	3	3
2	3	5
3	5	2
5	2	3
5	3	2
2	5	3
3	2	5

62 7 trips
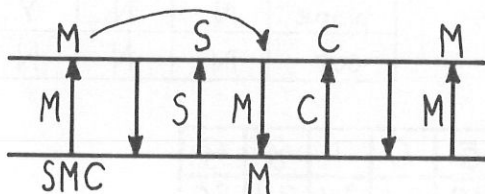
63 12

	80¢	70¢	40¢	25¢	15¢
1	1	1	0	0	0
2	1	0	1	0	2
3	1	0	0	1	3
4	0	1	2	0	0
5	0	1	1	1	1
6	0	1	0	2	2
7	0	0	3	0	2
8	0	0	2	1	3
9	0	0	1	2	4
10	0	0	0	6	0
11	0	0	0	3	5
12	0	0	0	0	10

64
Father		Mother
Brad	Baby	Bernice
Betty	Bow-Wow	Grandmother

65 Ricky is 2 rows south and 2 rows east of the cake.
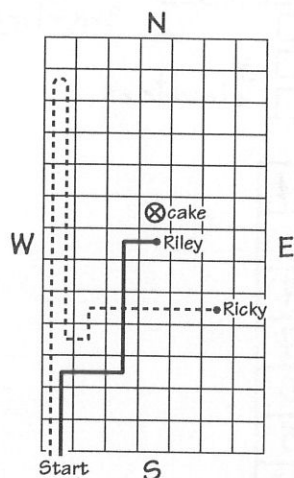
66 95 adults, 120 students

120	students
+95	adults
215	total

120
×.50
$60.00 + $190.00 = $250.00

95
×2
$190.00

67 576

2nd level

Platform

1st level

Street level

gates

escalators, stairways, elevators

$2 \times 6 \times 8 \times 6 = 576$
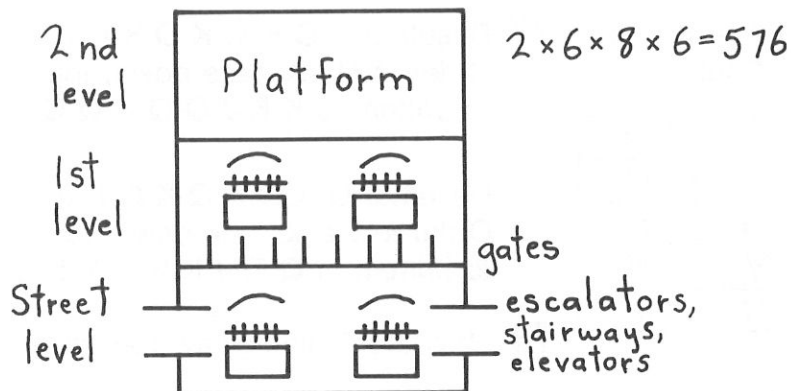
68 728

Level	(Difference between levels)	Number of boxes on level	Dimension of rectangle
1	(4)	2	1×2
2	(6)	6	2×3
3	(8)	12	3×4
4	(10)	20	4×5
5	(12)	30	5×6
6	(14)	42	6×7
7	(16)	56	7×8
8	(18)	72	8×9
9	(20)	90	9×10
10	(22)	110	10×11
11	(24)	132	11×12
12		156	12×13
		728	

69 30 cans of teal blue,
18 cans of lemon yellow,
12 cans of eggshell white

blue	5	10	15	20	25	30
yellow	3	6	9	12	15	18
white	2	4	6	8	10	12
total	10	20	30	40	50	60

70 $100.00

$13.20 = ⅓
$13.20 = ⅓ $]$ = ½ of money left
$13.20 = ⅓

$13.20 × 3 $39.60 = ½ of money left
$39.60 × 2 $79.20 = money left
$79.20 + 20.80 = $100.00

71 Maliki—30 yards, Sukia—60 yards,
Monique—56 yards, Henry—3 yards,
T.J.—33 yards

S•5

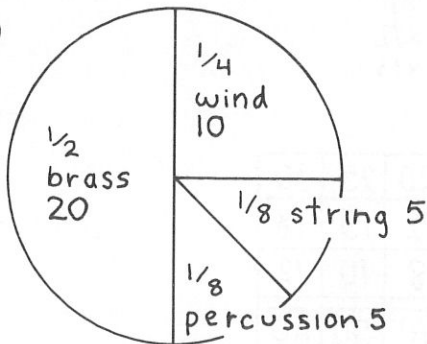**72** mechanic—John, veterinarian—Winnie, rock star—Tish, engineer—Paul

	John	Winnie	Tish	Paul
mechanic	Y	N	N	N
vet.	N	Y	N	N
star	N	N	Y	N
engineer	N	N	N	Y

73 465

Number of steps	Number of cubes
1	1 +2
2	3 +2
3	6 +3
4	10 +4
5	15 +5
6	21 +6
•	•
•	•
30	465

74 A grandmother, a mother, and a daughter; they each get $7.

75 40

1/4 wind 10
1/2 brass 20
1/8 string 5
1/8 percussion 5

76 15

Combinations of 50:
0 + 0 + 50
10 + 10 + 30
20 + 30 + 0
20 + 20 + 10

1st shot	2nd shot	3rd shot
0	0	50
0	50	0
50	0	0
10	10	30
10	30	10
30	10	10
20	30	0
20	0	30
30	20	0
30	0	20
20	20	10
20	10	20
10	20	20

77 Result: J K Q K Q K Q K Q J
Order in stack, face down, top to bottom: J K K J Q Q K Q Q K

Result: J Q K Q K Q K Q K J
Order in stack, face down, top to bottom: J Q Q J K K Q K K Q

78 They are 7 tiles away from their front door.

Van Gogh
Front door
Monet
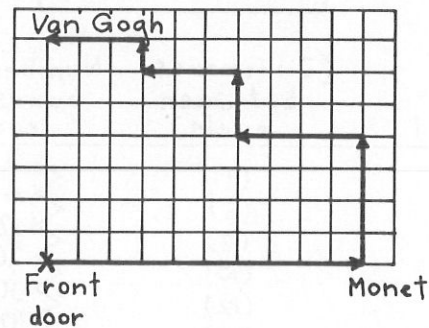
79 12th day, 19 moths, 738 worms

day	1	2	3	4	5	6
moths	3	9	5	11	7	13
worms	4	12	6	18	12	36
spiders	17	15	14	12	11	9

7	8	9	10	11	12
9	15	11	17	13	19
30	90	84	252	246	738
8	6	5	3	2	0

80 3 minutes

81 baseball caps only—2, red shorts only—1, tennis shoes only—1

Caps
Shorts
2 0 1
2 5 1
1
Shoes
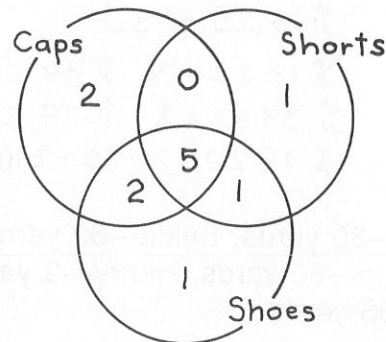
82 4 mugs at $2.50, 8 mugs at $5.75

$2.50			$5.75	
1	$2.50	1	$5.75	
2	5.00	2	11.50	
3	7.50	3	17.25	
(4)	(10.00)	4	23.00	
5	12.50	5	28.75	
6	15.00	6	34.50	
7	17.50	7	40.25	
8	20.00	(8)	(46.00)	

83

```
              Melvin   Tybony
        ┌─────────────────────────┐
 Dad    │                         │  Dad
  or    │                         │   or
 Mom    │                         │  Mom
        └─────────────────────────┘
              Crystal  Darrell
```
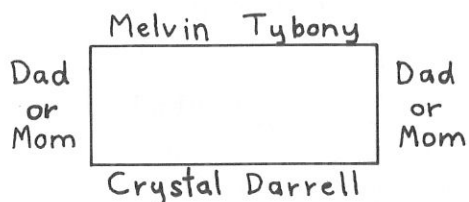
84 $4,410

times	bonus	total bonus earned
1	$ 10	$ 10
2	30	40
3	50	90
4	70	160
⋮	⋮	⋮
21		4,410

number in 1st column squared × 10
= number in 3rd column

85 goldfish—Benny, rabbit—Kenny,
chicken—Brian, raccoon—Paulette, parrot—Priscilla

	goldfish	rabbit	chicken	raccoon	parrot
Brian	N	N	Y	N	N
Paulette	N	N	N	Y	N
Priscilla	N	N	N	N	Y
Benny	Y	N	N	N	N
Kenny	N	Y	N	N	N

86 190

cells	passages	
2	1	> 2
3	3	> 3
4	6	> 4
5	10	> 5
6	15	> 6
7	21	
⋮	⋮	
20	190	

87 125

|	
1 stays 1	(1 × 1 × 1)
2 becomes 8	(2 × 2 × 2)
3 becomes 27	(3 × 3 × 3)
4 becomes 64	(4 × 4 × 4)
5 becomes 125	(5 × 5 × 5)

88 2

After 6,000 miles, take off 2 tires and put on spares. After 6,000 more miles, put the two original tires on in place of the two that have been on all the time.

89 Table 1: energy   water cons.   rock   constellation
Table 2: weather   rocket   soil test planets
   or
Table 1: constellation rock   water cons. energy
Table 2: planets   soil test   rocket   weather

90 55
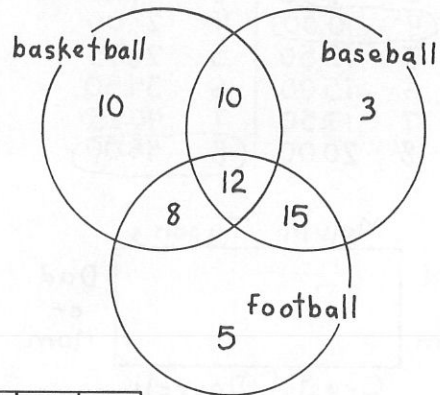
91 7

day	1	2	3	4	5	6	7	8	9	10	11	12
deliver			X			X			X			X
fold				X				X				X
collect						X						X

all 3 jobs every 12 days; 7 times in 84 days

92 72
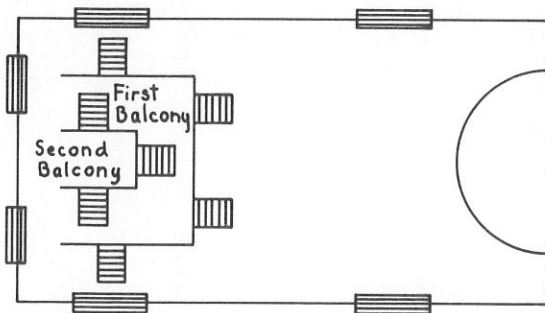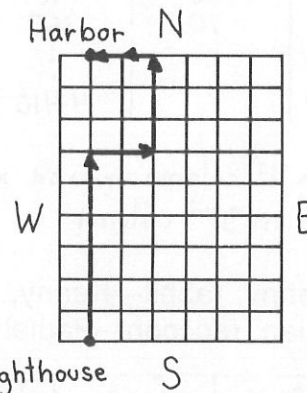
93 Christopher and Charles,
Alexander and Adam

94 128
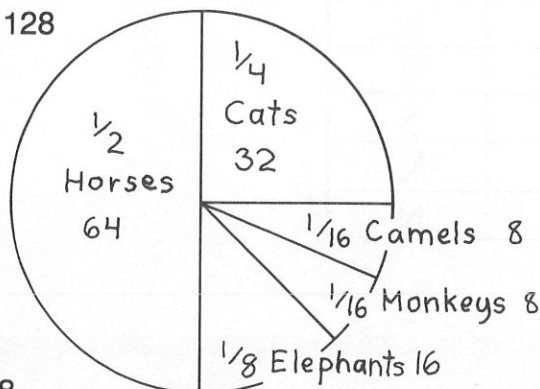
95 8

week	1	2	3	4	5	6	7	8
cans	4	12	20	28	36	44	52	60
total cans	4	16	36	64	100	144	196	256

96 7

97 10

98 285

1 by 1	squares	81	(9^2)
2 by 2	squares	64	(8^2)
3 by 3	squares	49	(7^2)
4 by 4	squares	36	(6^2)
5 by 5	squares	25	(5^2)
6 by 6	squares	16	(4^2)
7 by 7	squares	9	(3^2)
8 by 8	squares	4	(2^2)
9 by 9	squares	1	(1^2)

99 3

basketball / leadership / track Venn diagram: basketball 3, overlap 8, leadership 5, 5, 7, track 2

105 10

hour	1	2	3	4	5
puffs	12	13	15	18	22
squares	11	22	12	24	14
mounds	7	8	11	16	23

hour	6	7	8	9	10
puffs	27	33	40	48	57
squares	28	18	36	26	52
mounds	32	43	56	71	88

197 total
(over 15 dozen)

106 5 wagons, 12 Big Wheels, 14 bicycles

100 15

Combinations
of 8:

2 + 2 + 4
2 + 6 + 0
4 + 4 + 0
8 + 0 + 0
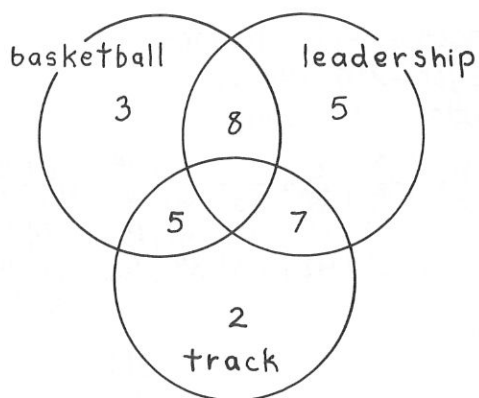
1st attempt	2nd attempt	3rd attempt
2	2	4
2	4	2
4	2	2
2	6	0
2	0	6
0	2	6
0	6	2
6	2	0
6	0	2
4	4	0
4	0	4
0	4	4
8	0	0
0	8	0
0	0	8

107 $1 bills—8, $5 bills—8, $10 bills—4

$1 number	am't.	$5 number	am't
1	1	1	5
2	2	2	10
3	3	3	15
4	4	4	20
5	5	5	25
6	6	6	30
7	7	7	35
8	8	8	40

$10 number	am't
1	10
2	20
3	30
4	40

101 A, 5, J, 2, 8, 6, 3, Q, 10, 4, 7, 9

102 T W E N T Y C A R S

103 Yankees—120, Giants—120,
Phillies—60, Tigers—30,
Padres—30; 360 cards total

Pie chart: 1/3 Yankees, 1/3 Giants, 1/6 Phillies, 1/12 Padres, 1/12 Tigers

108 46 guppies, 12 puppies, 5 frogs
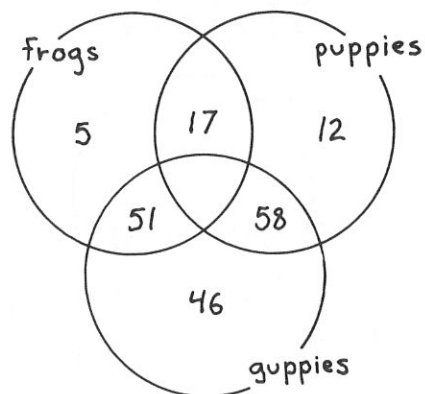
frogs / puppies / guppies Venn diagram: frogs 5, 17, puppies 12, 51, 58, guppies 46

104 Zeno—148 miles, Orb—74 miles,
Yurko—222 miles, Sam—444 miles

109 9

Number line: 2NC, NC, 2NC, NC, 2NC, NC, 2C, NC, 2NC with NC, NC, NC, CC markers

110 70

1 block = 2 paths   6 paths   20 paths

111 Giselle's team—54, Sara's team—30, Sue-Mee's team—12

112 1,240

marker	number of miles	
1	1	1
2	5	1+4
3	14	1+4+9
4	30	1+4+9+16
5	55	1+4+9+16+25
.	.	.
.	.	.
15	1,240	1+4+9+16+25+36+ 49+64+81+100+121+ 144+169+196+225

113 16

12¢	10¢	8¢	4¢
3	1	0	0
2	1	0	3
2	1	1	1
1	3	0	1
1	1	3	0
1	1	0	6
1	1	2	2
1	1	1	4
0	3	2	0
0	3	1	2
0	3	0	4
0	1	4	1
0	1	3	3
0	1	2	5
0	1	1	7
0	1	0	9

114 22

Sharon
26
54   32
28   34   6
Darrin   Aaron

115 chocolate—105 pounds, sugar—210 pounds, butter—147 pounds

days	3	6	9	12	15	18	21
choc.	15	30	45	60	75	90	105
sugar	30	60	90	120	150	180	210
butter	21	42	63	84	105	126	147

total = 462

116 50

GF   GM

SON DTR-IN-LAW SON DTR-IN-LAW DAUGHTER SON-IN-LAW DAUGHTER SON-IN-LAW DAUGHTER SON-IN-LAW DAUGHTER SON-IN-LAW

SS DDDD SS DDDD SS DDDD SS DDDD SS DDDD SS DDDD

117 5th day

day	1	2	3	4	5
#missing	2	6	18	54	162
#left	458	452	434	380	218

118 146

42
1st 6
homerooms
52   36 → next 4
8   8 → last
next 6   next 2

119 Jenny and Carl, Becky and T.J., Blanca and Christopher, Marisol and Peter

120 $276,000,000.00

satellites	#connections
1	0 >1
2	1 >2
3	3 >3
4	6 >4
5	10 >5
6	15 >6
7	21 >7
8	28
.	.
.	.
23	276