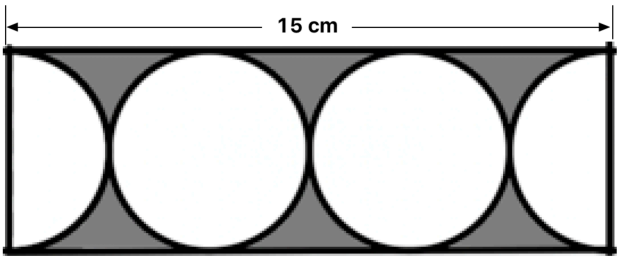
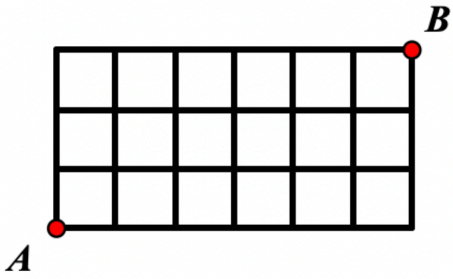
Problem Solving

1. A certain farmer has an orange grove. In his grove there are 120 trees. Each tree ordinarily produces 650 oranges. He is interested in raising his orange production and knows that because of lost space and sunlight, every additional tree that he plants will cause a reduction of 5 oranges from each tree. What is the maximum number of oranges that he will be able to produce in his grove, and how many trees will he need to reach this maximum.

2. Find the area of the shaded region.



3. How many paths to go from ***A*** to ***B***?



4. Find the sum: $24 + 25 + 26 + \ldots + 997 + 998 + 999 + 1000$

5. When $(1{,}234)^{23}$ is completely multiplied out, what will the number be in the ones place?

6. Angelina and her friends formed a club. They wanted to assign a unique 4-digit secret code number to each member of the club. They decided to use the digits 1, 3, 7, and 9 for their numbering system and each of these digits can appear only once in every secret code number (i.e. 1379 is a valid number, but 1133 is not a valid number). What is the maximum number of members who could join the club if everyone is to be assigned a unique secret code number?

7. You have a list of 7 numbers. The average of the numbers is 9. If you take away one of the numbers, the average of the numbers is 8. What number did you take away?

8. There are a total of 12 bicycles and tricycles at the park. Together they have a total of 29 wheels. How many are bicycles and how many are tricycles?

9. How many squares are there on a standard 8 x 8 checkerboard?

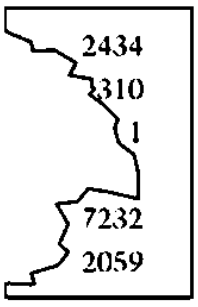
10. Terri, Dave, Emily, and Thad are running for Class President. When the election results came in, Thad received more votes than Emily. Dave received more votes than Thad. Terri received less votes than Emily. Who was elected Class President?

11. **a)** How many toothpicks will be required to build a row of 50 such toothpick houses?

b) How many such toothpick houses could be built if 500 toothpicks are available?

12. There was a war on the planet Mxyzptlk. It lasted 1,000 days. If the planet Mxyzptlk uses the same calendar that we use on Earth, and the 1,000-day war started on a Monday, what day of the week did the 1,000-day war end?

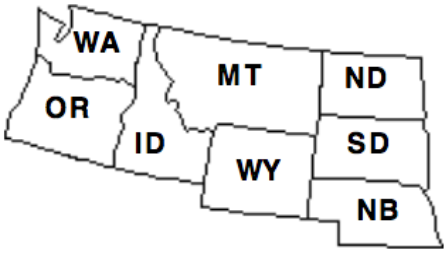
13. Al found the torn piece of paper shown below. Six numbers originally appeared in a column on this paper. The fourth number from the top of the column had been completely torn away. Al wondered whether the sum of the six numbers was odd or even.



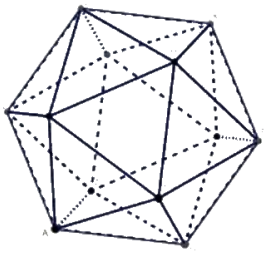
Give an example of a number that could be the fourth number in the column if the sum of the six numbers is an odd number.

14. How many degrees are in the acute angle formed by the clock hands at 10:00 ?

15. You intend to fly from Omaha, Nebraska and then travel by car to tour the states below. You wish to only cross the borders between neighboring states only once. What would be your itinerary?



16. How many edges are in the solid pictured here?



17. If you continue with the pattern shown below, what would the 20^{th} figure look like?

Figure 1	Figure 2	Figure 3	Figure 4
o o	o o o o o o	o o o o o o o o o o o o	o o o o o o o o o o o o o o o o o o o o

18. A manufacturer has a quantity of wire in lengths of 105, 120, 135, and 165 feet, respectively, which he wishes to cut without waste into the longest possible equal lengths. What must be the length of each of these?

19. Find the rational number exactly **midway** between 0.875 and $\frac{8}{9}$.

20. Express $\frac{11}{16}$ as the sum of unique "unit" fractions.