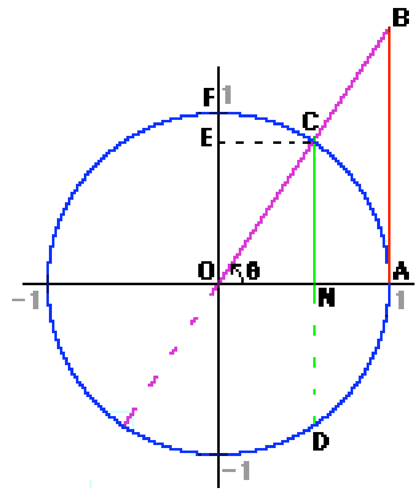
Geometry and Trigonometry

The names sine, secant and tangent are derived from the geometric terms associated with certain lines in the diagram below. If the radius of the circle is taken to be one unit, it follows that

$$\frac{\overline{CN}}{1} = \sin\theta, \text{ or } \overline{CN} = \sin\theta$$

$$\frac{\overline{AB}}{1} = \tan\theta, \text{ or } \overline{AB} = \tan\theta$$

$$\frac{\overline{OB}}{1} = \sec\theta, \text{ or } \overline{OB} = \sec\theta$$



The geometric narratives for these lines are-

- The <u>sine of an arc</u> is a straight line drawn from one end of that arc, perpendicular to a diameter passing through the other end of the same arc.
- The <u>tangent of an arc</u> is a straight line drawn from one extremity, perpendicular to the diameter, and terminated by a straight line drawn through the center of the circle and the other extremity of that arc.
- The <u>secant of an arc</u> is a straight line drawn from the center of the circle, and produced till it meets the tangent.

Now $\overline{CN}$ is half of the chord $\overline{CD}$, and early expressions for the sines of angles were really lengths of the chords in a circle. The Arabic word for chord was translated into the Latin *sinus*. The lengths $\overline{AB}$ and $\overline{OB}$ are portions of the *tangent* and the *secant* to the circle. The trigonometric functions bearing these names were developed later than the sine, and they merely assumed the corresponding geometric names.

The <u>co</u>functions received their names from the fact that a*ny trigonometric function of an acute angle is equal in value to the cofunction of the* ***<u>co</u>****mplementary angle*.

There are other(old) functions that can also be reckoned. The versed sine or *versine* of an angle (or arc) is that part of the diameter between the sine and the arc *i.e.*, $\text{vers}\theta = \overline{NA} = 1 - \cos\theta$. The coversed sine or *coversine* is the versine of the complement of an angle (or arc). Here $\text{covers}\theta = \overline{EF} = 1 - \sin\theta$. The *haversine* is one-half of the versine. $\text{hav}\theta = \frac{1}{2}(1 - \cos\theta)$.