2016 Marywood Mathematics Contest

Level I

Sponsored by

iMACS

The Student Mathematics and Computer Science Club of

Marywood University

and

Mu Alpha Theta

March 19, 2016

Directions:

1. This exam consists of 40 questions on 5 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

7. May the force be with you.

Please do NOT open the test until you are told to do so.

1. If x is an odd integer, which of the following is even?

    A. $2x + 1$ **B.** $2x + 3$ **C.** $x + x - 1$ **D.** $2(x + 1)$ **E.** None of these

2. The average of the five numbers 8, 9, 10, 11, 12 is

    A. 8   **B.** 9   **C.** 10   **D.** 11   **E.** None of these

3. If 7:30 a.m. was 16 minutes ago, in how many minutes will it be 8:00 a.m.?

    A. 12   **B.** 14   **C.** 16   **D.** 24   **E.** None of these

4. When a number is tripled, then decreased by 5, the result is 16. What is the original number?

    A. 9   **B.** 5   **C.** 7   **D.** 11   **E.** None of these

5. The average of 7 and 11 is a. The average of a and b is 11. What is the value of b?

    A. 13   **B.** 11   **C.** 10   **D.** 9   **E.** None of these

6. Which number is irrational?

    A. $9^{1/2}$ **C.** 3.14   **E.** None of these
    B. $10^{1/2}$ **D.** 1/3

7. If the length of the diagonal of the square is $3\sqrt{2}$, what is the area of the square?

    A. 9   **B.** 18   **C.** 6   **D.** 12   **E.** None of these

8. $\triangle ABC$ and $\triangle A'B'C'$ are similar triangles. Side AB and $A'B'$ are of length 2 and 6, respectively. Side BC and AC are of length 3 and 7 respectively. What is the length of side $B'C'$?

    A. 7   **B.** 5   **C.** 10   **D.** 1   **E.** None of these

9. Which of the following is NOT a valid reason for proving two congruent triangles?

    A. SAS(Side-Angle-Side)   **C.** SAA(Side-Angle-Angle)   **E.** These are all valid.
    B. SSS(Side-Side-Side)   **D.** SSA(Side-Side-Angle)

10. Solve: $-4 < 4 + 3x \leq 7$

A. $0 < x \leq 1$ **C.** $-8/3 \leq x < 1$ **E.** None of these

B. $-1 < x \leq 8/3$ **D.** $-8/3 < x \leq 1$

11. Oil is pumped into a non-empty tank at a changing rate. The volume of oil in the tank doubles every minute and the tank is filled in 12 minutes. How many minutes did it take for the tank to be half full?

A. 8   **B.** 10   **C.** 12   **D.** 14   **E.** None of these

12. Suppose $a, b,$ and c are integers satisfying $2^a \cdot 2^b \cdot 2^c = 128$. Find $a + b + c$.

A. 6   **B.** 7   **C.** 8   **D.** 9   **E.** None of these

13. The largest possible number of Sundays in a year is

A. 51   **B.** 53   **C.** 60   **D.** 52   **E.** None of these

14. What is the difference between the sum of all even integers from 1 to 100 and the sum of all odd integers from 1 to 100?

A. 50   **B.** 100   **C.** 150   **D.** 1050   **E.** None of these

15. If x and y are positive integers with $x + y = 10$, then the largest possible value of xy is

A. 1   **B.** 10   **C.** 25   **D.** 100   **E.** None of these

16. Find the volume of the loaf of bread given that there are 18 slices, each of which are one half-inch thick and have a cross-sectional area of 13 in^2.

A. 468 in^3 **B.** 234 in^3 **C.** 117 in^3 **D.** 36 in^3 **E.** None of these

17. If two men can paint two rooms in two days, how long does it take one man to paint one room?

A. Half a day.   **B.** One day.   **C.** Four days.   **D.** Five days.   **E.** None of these

18. Which point is the intersection of the lines $3x + 2y = 17$ and $x - 4y = 1$?

A. $(1, 5)$ **B.** $(-1, 5)$ **C.** $(5, 1)$ **D.** $(5, -1)$ **E.** None of these

19. Line L passes through the points $(3, 1)$, and $(4, -3)$. Find b so that the line L passes through the point $(-4, b)$

    A. -4   **B.** 12   **C.** 13   **D.** 29   **E.** None of these

20. For what value of k are the lines $2x + 3y = 4k$ and $x - 2ky = 7$ perpendicular?

    A. $-3/4$ **B.** $1/6$ **C.** $-1/3$ **D.** $1/2$ **E.** None of these

21. Let $n(A)$ denote the number of elements in the set A. If $n(A) = 32, n(B) = 93$ and $n(A \cup B) = 109$, what is $n(A \cap B)$?

    A. 48   **B.** 8   **C.** 18   **D.** 16   **E.** None of these

22. A line containing the point $(6, -1)$ is perpendicular to the line $y = 3x + 2$. What is the equation of the line?

    A. $y = 3x - 19$ **C.** $y = -\dfrac{1}{3}x + 1$ **D.** $y = \dfrac{1}{3}x - 3$

    B. $y = -3x + 17$ **E.** None of these

23. A hockey team has 6 more red helmets than blue helmets. The ratio of red helmets to blue helmets is $5 : 3$. The total number of red helmets and blue helmets is

    A. 24   **B.** 22   **C.** 20   **D.** 18   **E.** None of these

24. What is the next term in the sequence $1, 8, 27, 64, \cdots$?

    A. 75   **B.** 125   **C.** 325   **D.** 625   **E.** None of these

25. Solve for x: $(x + 1)^{3/2} = 27$

    A. 8   **B.** 9   **C.** 5   **D.** 10   **E.** None of these

26. A woman has \$ 2.15 in change in her purse, comprised entirely of dimes and quarters. Given that there are more quarters than dimes in her purse, what is the total number of coins?

    A. 8   **B.** 9   **C.** 10   **D.** 11   **E.** None of these

27. A certain cylindrical pencil is 20cm long, and has a diameter of 1 cm. The diameter of the lead is 0.5 cm. What is the volume, in cubic centimeters, of the wood of the pencil?

    A. 20π **B.** $15\pi/4$ **C.** 5π **D.** 15π **E.** None of these

28. If $24x = 8y + 4$, determine the value of the expression $6y - 18x + 13$.

    A. 13   **B.** 10   **C.** 4   **D.** 16   **E.** None of these

29. What is the greatest common factor of: $3x^2y^4, 9x^3y^2, 12xy^3$

    A. $36x^2y^3$ **B.** $3x^3y^3$ **C.** $9xy^2$ **D.** $3x^2y^3$ **E.** None of these

30. How many real solutions does the equation $x^5 + 2x^3 + 8x^2 + 16 = 0$ have?

    A. 0   **B.** 2   **C.** 4   **D.** 5   **E.** None of these

31. Two fair six-sided dice are rolled. What is the probability that the results sum to 5 or 7?

    A. 5/18   **B.** 2/9   **C.** 5/9   **D.** 7/18   **E.** None of these

32. Simplify: $\frac{2^{2016} - 2^{2015}}{128}$

    A. 2^{2009} **B.** 128^{2015} **C.** $1/64$ **D.** $2^{2001} - 2^{2000}$ **E.** None of these

33. The least whole number greater than 1 that is both a square and a cube is 64. What is the least whole number greater than 1 that is a square, cube, and a fourth power?

    A. 4096   **B.** 256   **C.** 243   **D.** 19683   **E.** None of these

34. If $x + y = 5$ and $x^2 + y^2 = 63$, determine the value of xy.

    A. -9 **B.** 19   **C.** 57   **D.** 38   **E.** None of these

35. A class has three girls and three boys. These students line up at random, one after another. What is the probability that no boy is right next to another boy, and no girl is right next to another girl?

    A. 1/6   **B.** 1/20   **C.** 3/10   **D.** 1/10   **E.** None of these

36. The number of pairs (m, n) such that $2^m - 2^n = 63$ in which m and n are nonnegative integers is

A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** None of these

37. Compute the smallest positive integer n such that $n^2 + n^0 + n^1 + n^3$ is a multiple of 17.

A. 4   **B.** 6   **C.** 5   **D.** 17   **E.** None of these

38. On a test the passing students had an average of 83, while the failing students had an average of 55. If the overall class average was 76, what percent of the class passed?

A. 44%   **B.** 68%   **C.** 75%   **D.** 72%   **E.** None of these

39. If you answered all 40 questions on this contest at random, what is the probability that you would get exactly one question right?

A. $40(4/5)^{39}$ **C.** $5(4/5)^{40}$ **E.** None of these
B. $1/5 + (4/5)^{39}$ **D.** $8(4/5)^{39}$

40. A rubber ball is dropped from a height of 20 feet. Every time it hits the ground, it bounces back up 50% of its previous height. How far does the ball travel before it comes to rest? (Assume the ball only moves up and down. Thus at the top of the first bounce, it has traveled $20 + 14 = 34$ feet.)

A. 20 feet   **B.** 40 feet   **C.** 60 feet   **D.** 80 feet   **E.** None of these

This page left blank for your use.

2016 Marywood Mathematics Contest

Level II

Sponsored by

iMACS

The Marywood Math and Computer Science Club

Marywood University

and

Mu Alpha Theta

March 19, 2016

Directions:

1. This exam consists of 40 questions on 6 pages. Please check to make sure that you have all the pages.

2. No calculator or any other electronic device is allowed on this exam.

3. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess. Your score will be the number of correct responses.

4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark **only one answer** for each question. Only those answers on the answer sheet will be counted.

5. There is a sheet of blank paper on the last page which you can tear off and use as scratch paper. You may also use the back of the pages.

6. NOTE: **In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.**

7. May the force be with you.

Please do NOT open the test until you are told to do so.

1. If x is an odd integer, which of the following is even?

    A. $2x + 1$ **B.** $2x + 3$ **C.** $x + x - 1$ **D.** $2(x + 1)$ **E.** None of these

2. Find t so that the three points are collinear: $(-2, 5)$, $(0, t)$, $(1, 1)$

    A. $\dfrac{4}{3}$ **B.** $\dfrac{5}{3}$ **C.** $\dfrac{7}{3}$ **D.** $\dfrac{8}{3}$ **E.** None of these

3. Find the distance between the points $(5, 4)$ and $(2, 8)$.

    A. 5   **B.**   **C.**   **D.**   **E.** None of these

4. If $a \triangle b = a^b + b^a$, then $(2 \triangle 3) \triangle 1 =$

    A. 12   **B.** 18   **C.** 22   **D.** 48   **E.** None of these

5. An explorer is searching for buried treasure. To reach the region where the treasure is buried, she must pass through three gates, leaving half her money at each gate. With how much money must she begin in order to have \$144 left to buy supplies when she reaches the treasure?

    A. \$288   **B.** \$1,050   **C.** \$1,152   **D.** \$2,304   **E.** None of these

6. If $\sin \theta = \dfrac{\sqrt{3}}{2}$, then $\theta =$

    A. $\dfrac{\pi}{2}$ **B.** $\dfrac{\pi}{3}$ **C.** $\dfrac{\pi}{4}$ **D.** $\dfrac{\pi}{6}$ **E.** None of these

7. How many unique real solutions does the equation $x^3 - 4x = 0$ have?

    A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** None of these

8. You toss a fair coin 5 times. What is the probability that the you do not get 5 tails.

    A. $\dfrac{1}{32}$ **B.** $\dfrac{1}{5}$ **C.** $\dfrac{31}{32}$ **D.** $\dfrac{1}{10}$ **E.** None of these

9. A room has six doorways. In how many ways can a person enter the room and then leave the room without using the same door?

    A. 5   **B.** 6   **C.** 25   **D.** 36   **E.** None of these

10. If AB is perpendicular to BC, BC is perpendicular to CD, $AB = 8$, $BC = 5$, and $CD = 4$, then what is the shortest path from A to D?
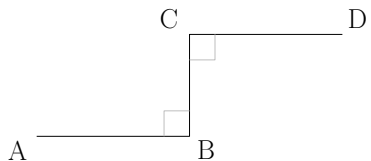
    A. 12   **B.** 13   **C.** 14   **D.** 15   **E.** None of these

11. If -3 is a solution to $x^2 + hx + 27 = 0$, then $h =$

    A. 12   **B.** 13   **C.** 14   **D.** 15   **E.** None of these

12. $\log_2\left(\dfrac{1}{8}\right) =$

    A. $\dfrac{1}{3}$ **B.** $-\dfrac{1}{3}$ **C.** 3   **D.** -3 **E.** None of these

13. If $|x - 3| \le 4$, then

    A. $1 \le x \le 7$ **C.** $-1 < x < 7$ **E.** None of these
    B. $-7 \le x \le -1$ **D.** $-7 < x < 1$

14. The domain of the function $f(x) = \dfrac{1}{\sqrt{3 - x}}$ is

    A. $x < 3$ **B.** $x \le 3$ **C.** $x > 3$ **D.** $x \ge 3$ **E.** None of these

15. How many cubic centimeters (cc) of water must be added to 100 cc of an 80% solution of boric acid to reduce it to a 50% solution?

    A. 30   **B.** 40   **C.** 50   **D.** 60   **E.** None of these

16. Express the following product as a reduced fraction

$$\left(1 - \frac{1}{2016}\right)\left(1 - \frac{1}{2015}\right)\left(1 - \frac{1}{2014}\right) \cdots \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{1}\right)$$

    A. $\dfrac{2015}{2016}$ **B.** $\dfrac{1}{2016}$ **C.** $\dfrac{503}{1008}$ **D.** $\dfrac{251}{504}$ **E.** None of these

17. Find the x-coordinate of the solution of the system

$$\begin{aligned} 3x + 4y &= 3 \\ x - 2y &= 6 \end{aligned}$$

A. 1 **B.** 2 **C.** 3 **D.** 4 **E.** None of these

18. $\dfrac{1}{\tan \theta} + \dfrac{1}{\cot \theta} =$

A. $\dfrac{1}{\sin \theta \cos \theta}$ **B.** $\dfrac{\sin \theta}{\cos \theta}$ **C.** $\sin \theta \cos \theta$ **D.** $\dfrac{\cos \theta}{\sin \theta}$ **E.** None of these

19. Solve $x^2 - 4x + 29 = 0$

A. $-4 \pm 2i$ **B.** $4 \pm 2i$ **C.** $-2 \pm 4i$ **D.** $2 \pm 4i$ **E.** None of these

20. $(5 - 3\sqrt{5})(3 + \sqrt{5}) =$

A. $8\sqrt{5}$ **B.** $-4\sqrt{5}$ **C.** $3\sqrt{5}$ **D.** 15 **E.** None of these

21. For all non-zero real numbers x and y such that $x - y = xy$, $\dfrac{1}{x} - \dfrac{1}{y} =$

A. $\dfrac{1}{xy}$ **B.** $\dfrac{1}{x-y}$ **C.** 1 **D.** -1 **E.** None of these

22. If $x^2 + 2x + n > 10$ for all real numbers x, then which of the following conditions must be true?

A. $n > 11$ **B.** $n = 10$ **C.** $n = 11$ **D.** $n < 10$ **E.** None of these

23. 12 knights of Camelot are gathered around a large round table. Out of this group, 1/2 belong to the "Sir Lancelot Fan Club," 1/3 belong to the "Sir Galahad" fan club, and 1/4 belong to both clubs. How many belong to neither fan club?

**A.** 2 **B.** 3 **C.** 4 **D.** 5 **E.** None of these

24. Marywood Field Hocky team won 30% of 60 consecutive games it played. Under the leadership of thier new captain, Susan, this team had a winning streak that raised its average to 50%. How many games did the team play in its winning streak to obtain this average?

A. 18 **B.** 24 **C.** 30 **D.** 36 **E.** None of these

3

25. If the mean of x, 3, $4x - 3$, $x + 4$, -16, 9, and $x - 4$ is 4, what is x?

    A. 6   **B.** 8   **C.** 5   **D.** 10   **E.** None of these

26. How many paths are there from A to B which only move right or up and follow the lines?
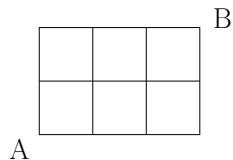
    A. 5   **B.** 6   **C.** 7   **D.** 10   **E.** None of these

27. The two congruent circles below are tangent to the sides of the rectangle and tangent to each other. If the radius of each is equal to 2, what is the area of the shaded region?
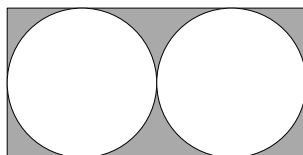
    A. $32 - 8\pi$ **B.** $8 - 4\pi$ **C.** $16\pi - 8$ **D.** $16(\pi - 1)$ **E.** None of these

28. The point $A = (2, 3)$ is reflected about the x-axis to a point B. Then B is reflected about the line $y = x$ to a point C. What is the area of the triangle ABC?

    A. 12   **B.** 18   **C.** 24   **D.** 30   **E.** None of these

29. This problem and the next deal with converting a number from one base to base 10. Recall that $523_{\text{base 10}} = 5 \cdot 10^2 + 2 \cdot 10 + 3 = 523$, while $523_{\text{base 6}} = 5 \cdot 6^2 + 2 \cdot 6 + 3 = 195$

    $231_{\text{base 5}} =$

    A. 66   **B.** 91   **C.** 30   **D.** 231   **E.** None of these

30. $101_{\text{base } (11_{\text{base 2}})} =$

    A. 10   **B.** 5   **C.** 123   **D.** 42   **E.** None of these

31. What is the difference between the sum of all even integers from 1 to 100 and the sum of all odd integers from 1 to 100?

    A. 1   **B.** 2   **C.** 0   **D.** 4   **E.** None of these

32. In a game of Gorf, three gligs and two glugs score 21 points, while two gligs and 5 glugs score 25 points. How many points do four gligs and three glugs score?

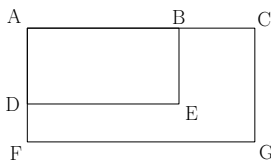   **A.** 28   **B.** 29   **C.** 30   **D.** 31   **E.** None of these

33. $(3 + 2i)^2 =$

    A. $13 - 12i$ **B.** $13 + 12i$ **C.** $9 - 4i$ **D.** $9 + 4i$ **E.** None of these

34. What is the smallest positive integer k such that $2x(kx - 4) - x^2 + 6 = 0$ has no real roots.

    A. 0   **B.** 1   **C.** 2   **D.** 3   **E.** None of these

35. $ACGF$ and $ABED$ are both rectangles with $AB = \dfrac{2}{3}AC$ and $AD = \dfrac{2}{3}AF$. What is the ratio of the area of $ACGF$ to the area of $ABED$?

A   B   C
D   E
F   G

    A. $\dfrac{1}{9}$ **B.** $\dfrac{2}{3}$ **C.** $\dfrac{4}{9}$ **D.** $\dfrac{2}{3}$ **E.** None of these

36. If $\log(\log(\log(\log(x)))) = 0$, where $\log(10) = 1$, then $x =$

    A. 10 **B.** $10^{10,000}$ **C.** $10^{10,000,000}$ **D.** $10^{10,000,000,000}$ **E.** None of these

37. How many positive divisors does 6! have?

    A. 4   **B.** 6   **C.** 10   **D.** 20   **E.** None of these

38. If b varies over all real numbers, upon which curve do the vertexes of the parabolas with equations $y = x^2 + bx$ lie?
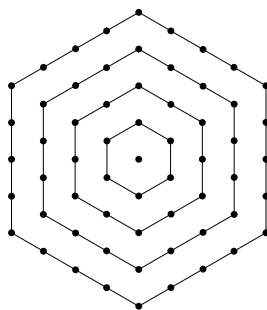
    A. a line   **B.** a circle   **C.** a parabola   **D.** a hyperbola   **E.** None of these

39. For positive integers n, it is know that $1 + 3 + 5 \cdots + (2n - 1) = n^2$.

    Evaluate $\dfrac{1 + 3 + 5 + \cdots + 739}{741 + 743 + \cdots + 1479}$

    A. $\dfrac{1}{2}$ **B.** $\dfrac{1}{3}$ **C.** $\dfrac{1}{4}$ **D.** $\dfrac{1}{5}$ **E.** None of these

40. Consider the hexagonal array shown below. Then n^{th} hex number is defined as the total number of dots in the first n layers. The first seven hex numbers are 1, 7, 19, 37, 61, 91, and 127.

Find the 50th hex number.

A. 294 **B.** 7,351 **C.** 7,650 **D.** 7,651 **E.** None of these

This page left blank for your use.