Given $\begin{array}{l} x'(t) = 3x - 9y \\ y'(t) = 4x - 3y \end{array}$ we have $A = \begin{pmatrix} 3 & -9 \\ 4 & -3 \end{pmatrix}$ $\begin{array}{l} \Rightarrow \lambda^2 + 27 = 0 \\ \Rightarrow \lambda_1 = 3\sqrt{3}i, \lambda_2 = -3\sqrt{3}i \end{array}$

$\alpha = 0$

$\beta = 3\sqrt{3}$

$$\vec{\eta} = \begin{pmatrix} 9 \\ 3 - 3\sqrt{3}i \end{pmatrix} = \begin{pmatrix} 9 + 0i \\ 3 - 3\sqrt{3}i \end{pmatrix} \mapsto \begin{pmatrix} 3 + 0i \\ 1 - \sqrt{3}i \end{pmatrix}$$

$$\begin{pmatrix} 3 + 0i \\ 1 - \sqrt{3}i \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix} i$$

$$\therefore \ \vec{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \ \textit{and} \ \vec{b} = \begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix}$$

We should use the following solution form:

$$\vec{X} = c_1\vec{U} + c_2\vec{V} \quad \text{where}$$

$$\vec{U} = e^{\alpha t}\left[(\cos\beta t)\vec{a} - (\sin\beta t)\vec{b}\right]$$
$$\vec{V} = e^{\alpha t}\left[(\sin\beta t)\vec{a} + (\cos\beta t)\vec{b}\right]$$

So

$$\begin{aligned} \vec{U} &= \left(\cos 3\sqrt{3}\,t\right)\vec{a} - \left(\sin 3\sqrt{3}\,t\right)\vec{b} \\ &= \left(\cos 3\sqrt{3}\,t\right)\begin{pmatrix} 3 \\ 1 \end{pmatrix} - \left(\sin 3\sqrt{3}\,t\right)\begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix} \\ &= \begin{pmatrix} 3\cos 3\sqrt{3}\,t \\ \cos 3\sqrt{3}\,t \end{pmatrix} - \begin{pmatrix} 0 \\ -\sqrt{3}\sin 3\sqrt{3}\,t \end{pmatrix} \\ &= \begin{pmatrix} 3\cos 3\sqrt{3}\,t \\ \cos 3\sqrt{3}\,t + \sqrt{3}\sin 3\sqrt{3}\,t \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \vec{V} &= \left(\sin 3\sqrt{3}\,t\right)\vec{a} + \left(\cos 3\sqrt{3}\,t\right)\vec{b} \\ &= \left(\sin 3\sqrt{3}\,t\right)\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \left(\cos 3\sqrt{3}\,t\right)\begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix} \\ &= \begin{pmatrix} 3\sin 3\sqrt{3}\,t \\ \sin 3\sqrt{3}\,t \end{pmatrix} + \begin{pmatrix} 0 \\ -\cos 3\sqrt{3}\,t \end{pmatrix} \\ &= \begin{pmatrix} 3\sin 3\sqrt{3}\,t \\ \sin 3\sqrt{3}\,t - \cos 3\sqrt{3}\,t \end{pmatrix} \end{aligned}$$

Then

$$\vec{X} = c_1\begin{pmatrix} 3\cos 3\sqrt{3}\,t \\ \cos 3\sqrt{3}\,t + \sqrt{3}\sin 3\sqrt{3}\,t \end{pmatrix} + c_2\begin{pmatrix} 3\sin 3\sqrt{3}\,t \\ \sin 3\sqrt{3}\,t - \cos 3\sqrt{3}\,t \end{pmatrix}$$ and therefore

$$x(t) = 3c_1\cos 3\sqrt{3}\,t + 3c_2\sin 3\sqrt{3}\,t$$
$$y(t) = c_1\cos 3\sqrt{3}\,t + \sqrt{3}c_1\sin 3\sqrt{3}\,t + c_2\sin 3\sqrt{3}\,t - c_2\cos 3\sqrt{3}\,t$$