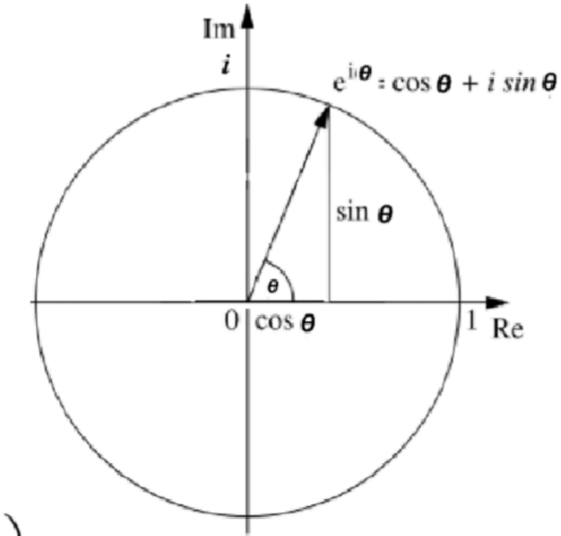
$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} \ \ldots$$



So

$$\begin{aligned}
e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \cdots \\
&= 1 + ix - \frac{(x)^2}{2!} - \frac{i(x)^3}{3!} + \frac{(x)^4}{4!} + \frac{i(x)^5}{5!} - \frac{(x)^6}{6!} - \frac{i(x)^7}{7!} + \cdots \\
&= \left(1 - \frac{(x)^2}{2!} + \frac{(x)^4}{4!} - \frac{(x)^6}{6!} + \cdots\right) + i\left(x - \frac{(x)^3}{3!} + \frac{(x)^5}{5!} - \frac{(x)^7}{7!} + \cdots\right) \\
&= \cos x + i \sin x
\end{aligned}$$

$x + iy$

$x = r\cos\theta \qquad y = r\sin\theta$

$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan\frac{y}{x}$

$re^{i\theta} = r\,\mathrm{cis}\,\theta = r(\cos\theta + i\sin\theta)$

$$r(\cos\theta + i\sin\theta)^n = r^n(\cos n\theta + i\sin n\theta)$$

$$r(\cos\theta + i\sin\theta)^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{1}{n}\theta + i\sin\frac{1}{n}\theta\right)$$

$$\cos(at) = RE(\cos at + i\sin at) = RE\left(e^{iat}\right)$$

$$\sin(at) = IM(\cos at + i\sin at) = IM\left(e^{iat}\right)$$

$$e^{iz} = \cos z + i \sin z$$

$$e^{-iz} = \cos z - i \sin z$$

$$\sin z = \frac{1}{2i}\left(e^{iz} - e^{-iz}\right)$$

$$\cos z = \tfrac{1}{2}\left(e^{iz} + e^{-iz}\right)$$

$$\int e^{4t}\cos 5t\,dt = \int e^{4t}\,RE\left(\cos 5t + i\sin 5t\right)dt$$

$$= RE\int e^{4t}e^{i5t}\,dt = RE\int e^{(4+i5)t}\,dt$$

$$= RE\left(\frac{1}{4+5i}\;e^{(4+i5)t}\right) = RE\left(\frac{4-5i}{41}\;e^{(4+i5)t}\right)$$

$$= RE\left(\left(\frac{4}{41}-\frac{5i}{41}\right)\;e^{4t}\left(\cos 5t + i\sin 5t\right)\right)$$

$$= \frac{1}{41}e^{4t}\left(4\cos 5t + 5\sin 5t\right)$$

$$\int e^{4t}\sin 5t\,dt = \int e^{4t}\,IM\left(\cos 5t + i\sin 5t\right)dt$$

$$= IM\int e^{4t}e^{i5t}\,dt = RE\int e^{(4+i5)t}\,dt$$

$$= IM\left(\frac{1}{4+5i}\;e^{(4+i5)t}\right) = IM\left(\frac{4-5i}{41}\;e^{(4+i5)t}\right)$$

$$= IM\left(\left(\frac{4}{41}-\frac{5i}{41}\right)\;e^{4t}\left(\cos 5t + i\sin 5t\right)\right)$$

$$= \frac{1}{41}e^{4t}\left(-5\cos 5t + 4\sin 5t\right)$$