MTH 211 **Test #2**
Summer 2009

Introduction to Ordinary Differential Equations

Name ________________________

1. Compute $(3 - 3i)^4$ by converting to exponential form. (Express your answer in the form $\lambda + \mu i$.)

$$\left(3\sqrt{2}\, e^{\frac{7}{4}\pi i}\right)^4 = -324 + 0i$$

2. Find the general solution to each of the following:

a) $y'' + 10y' + 21y = 0$

$$y(t) = c_1 e^{-3t} + c_2 e^{-7t}$$

b) $y'' - 3y' + 4y = 0$

$$y(t) = c_1 e^{\frac{3}{2}t} \cos\left(\frac{1}{2}\sqrt{7}\, t\right) + c_1 e^{\frac{3}{2}t} \sin\left(\frac{1}{2}\sqrt{7}\, t\right)$$

3. Find the solution to the initial value problem: $y'' - 8y' + 16y = 0$; $y(0) = 2,\ y'(0) = 1$.

$$y(t) = -\frac{1}{3} e^{4t} + \frac{7}{3} t\, e^{4t}$$

4. Solve the following by the method of undetermined coefficients:

a) $y'' - y' - 2y = 4x^2$ **b)** $y'' - y = 5\cos\sqrt{2}\,x$

$$y(t) = c_1 e^{2t} + c_2 e^{t} - 2t^2 + 2t - 3$$

$$y(t) = c_1 e^{t} + c_2 e^{-t} - \frac{5}{3}\cos\sqrt{2}\,t$$

5. Solve the following initial value problem: $y' - 5y = 3e^x - 2x + 1$; $y(0) = 1$, $y'(0) = 1$.

$$y(t) = c_1 e^{5t} - \frac{3}{4}e^{t} + \frac{2}{5}t - \frac{3}{25}$$

6. Given that $y_1(t) = e^t$ is a solution of $ty'' - (2t + 1)y' + (t + 1)y = 0$. Find a general solution and calculate the Wronksian of the component solutions.

Using reduction of order $y(t) = c_1 e^{2t} + c_2 e^{t}$, $W\left(e^{2t}, e^{t}\right) = e^{2t} - 2e^{3t}$