Ordinary Differential Equations

Name ______________________

All work must be shown *to receive credit for each problem. Please circle your final answers.*

1. $\mathcal{L}\left\{e^{t}\sin t + 5e^{-3t}\cos 2t\right\} = \frac{1}{(s-1)^2+1} + 5\left(\frac{s+3}{(s+3)^2+1}\right)$

2. $\mathcal{L}\left\{e^{5t}t^{3}\right\} = \frac{6}{(s-5)^4}$

3. $\mathcal{L}^{-1}\left\{\frac{s+6}{s^2+s-2}\right\} = \frac{7}{3}e^{-t} - \frac{4}{3}e^{-2t}$

4. $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2+4}\right\} = 2\cos 2t - \frac{1}{2}\sin 2t$

- In solving the differential equation $\frac{dy}{dt} + 3y = e^{-t};\ y(0)=1$ using Laplace transforms

5. $Y(s) = \frac{1}{(s+3)(s+1)} + \frac{1}{s+3}$

6. $y(t) = \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t}$

- In solving the differential equation $y'' + 2y' + 2y = 0;\ y(0)=0,\ y'(0)=1$ using Laplace transforms

7. $Y(s) = \frac{1}{s^2+2s+2}$

8. $y(t) = e^{-t}\sin t$

- In solving the differential equation $y'' + 4y = \sin t$ with $y(0)=1,\ y'(0)=0$ using Laplace transforms where

9. $Y(s) = \frac{1}{(s^2+1)(s^2+4)} - \frac{s}{s^2+4}$

10. $y(t) = \cos 2t - \frac{1}{6}\sin 2t + \frac{1}{3}\sin t$

11. Find the first seven (7) terms for the power series solution to $y'' + y = 0$ where $y(0)=a_o,\ y'(0)=a_1$

$$a_0\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \cdots\right) + a_1\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \cdots\right)$$

MTH 211 Test 3
Spring 2024

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1. $\mathcal{L}\left\{ e^{t}\sin t + 5e^{-3t}\cos 2t \right\} =$ use #19 and #20

2. $\mathcal{L}\left\{ \delta(t-1) + e^{t}u(t-1) + t^{2}e^{-2t} \right\} =$ use #26, #28, and #23

3. $\mathcal{L}^{-1}\left\{ \dfrac{2s-1}{s^{2}+4} \right\} =$ split the fraction and use #8 and #7

4. $\mathcal{L}^{-1}\left\{ \dfrac{e^{-\pi s}}{s} \right\} =$ use #25, $u(t-\pi)$

- In solving the differential equation $y'' - 5y' + 6y = te^{t};\ y(0)=2,\ y'(0)=6$ using Laplace transforms

5. $Y(s) =$

$$s^{2}Y - 5sY + 6Y = \frac{1}{(s-1)^{2}}$$

$$Y = \frac{1}{(s-2)(s-3)(s-1)^{2}}$$

6. $y(t) = \mathcal{L}^{-1}\left\{ \dfrac{\frac{3}{2}}{(s-1)} + \dfrac{1}{(s-1)^{2}} - \dfrac{1}{(s-2)} + \dfrac{\frac{1}{4}}{(s-3)} \right\} = \dfrac{3}{2}e^{t} - e^{-t} + \dfrac{1}{4}e^{3t} + te^{t}$

- In solving the differential equation $y'' + y = \delta(t-\pi);\ y(0)=0,\ y'(0)=0$ using Laplace transforms

7. $Y(s) =$

$$s^{2}Y + Y = e^{-\frac{\pi}{2}s}$$

$$Y = e^{-\frac{\pi}{2}s}\cdot\frac{1}{s^{2}+1}$$

8. $y(t) = u\left(t-\dfrac{\pi}{2}\right)\mathcal{L}^{-1}\left\{\dfrac{1}{s^{2}+1}\right\}\Bigg|_{t \leftarrow t-\frac{\pi}{2}} = -u\left(t-\dfrac{\pi}{2}\right)\cos t$

- In solving the differential equation $y'' + y = f(t)$ with $y(0)=0,\ y'(0)=0$ using Laplace transforms where $f(t)=\begin{cases}\sin t & 0 \le t < \frac{\pi}{2}\\ 0 & t \ge \frac{\pi}{2}\end{cases}$

$$y'' + y = \sin t - u\left(t-\frac{\pi}{2}\right)\sin t$$

$$s^2Y + Y = \frac{1}{s^2+1} - e^{-\frac{\pi}{2}s}\cdot\mathcal{L}\left\{\sin\left(t+\frac{\pi}{2}\right)\right\} = \frac{1}{s^2+1} - e^{-\frac{\pi}{2}s}\cdot\mathcal{L}\{\cos t\}$$

9. $Y(s)$ =

$$Y = \frac{1}{\left(s^2+1^2\right)^2} - e^{-\frac{\pi}{2}s}\cdot\frac{s}{\left(s^2+1^2\right)^2}$$

10. $y(t) = \frac{1}{2}\sin t - \frac{1}{2}t\cos t - u\left(t-\frac{\pi}{2}\right)\mathcal{L}^{-1}\left\{\frac{s}{\left(s^2+1^2\right)^2}\right\}\Bigg|_{t\leftarrow t-\frac{\pi}{2}}$ use #9

$$= \frac{1}{2}\sin t - \frac{1}{2}t\cos t - u\left(t-\frac{\pi}{2}\right)2(t\ \sin t)\Big|_{t\leftarrow t-\frac{\pi}{2}}$$

$$= \frac{1}{2}\sin t - \frac{1}{2}t\cos t - 2u\left(t-\frac{\pi}{2}\right)\left(t-\frac{\pi}{2}\right)\sin\left(t-\frac{\pi}{2}\right)$$

$$= \frac{1}{2}\sin t - \frac{1}{2}t\cos t + 2u\left(t-\frac{\pi}{2}\right)\left(t-\frac{\pi}{2}\right)\cos t$$