$$x'(t) = x + y - 2z$$
$$y'(t) = -x + 2y + z$$
$$z'(t) = y - z$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det \begin{pmatrix} 1-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{pmatrix} = \vec{0}$$

$$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$$
$$(\lambda - 1)(\lambda + 1)(\lambda - 2) = 0$$
$$\lambda_1 = 1, \quad \lambda_2 = -1, \quad \lambda_3 = 2$$

for $\lambda_1 = 1$:

$$\begin{pmatrix} 0 & 1 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = \vec{0} \mapsto \begin{matrix} \eta_2 - 2\eta_3 = 0 \\ -\eta_1 + \eta_2 + \eta_3 = \\ \eta_2 - 2\eta_3 = \end{matrix}$$

$$\vec{\eta}_1 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

for $\lambda_2 = -1$:

$$\begin{pmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = \vec{0} \mapsto \begin{matrix} 2\eta_1 + \eta_2 - 2\eta_3 = \\ -\eta_1 + 3\eta_2 + \eta_3 = \\ \eta_2 = 0 \end{matrix}$$

$$\vec{\eta}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

for $\lambda_3 = 2$

$$\begin{pmatrix} -1 & 1 & -2 \\ -1 & 0 & 1 \\ 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = \vec{0} \mapsto \begin{matrix} -\eta_1 + \eta_2 - 2\eta_3 = \\ -\eta_1 + \eta_3 = 0 \\ \eta_2 - 3\eta_3 = 0 \end{matrix}$$

$$\vec{\eta}_3 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

$$\vec{X} = c_1 e^{\lambda_1 t} \vec{\eta}_1 + c_2 e^{\lambda_2 t} \vec{\eta}_2 + c_3 e^{\lambda_3 t} \eta_3$$

$$x(t) = 3c_1 e^{t} + 2c_2 e^{-t} + c_3 e^{2t}$$
$$y(t) = c_1 e^{t} + c_3 e^{2t}$$
$$z(t) = c_1 e^{t} + 3c_2 e^{-t} + c_3 e^{2t}$$