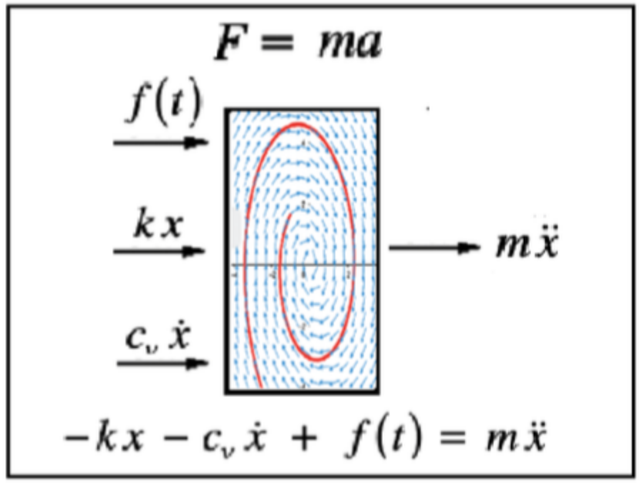


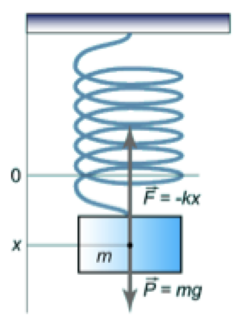
$$A\frac{d^2y}{dt^2} \quad + \quad B\frac{dy}{dt} \quad + \quad Cy = \quad f(t)$$

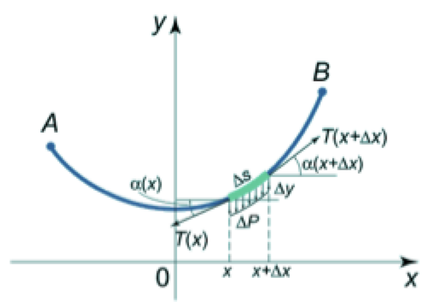
acceleration
curve bending

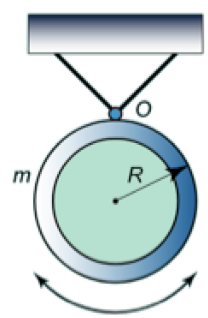
damping
friction
resistance

restoring force

"forcing" function







mth_211

$$Ay'' + By' + Cy = f(t)$$

constant coefficients

"forcing" function

Forced response to the impulse function :

$$y'' + By' + Cy = \delta(t)$$

$$y'' + By' + Cy = 0 \qquad y(0) = 0 \text{ and } y'(0) = 1$$

$$s^2 + Bs + C = 0 \Rightarrow s_1, s_2$$

$$y(t) = e^{s_1 t} - e^{s_2 t} \quad \text{since } y(0) = 0$$

$$y(t) = \frac{e^{s_1 t} - e^{s_2 t}}{s_1 - s_2} \quad \text{since } y'(0) = 1$$

No damping: $y(t) = \frac{1}{\beta}\sin\beta t$

Underdamping: $y(t) = e^{-\frac{1}{2}Bt}\frac{1}{\beta}\sin\beta t$

Critical damping: $y(t) = te^{-\frac{1}{2}Bt}$

$$Ay'' + By' + Cy = f(t)$$

constant coefficients

"forcing" function

Forced response to the step function :

$$y'' + By' + Cy = u(t)$$

$$y(t) = 1 + \frac{e^{s_1 t} - e^{s_2 t}}{s_1 - s_2}$$

$$Ay'' + By' + Cy = f(t)$$

constant coefficients

"forcing" function

Variation of Parameters Revisited :

$$y_p(t) = \int_0^t g(t-T)\,f(T)\,dT \quad \text{where } g(t) \text{ is the impulse response}$$

```
Simplify[DSolveValue[y''[t] + y[t] == Cos[t],
  y[t], t]]
```

$$\frac{1}{2}\left(\text{Cos}[t] + 2\,c_1\,\text{Cos}[t] + t\,\text{Sin}[t] + 2\,c_2\,\text{Sin}[t]\right)$$

$$\int_0^t (\text{Sin}[t-T] * \text{Cos}[T])\,dT$$

$$\frac{1}{2}\,t\,\text{Sin}[t]$$

$$Ay'' + By' + Cy = f(t)$$

constant coefficients (A, B, C)

"forcing" function ($f(t)$)

Variation of Parameters Revisited :

$$y_p(t) = \int_0^t g(t-T)\,f(T)\,dT \quad \text{where } g(t) \text{ is the impulse response}$$

```
DSolveValue[y''[t] - 10*y'[t] + 25*y[t] ==
  6*e^(5*t), y[t], t]
```

$$3\,e^{5t}\,t^2 + e^{5t}\,c_1 + e^{5t}\,t\,c_2$$

```
Expand[
 Simplify[∫_0^t ((t - T)*e^(5*(t-T))*6*e^(5*T)) dT]]
```

$$3\,e^{5t}\,t^2$$

$$Ay'' + By' + Cy = f(t)$$

constant coefficients

"forcing" function

Variation of Parameters Revisited :

$$y_p(t) = \int_0^t g(t-T)f(T)\,dT \quad \text{where } g(t) \text{ is the impulse response}$$

```
Expand[
 Simplify[
  DSolveValue[y''[t] + 5 * y'[t] + 6 * y[t] ==
    Sin[2 * t], y[t], t]]]
```

$$e^{-3t} c_1 + e^{-2t} c_2 - \frac{5}{52} \text{Cos}[2t] + \frac{1}{52} \text{Sin}[2t]$$

$$\text{Expand}\left[\text{Simplify}\left[\int_0^t \left(\frac{e^{-2(t-T)} - e^{-3(t-T)}}{2-3}\right) * \text{Sin}[2*T]\, dT\right]\right]$$

$$\frac{2e^{-3t}}{13} - \frac{e^{-2t}}{4} + \frac{5}{52} \text{Cos}[2t] - \frac{1}{52} \text{Sin}[2t]$$