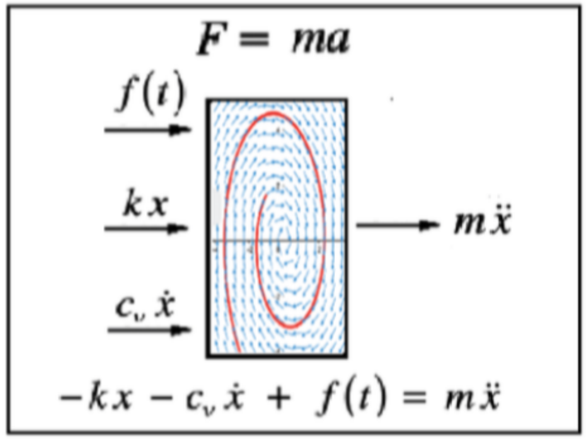


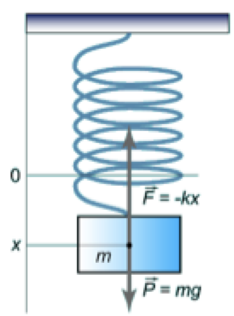
$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = f(t)$$

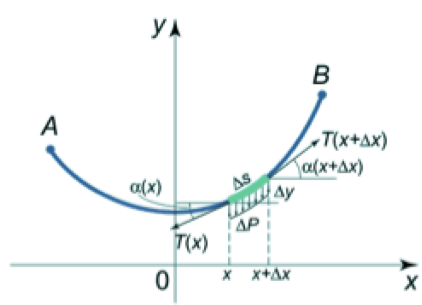
acceleration
curve bending

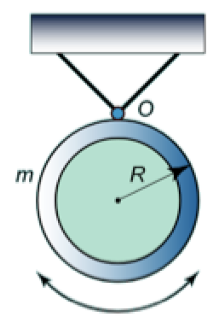
damping
friction
resistance

restoring force

"forcing" function







mth_211

Mechanical and Electrical Models

- *Hooke's Law spring*

$$y'' + \frac{c}{m}y' + \frac{k}{m}y = \frac{1}{m}f(t)$$

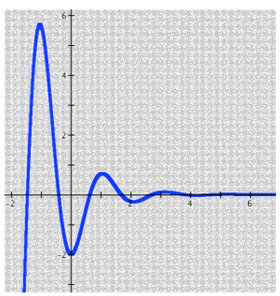
Displacement $y(t)$, mass m, damping constant c, spring constant k, external force $f(t)$.

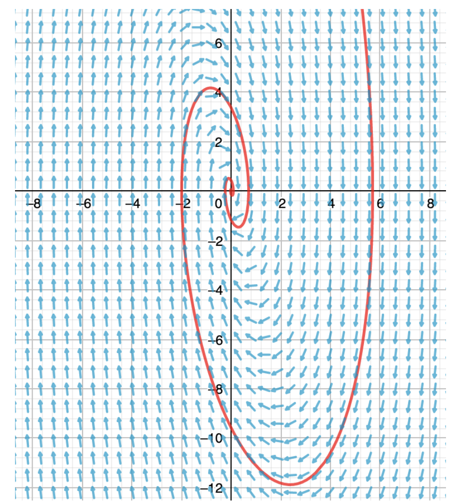
A mass weighing 16 pounds is attached to a 5-foot-long spring. At equilibrium the spring measures 8.2 feet. If the mass is initially released from rest at a point 2 feet above the equilibrium position, find the displacements y(t) if it is further known that the surrounding medium offers a resistance numerically equal to the instantaneous velocity.

$m = 16/32 = 1/2$ $\quad$ $8.2 - 5 = 3.2$ and $16 = k(3.2)$ by Hooke's Law, so $k = 5$.

Thus,

$$y'' + (2)\,y' + 10y = 0\ ; \quad y(0) = -2,\ y'(0) = 0$$





Mechanical and Electrical Models

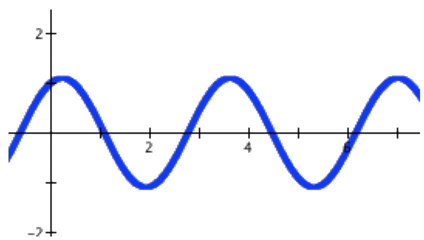
- *Linearized pendulum*

$$\theta'' + \frac{c}{m}\theta' + \frac{g}{L}\theta = \frac{1}{mL}h(t)$$

Angular displacement $\theta(t)$, mass m, damping constant c, length L, gravitational constant g, external tangential force $h(t)$.

Given a linearized and undriven pendulum with a mass of 2 kg, if $c = 0$ and $L = 2$ m, find the angular displacement at time $t = 3$ s. $\theta(0) = 1$ and $\theta'(0) = 1$

$$\theta'' + 4.9\,\theta = 0\,; \quad \theta(0) = 1, \;\; \theta'(0) = 0$$



$$\boldsymbol{\theta}\ (\mathbf{t}) = \cos\left(\frac{7}{10}t\sqrt{7}\right) + \frac{1}{7}\sqrt{10}\cdot\sin\left(\frac{7}{10}t\sqrt{7}\right)$$

Mechanical and Electrical Models

- *Current in RLC circuit*

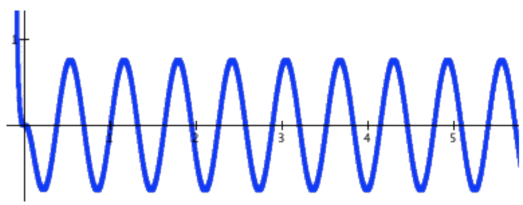
$$I'' + \frac{R}{L}I' + \frac{1}{CL}I = \frac{1}{L}E'(t)$$

Current $I(t)$, inductance L, resistance R, capacitance C, rate of change, $E'(t)$ of the driving voltage $E(t)$

Find the current at time t if $R = 40$ ohms, $L = 1$ henry, $C = 0.0016$ farads and $E = 5\cos 10t$. The initial current is 0.

$$I'' + 40\,I' + 625\,I = -50\sin 10\,t$$

$$I(t) = -\frac{20}{2091}e^{-20t}\left(-48e^{20t}\cos(10t) + 48\cos(15t) + 63e^{20t}\sin(10t) + 22\sin(15t)\right)$$



Mechanical and Electrical Models

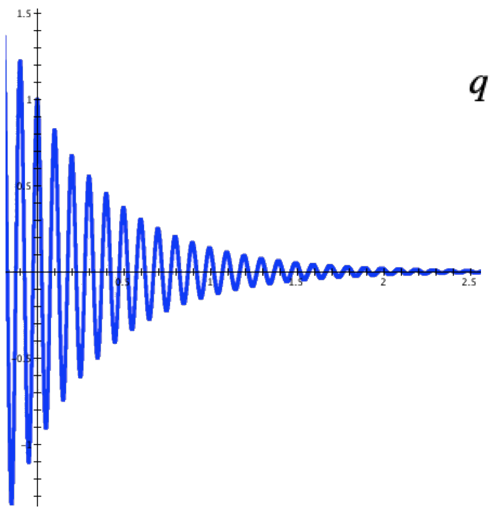
- *Charge on capacitor in RLC circuit*

$$q'' + \frac{R}{L}q' + \frac{1}{CL}q = \frac{1}{L}E(t)$$

Charge $q(t)$, inductance L, resistance R, capacitance C, rate of change, driving voltage $E(t)$

Find the charge $q(t)$ on the capacitor in an RLC-series circuit when $L = 0.25$ henry, $R = 10$ ohms , $C = 0.001$ farad , $E(t) = 0$, $q(0) = q_0$ coulombs, and $i(0) = 0$.

$$q'' + 40q' + 4000q = 0; \quad q(0) = q_0, \quad q'(0) = 0$$



$$q(t) = \frac{1}{60}e^{-20t}\left(60q_0\cos 60t + (1 + 20q_0)\sin 60t\right)$$

Mechanical and Electrical Models

1. An 8-lb weight stretches a spring 6 ft, thereby reaching its equilibrium position. Assuming a damping constant for the system of 4 lb/(ft/sec), the weight is pulled down 3 inches below its equilibrium position and given a downward velocity of 2 ft/sec. When will the mass attain its maximum displacement below equilibrium? What is the maximum displacement? Show the graph of the motion.

2. A capacitor whose capacitance is $\frac{2}{1010}$ farad, an inductor whose coefficient of inductance is $\frac{1}{20}$ henry, and a resistor whose resistance is 1 ohm are connected in series. If at $t = 0$, $i = 0$ and the charge on the capacitor is 1 coulomb, find the charge and the current in the circuit due to the discharge of the capacitor when $t = 0.01$ second.

3. An RLC circuit connected in series has a resistance of 5 ohms, an inductance of 0.05 henry, a capacitor of 0.004 farad, and an applied alternating emf of 200 cos 100t volts. Find an expression for the current.

continued on next slide →

Mechanical and Electrical Models

4. A helical spring is stretched 32 inches by an object weighing 2 pounds, and brought to rest. It is then given an additional pull of 1 ft and released. If the spring is immersed in a medium whose coefficient of resistance is W, find the equation of motion of the object. Assume the resisting force is proportional to the first power of the velocity. Also provide a graph of the motion.

5. A simple linearized unforced pendulum of length 2 ft is released from the position $\theta = \dfrac{1}{12}$ rad at time $t = 0$.

(a) Find the value of θ when $t = \dfrac{\pi}{16}$.

(b) When and with what angular velocity does the pendulum cross the equilibrium position?

6. A particle moves in a straight line in accordance with $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} - 16x = 0$

At $t = 0$, the particle is at $x = 2$ ft and moving to the left with a velocity of 10 ft/sec.

(a) When will the particle change direction and go to the right?

(b) Will it ever change direction again?