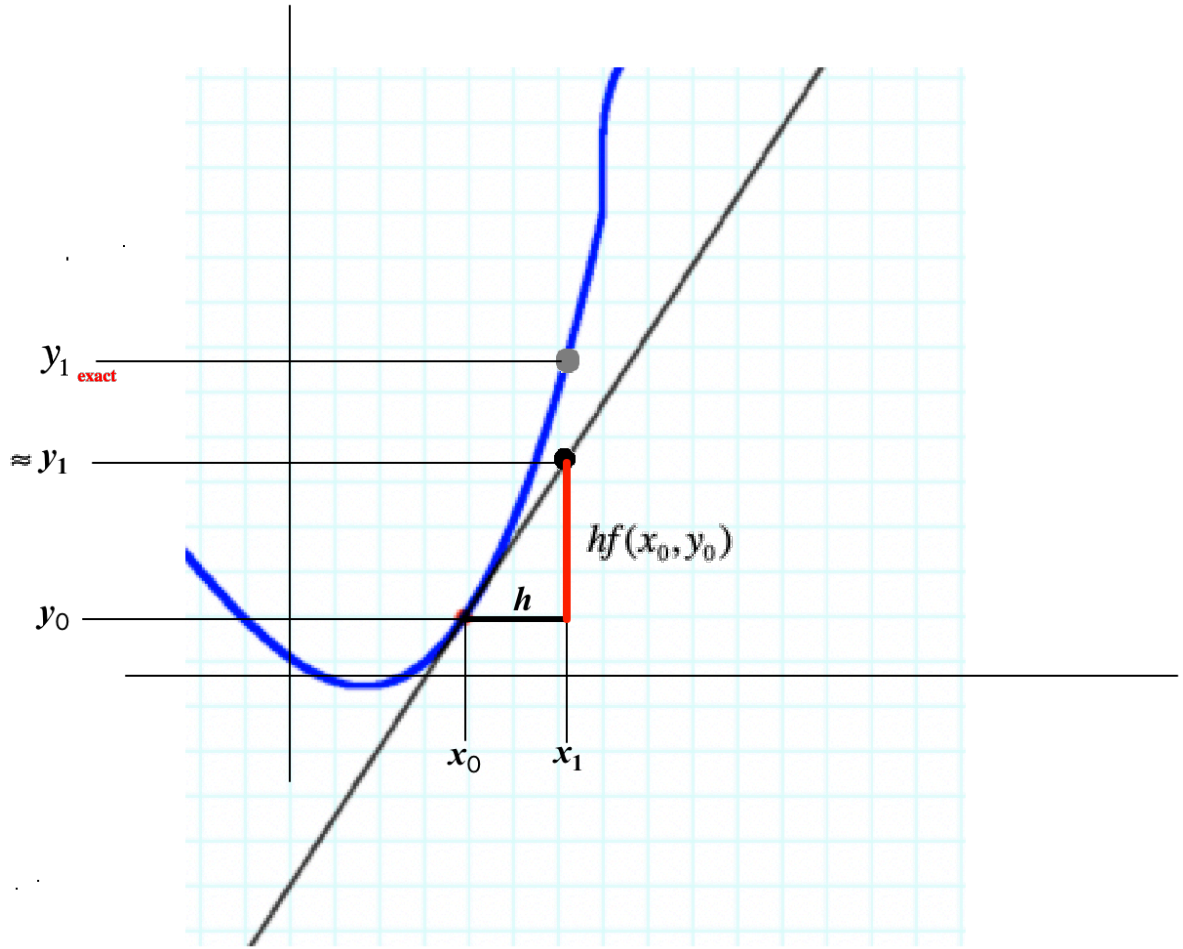
Numerical Methods
(Euler's Method)



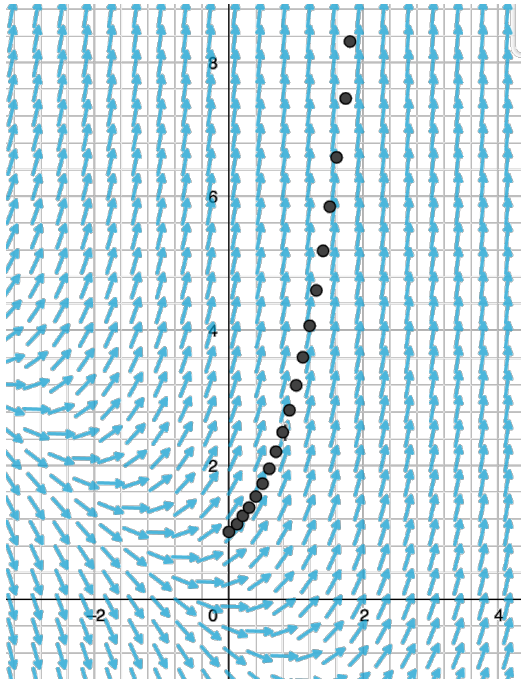
$$\frac{h \cdot f(x,y)}{h}$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Numerical Methods
(Euler's Method)

	y'=y+x	
0.1		
	x	y
	0.0	1.0000000
	0.1	1.1000000
	0.2	1.2200000
	0.3	1.3620000
	0.4	1.5282000
	0.5	1.7210200
	0.6	1.9431220
	0.7	2.1974342
	0.8	2.4871776
	0.9	2.8158954
	1.0	3.1874849
	1.1	3.6062334
	1.2	4.0768568
	1.3	4.6045424
	1.4	5.1949967
	1.5	5.8544963
	1.6	6.5899460
	1.7	7.4089406
	1.8	8.3198346
	1.9	9.3318181
	2.0	10.4549999

		y'=y+x	
h ->	0.1		
		x	y
		0	1
		=C4+$B$2	=D4+$B$2*((C4)+D4)
		=C5+$B$2	=D5+$B$2*((C5)+D5)
		=C6+$B$2	=D6+$B$2*((C6)+D6)
		=C7+$B$2	=D7+$B$2*((C7)+D7)
		=C8+$B$2	=D8+$B$2*((C8)+D8)
		=C9+$B$2	=D9+$B$2*((C9)+D9)
		=C10+$B$2	=D10+$B$2*((C10)+D10)
		=C11+$B$2	=D11+$B$2*((C11)+D11)
		=C12+$B$2	=D12+$B$2*((C12)+D12)
		=C13+$B$2	=D13+$B$2*((C13)+D13)
		=C14+$B$2	=D14+$B$2*((C14)+D14)
		=C15+$B$2	=D15+$B$2*((C15)+D15)
		=C16+$B$2	=D16+$B$2*((C16)+D16)
		=C17+$B$2	=D17+$B$2*((C17)+D17)
		=C18+$B$2	=D18+$B$2*((C18)+D18)
		=C19+$B$2	=D19+$B$2*((C19)+D19)
		=C20+$B$2	=D20+$B$2*((C20)+D20)
		=C21+$B$2	=D21+$B$2*((C21)+D21)
		=C22+$B$2	=D22+$B$2*((C22)+D22)
		=C23+$B$2	=D23+$B$2*((C23)+D23)



Numerical Methods
(Euler's Method)

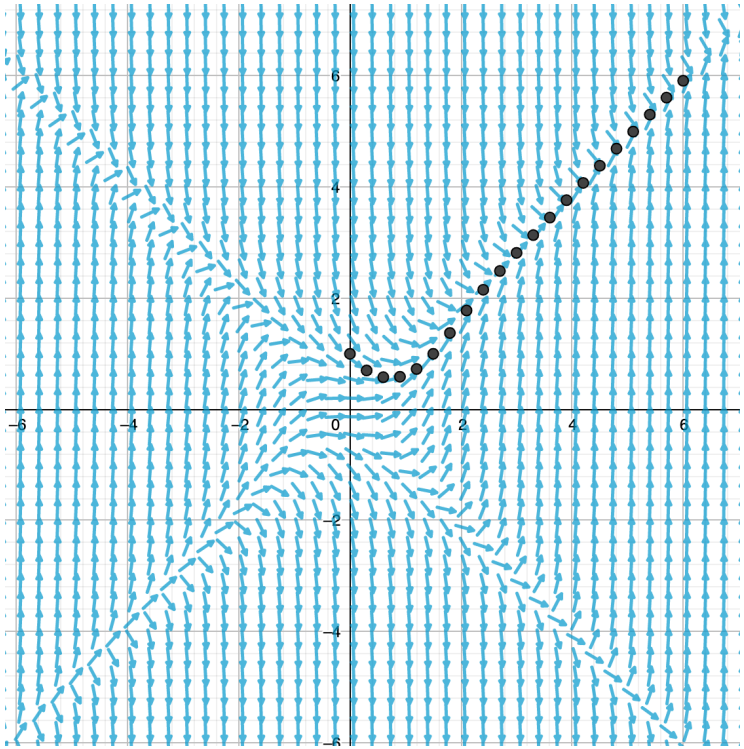
```
% Euler's Method
% Initial conditions and setup
clc
%% h = (enter your step size here);  % step size
h = 0.1;
%% x = (enter the starting value of x here):h:(enter the ending value of x here);  % the range of x
x = 0:h:2.0;
y = zeros(size(x));  % allocate the result y
%% y(1) = (enter the starting value of y here);  % the initial y value
y(1) = 1.0;
n = numel(y);  % the number of y values
% The loop to solve the DE
for i=1:n-1
    %% f = the expression for y' in your DE
    f = y(i)+x(i);
    y(i+1) = y(i) + h * f;
    fprintf(' %6.4f \t %6.4f   \n',x(i),y(i));
end
```

0.0000	1.0000
0.1000	1.1000
0.2000	1.2200
0.3000	1.3620
0.4000	1.5282
0.5000	1.7210
0.6000	1.9431
0.7000	2.1974
0.8000	2.4872
0.9000	2.8159
1.0000	3.1875
1.1000	3.6062
1.2000	4.0769
1.3000	4.6045
1.4000	5.1950
1.5000	5.8545
1.6000	6.5899
1.7000	7.4089
1.8000	8.3198
1.9000	9.3318

Numerical Methods
(Euler's Method)

$$y' = x^2 - y^2 ; \quad y(0) = 1$$

$$y = \left(\frac{\left(\frac{1+i}{2}\right)(1+i)\, J_\alpha\left(-\frac{3}{4}, \frac{ix^2}{2}\right)\Gamma\left(\frac{1}{4}\right) + ix^2\sqrt{2}\, J_\alpha\left(-\frac{5}{4}, \frac{ix^2}{2}\right)\Gamma\left(\frac{3}{4}\right) + \sqrt{2}\, J_\alpha\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\Gamma\left(\frac{3}{4}\right) - ix^2\sqrt{2}\, J_\alpha\left(\frac{3}{4}, \frac{ix^2}{2}\right)\Gamma\left(\frac{3}{4}\right)}{\left(x\left(J_\alpha\left(\frac{1}{4}, \frac{ix^2}{2}\right)\Gamma\left(\frac{1}{4}\right) + (1+i)\sqrt{2}\, J_\alpha\left(-\frac{1}{4}, \frac{ix^2}{2}\right)\Gamma\left(\frac{3}{4}\right)\right)\right)} \right)$$



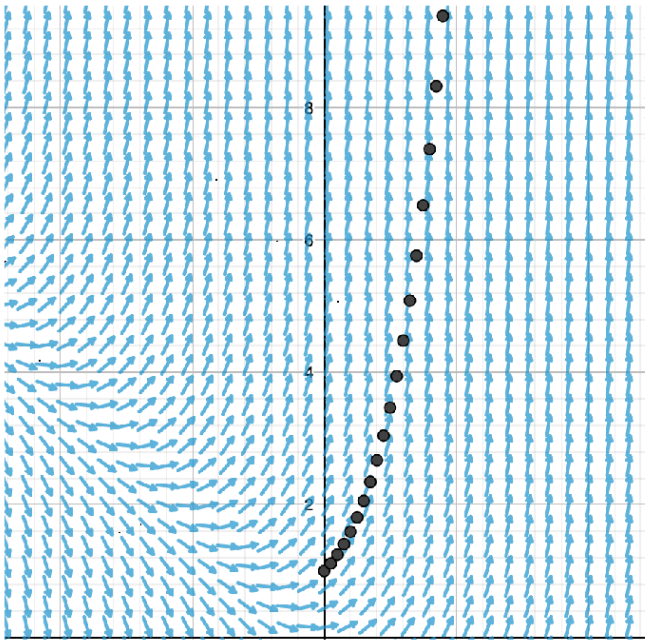
	x^2 - y^2	
0.3		
	x	y
	0	1
	0.3	0.7
	0.6	0.58
	0.9	0.59
	1.2	0.73
	1.5	1
	1.8	1.38
	2.1	1.78
	2.4	2.15
	2.7	2.49
	3	2.82
	3.3	3.14
	3.6	3.45
	3.9	3.76
	4.2	4.08
	4.5	4.38
	4.8	4.69
	5.1	5
	5.4	5.3
	5.7	5.61
	6	5.91

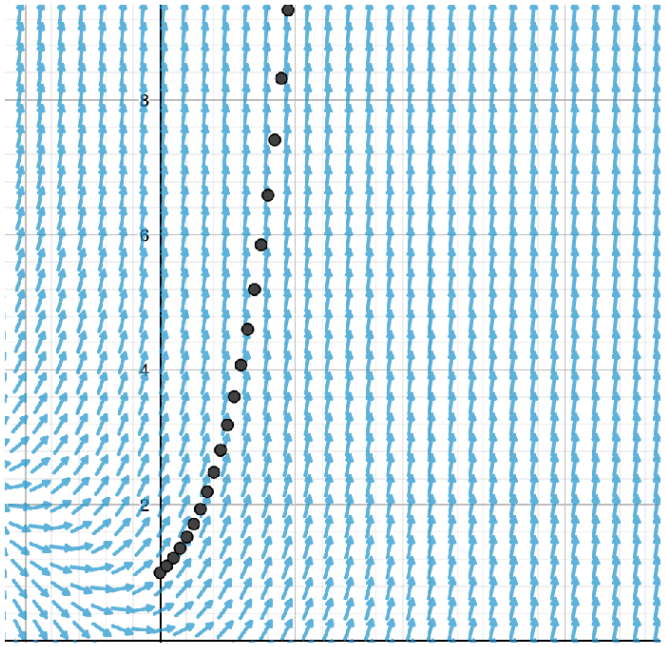
Numerical Methods
(Euler's Method)

```
% Calculates ODE using Runge-Kutta 4th order method


clc;                                % Clears the screen
clear all;

h=0.1;                              % step size
x = 0:h:2;                          % Calculates upto y(2)
y = zeros(1,length(x));
y(1) = 1;                           % initial condition
F_xy = @(t,r)t+r;                   % change the function as you desire

for i=1:(length(x)-1)
    k_1 = F_xy(x(i),y(i));
    k_2 = F_xy(x(i)+0.5*h,y(i)+0.5*h*k_1);
    k_3 = F_xy((x(i)+0.5*h),(y(i)+0.5*h*k_2));
    k_4 = F_xy((x(i)+h),(y(i)+k_3*h));

    y(i+1) = y(i) + (1/6)*(k_1+2*k_2+2*k_3+k_4)*h;        % main R-K equation
    fprintf('%6.4f\t %6.4f \n',x(i),y(i));
end
```

0.0000	1.0000
0.1000	1.1103
0.2000	1.2428
0.3000	1.3997
0.4000	1.5836
0.5000	1.7974
0.6000	2.0442
0.7000	2.3275
0.8000	2.6511
0.9000	3.0192
1.0000	3.4366
1.1000	3.9083
1.2000	4.4402
1.3000	5.0386
1.4000	5.7104
1.5000	6.4634
1.6000	7.3061
1.7000	8.2479
1.8000	9.2993
1.9000	10.4718

Numerical Methods
(Improved Euler Method, Runge-Kutta)





Euler err	Heun err	R-K(3) err	R-K er
0	0	0	
0.01034184	0.00034184	0.00184150	0.001
0.02280552	0.00125552	0.00387576	0.003
0.03771762	0.00280487	0.00612298	0.006
0.05544940	0.00506081	0.00860544	0.008
0.07642254	0.00810215	0.01134776	0.011
0.10111560	0.01201647	0.01437714	0.014
0.13007121	0.01690106	0.01772365	0.017
0.16390424	0.02286405	0.02142046	0.021
0.20331084	0.03002555	0.02550426	0.025
0.24907874	0.03851901	0.03001554	0.030
0.30209864	0.04849271	0.03499906	0.035
0.36337709	0.06011138	0.04050425	0.040
0.43405091	0.07355801	0.04658572	0.046
0.51540326	0.08903590	0.05330378	0.053
0.60888180	0.10677088	0.06072508	0.060
0.71611888	0.12701383	0.06892323	0.069
0.83895421	0.15004341	0.07797954	0.078
0.97946030	0.17616916	0.08798385	0.088
1.13997079	0.20573491	0.09903538	0.099
1.32311230	0.23912255	0.11124375	0.111

- Find the numerical solution to $\frac{dy}{dx} = y^x$ using Euler's method.