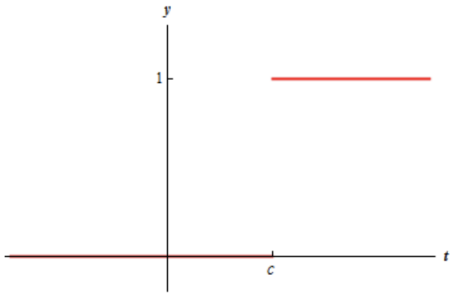
Growth and Decay
(Heaviside Function)

The function is the Heaviside function and is defined as,

$$u_c(t) = \begin{cases} 0 & \text{if } t < c \\ 1 & \text{if } t \geq c \end{cases}$$

Here is a graph of the Heaviside function.



Heaviside functions are often called step functions. Here is some alternate notation for Heaviside functions.

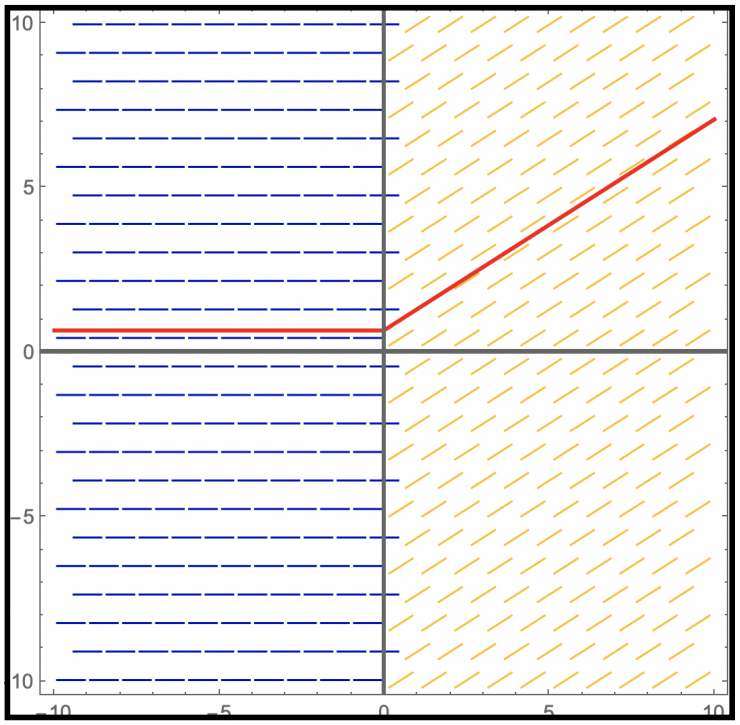
$$u_c(t) = u(t-c) = H(t-c)$$

$$\frac{d}{dt}\big(u(t)\big) = \delta(t)$$

Growth and Decay
(Dirac Delta Function)

$$\frac{dy}{dt} = a\,u(t-c)$$

$$y(t) = C + a(t-c)\,u(t-c)$$



Growth and Decay
(Dirac Delta Function)

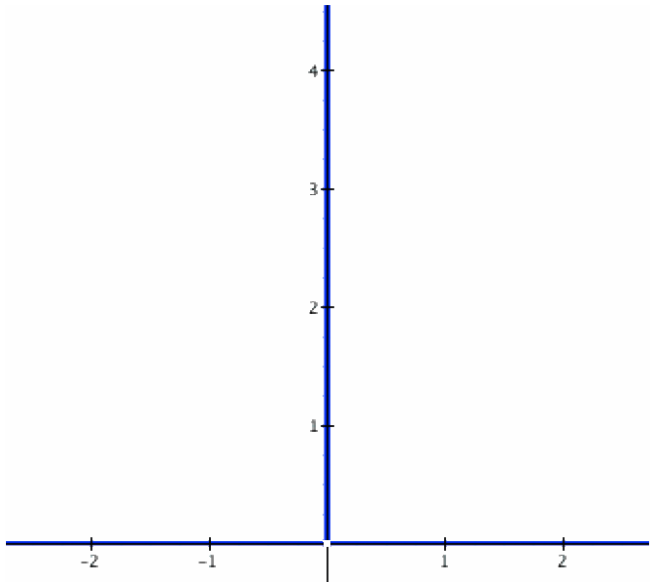
$$\frac{d}{dt}\big(u(t)\big) = \delta(t)$$

Dirac Delta Function

1. $\delta(t-a) = 0, \quad t \neq a$

2. $\displaystyle\int_{a-\varepsilon}^{a+\varepsilon} \delta(t-a)\, dt = 1, \quad \varepsilon > 0$

3. $\displaystyle\int_{a-\varepsilon}^{a+\varepsilon} f(t)\,\delta(t-a)\, dt = f(a), \qquad \varepsilon > 0$

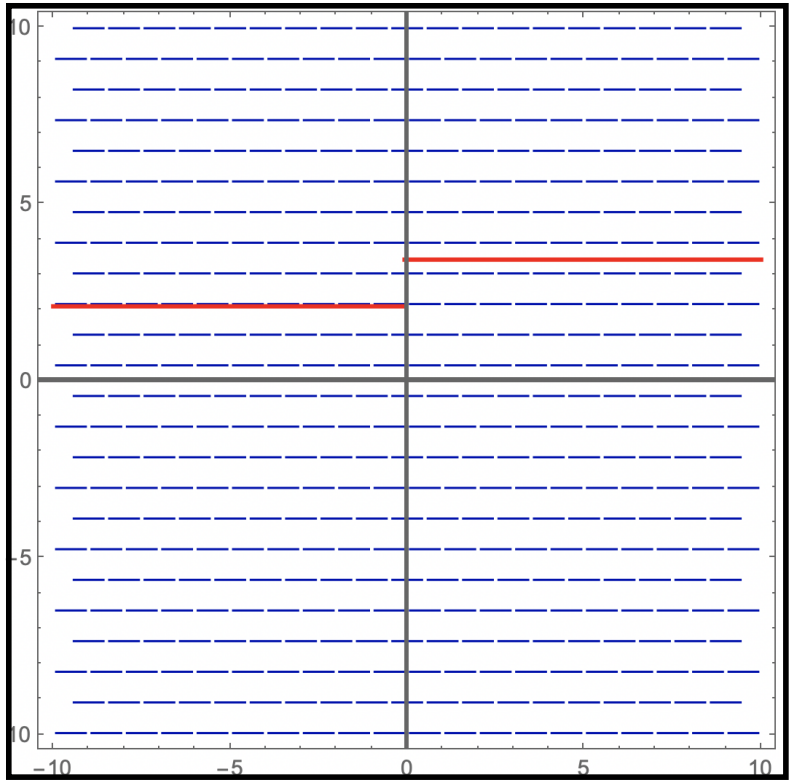


$$\int_{-\infty}^{\infty} \delta(t)\, dt = u(t)\Big|_{-\infty}^{\infty} = 1 - 0 = 1$$

Growth and Decay
(Dirac Delta Function)

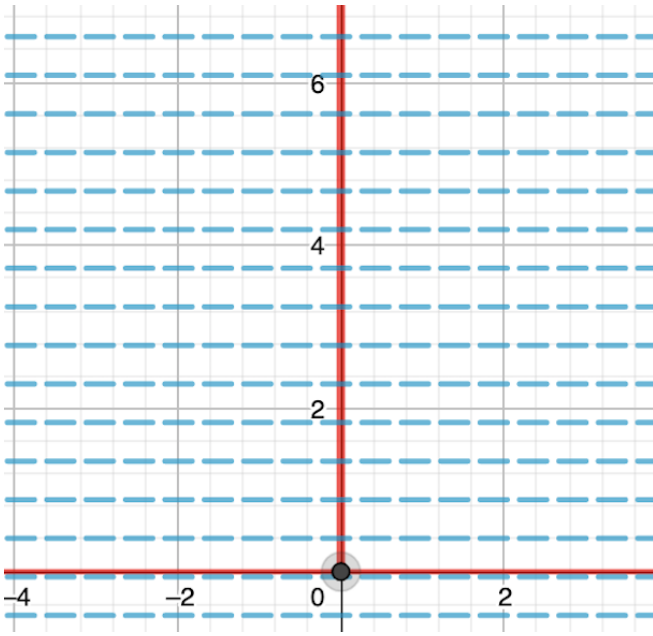
$$\frac{dy}{dt} = a\delta(t-c)$$

$$y(t) = C + au(t-c)$$



Growth and Decay
(Dirac Delta Function)

$$\int_{-\infty}^{\infty} \delta(t)\, dt = u(t)\Big|_{-\infty}^{\infty} = 1 - 0 = 1$$



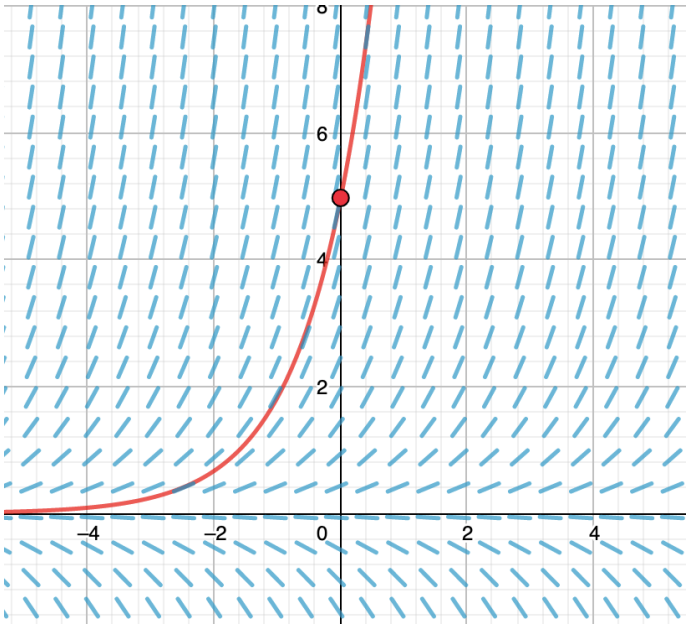
$$\int_{-\infty}^{\infty} \delta(t)\, f(t)\, dt = f(0)$$

$$\int_{-\infty}^{\infty} \delta(t)\, e^{t}\, dt = e^{0} = 1$$

$$\int_{-\infty}^{\infty} \delta(t)\, \sin t\, dt = 0$$

$$\int_{-\infty}^{\infty} \delta(t - T)\, e^{t}\, dt = e^{t-T}$$

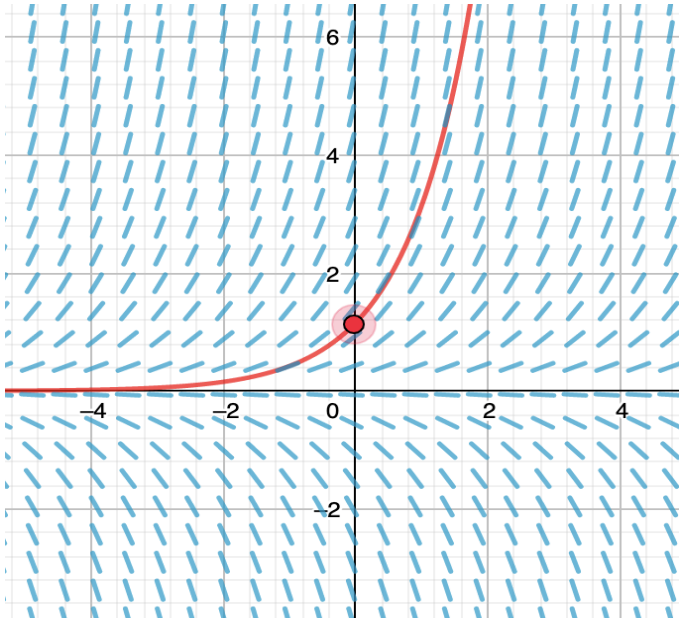
Growth and Decay



$$\frac{dy}{dt} = ay + u(t-c)$$

$$y(t) = Ce^{at} + e^{a(t-c)}u(t-c)$$

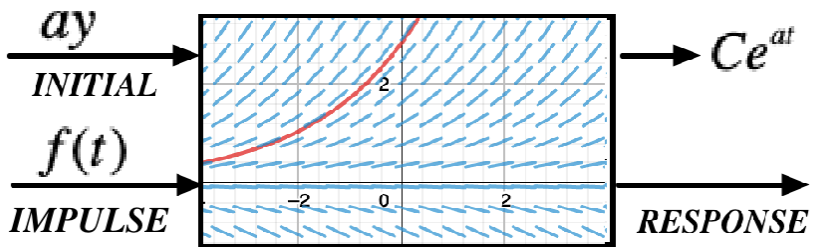
Growth and Decay



$$\frac{dy}{dt} = ay + \delta(t-c)$$

$$y(t) = Ce^{at} + e^{a(t-c)}u(t-c)$$

Growth and Decay
(Systems Model)



- $\dfrac{dy}{dt} = ay + \cos bt \longrightarrow y(t) = Ce^{at} + \dfrac{1}{a^2+b^2}\left(b\sin bt - a\cos bt\right)$
- $\dfrac{dy}{dt} = ay + t^n \longrightarrow y = Ce^{at} + O\left(t^n\right)$
- $\dfrac{dy}{dt} = ay + b\ln t \longrightarrow y(t) = Ce^{at} + \left(\dfrac{b}{a}\mathrm{Ei}(-at) - e^{-at}\ln t\right)$
- $\dfrac{dy}{dt} = ay + b \longrightarrow y(t) = Ce^{at} - \dfrac{b}{a}$
- $\dfrac{dy}{dt} = ay + e^{bt} \longrightarrow y(t) = Ce^{at} + \dfrac{1}{b-a}e^{bt}$
- $\dfrac{dy}{dt} = ay + e^{at} \longrightarrow y(t) = Ce^{at} + te^{at}$
- $\dfrac{dy}{dt} = ay + u(t-T) \longrightarrow y(t) = Ce^{at} + e^{a(t-T)}u(t-T)$
- $\dfrac{dy}{dt} = ay + \delta(t-T) \longrightarrow y(t) = Ce^{at} + e^{a(t-T)}u(t-T)$