Math 112 Test #4 Fall 1996

1. Find the sum of $\sum_{n\geq 2} \frac{2^n}{3^{n+1}}$.

2. For each of the following, tell whether the series converges or diverges. Explain all answers.

a) $\sum_{n\geq 2} \frac{1}{n\sqrt{n}}$

b) $\sum_{n\geq 1} \frac{n}{n^2 + 3n - 2}$

c) $\sum_{n\geq 1} \frac{1}{n^2 + 2}$

d) $\sum_{n\geq 2} \frac{3^n}{n^3}$

e) $\sum_{n\geq 2} \frac{(\ln n)^3}{n}$

f) $\sum_{n\geq 1} (-1)^n \frac{n!}{5^n}$

g) $\sum_{n\geq 1} \frac{(2n)!}{n!\,(2n)^n}$

h) $\sum_{n\geq 1} (-1)^n \sin(\frac{1}{n})$

3. Find the interval of convergence for $\sum_{n\geq 0} \frac{3^n}{5^{2n}} x^{3n}$.

4. We could use L'Hopital's Rule to calculate $\lim_{x\to 0} \frac{\sin x - (x - \frac{x^3}{3!} + \frac{x^5}{5!})}{x^7}$. It is much easier however, if we replace sin x by it's Taylor series. Calculate the limit using the Taylor series substitution.

5. **a)** Find the Taylor Series associated with $f(x) = e^x$ about $x_0=1$.
b) For what values of x does the series in **a)** converge (Explain your answer)?

- *At the end of the test, fold this test paper and insert it at the* ***end*** *of your test booklet* -.