

Binomial Series Homework

1. Compute each of the following binomial coefficients

a) $\binom{3}{2}$

e) $\binom{\frac{1}{2}}{n}$

b) $\binom{5}{3}$

f) $\binom{-\frac{1}{4}}{n}$

c) $\binom{-1}{2}$

d) $\binom{\frac{2}{3}}{6}$

2. Compute (and simplify) each of the following binomial series and give the radius of convergence.

a) $\sqrt{1+x}$ Ans: $1 + \frac{x}{2} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-3)}{2^n n!} x^n, R=1$

b) $\frac{1}{(1+x)^4}$ Ans: $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)(n+3)}{6} x^n, R=1$

c) $\frac{1}{(2+x)^3}$ Ans: $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n, R=2$

d) $\sqrt[3]{1+x^2}$ Ans: $1 + \frac{x^2}{3} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n-4)}{3^n n!} x^{2n}, R=1$

e) $\frac{x}{\sqrt{4+x^2}}$ Ans: $\frac{x}{2} + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^{3n+1} n!} x^{2n+1}, R=2$

f) $\frac{x^2}{\sqrt{2+x}}$ Ans: $\frac{x^2}{\sqrt{2}} + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^{2n+1/2} n!} x^{n+2}, R=2$

g) $\frac{1}{\sqrt[3]{8+x}}$ Ans: $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 4 \cdot 7 \cdot \dots \cdot (3n-2)}{24^n n!} x^n, R=8$

3. Find the power series for $\sin^{-1} x$ using the following steps:

a) Find the binomial series for $\frac{1}{\sqrt{1-x^2}}$ Ans: $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^n n!} x^{2n}$

b) Integrate it to find the power series for $\sin^{-1} x$

Ans: $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{(2n+1) 2^n n!} x^{2n+1}$