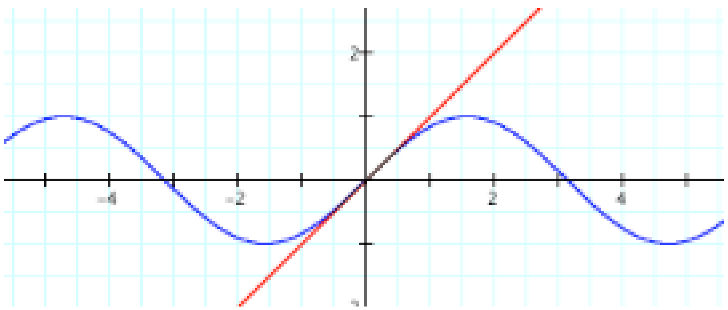
Linear approximation by differentials :

$$\frac{dy}{dx} = f'(x) \quad \Rightarrow \; dy \approx f'(x)\; dx \text{ , so } f(x) \approx f(x) + f'(x)\; dx$$

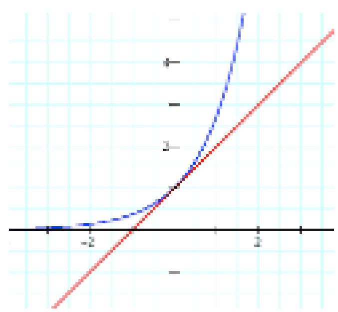
Let $f(x) = \sin x$ so $\sin x \approx \sin x + \cos x \;\; (x - x_0)$. Checking “around 0” we have

$$\begin{aligned} \sin 0 &\approx \sin 0 + \cos 0 \;\; (x - 0) \\ &\approx \;\; 0 \;\; + \;\; 1\,(x) \;\; \approx \;\; x \end{aligned}$$



Similarly, $\cos 0 \approx \cos 0 - \sin 0 \;\; (x - 0)$ and $e^0 \approx e^0 + e^0 \;\; (x - 0)$

$\approx \;\; 1 \;\; - \;\; 0\,(x) \;\; \approx \;\; 1$ $\qquad \approx \;\; 1 \;\; + \;\; 1\,(x) \;\; \approx \;\; 1 + x$



Taylor's Formula and Maclaurin's Series:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + \cdots$$

$$f'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 + \cdots$$

$$f''(x) = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + 30a_6x^4 + \cdots$$

$$f'''(x) = 6a_3 + 24a_4x + 60a_5x^2 + 120a_6x^3 + \cdots$$

$$f^{iv}(x) = 24a_4 + 120a_5x + 360a_6x^2 + \cdots$$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}(x)^k$$

$f(0) = a_0$

$$\Rightarrow a_0 = \frac{f(0)}{0!}$$

$f'(0) = a_1$

$$\Rightarrow a_1 = \frac{f'(0)}{1!}$$

$f''(0) = 2a_2$

$$\Rightarrow a_2 = \frac{f''(0)}{2!}$$

$f'''(0) = 6a_3$

$$\Rightarrow a_3 = \frac{f'''(0)}{3!}$$

$f^{iv}(0) = 24a_4$

$$\Rightarrow a_4 = \frac{f^{iv}(0)}{4!}$$

$\vdots$

$f^{(n)}(0) = n!a_n$

$$\Rightarrow a_n = \frac{f^{(n)}(0)}{n!}$$