MTH 112 Test 1
Fall 2020

Name ____________________ **Calculus II**

When finished submit your answers at **https://pryor.mathcs.wilkes.edu/submissions**.
If you feel the answer is none of the choices given, submit no answer to the question.

1. Given $f(x) = \int \frac{e}{x^2}\,dx$ where $f(e) = \pi$, find C.

$\int \frac{e}{x^2}\,dx = \int ex^{-2}\,dx = -ex^{-1} + C = -\frac{e}{x} + C.$ Since $f(e) = \pi = -\frac{e}{x} + C \Rightarrow \quad C = \pi + 1$

2. What would be the first step to find $f(x) = \int \frac{2x+5}{x^2+9}\,dx$? Decompose into two fractions.

$f(x) = \int \frac{2x+5}{x^2+9}\,dx = f(x) = \int \frac{2x}{x^2+9} + \frac{5}{x^2+9}\,dx$

3. Which function below is equivalent to $f(x) = \int_{x=0}^{x=4} \frac{dx}{1+\sqrt{x}}$ using an appropriate substitution of u for x? Let $u^2 = x$, $2udu = dx$, $u = \sqrt{x}$. $\Rightarrow$ $f(u) = \int_{u=\sqrt{x}=\sqrt{0}=0}^{u=\sqrt{x}=\sqrt{4}=2} \frac{2udu}{1+u}$

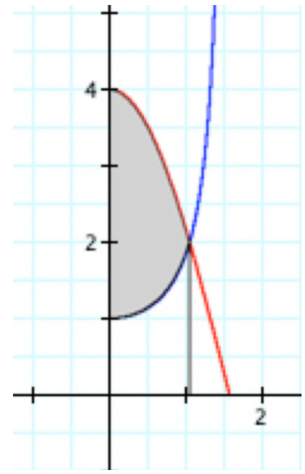
a. $f(u) = \int_{u=0}^{u=2} u^2 + 1\,du$

b. $f(u) = \frac{1}{2}\int_{u=0}^{u=4} 1 + \frac{1}{1+u}\,du$

c. $f(u) = 2\int_{u=0}^{u=2} \frac{u}{u+1}\,du$

d. $f(u) = \int_{u=0}^{u=2} \frac{u+1}{u}\,du$

4. What is the area between $f(x)=4\cos x$ and $g(x)=\sec x$?



a. $\frac{\sqrt{3}}{2} - \ln\left|\frac{\sqrt{3}}{2} - \sqrt{3}\right|$

b. $\ln\left|\frac{\sqrt{3}}{2}\right| - \frac{\sqrt{3}}{2}$

c. $\frac{\sqrt{3}}{2}$

d. 1

$$\int_{x=0}^{x=\frac{\pi}{3}} 4\cos x - \sec x \, dx = -4\sin x - \ln|\sec x + \tan x| \Big|_0^{\frac{\pi}{3}} = -2\sqrt{3} - \ln\left|2 + \sqrt{3}\right|$$

5. Given $f(x)=\int \sin^2 6x \cos 6x \, dx$, what is $f\left(\frac{\pi}{3}\right)$ if $f(0)=\pi$?

$f(x)=\int \sin^2 6x \cos 6x \, dx = \frac{1}{18}\sin^3 6x + C$. Since $f(0)=\pi$, we have $C=\pi$.

So then $\frac{1}{18}\sin^3 6\left(\frac{\pi}{3}\right) + \pi = \pi$

a. π **b.** 0 **c.** 2π **d.** $-\pi$

6. How would you rewrite and simplify $\sqrt{e^{x+3}}$ in order to evaluate $f(x)=\int \sqrt{e^{x+3}} \, dx$?

$$\sqrt{e^{x+3}} = \left(e^{x+3}\right)^{\frac{1}{2}} = e^{\frac{1}{2}x + \frac{3}{2}}$$

7. The electric current in a certain inductor is given by $i(t) = 8\int \frac{dt}{100+t^2}$, what would be the current at $t = 10$ if $i(0) = 10$?

$i(t) = 8\int \frac{dt}{100+t^2} = \frac{8}{100}\int \frac{dt}{1+\left(\frac{t}{10}\right)^2} = \frac{4}{5}\arctan\frac{t}{10} + C$. Since $i(0)$=10 we have

$i(t) = \frac{4}{5}\arctan\frac{t}{10} + 10$ and $\therefore$ $i(10) = \frac{\pi}{5} + 10$

a. $\frac{\pi}{40}$ **b.** $\frac{\pi}{4}$ **c.** $\frac{\pi}{400}$ **d.** $\frac{2\pi}{5}$

8. Given the differential equation $\frac{dy}{dx} = \frac{x}{1+x^2}$, what is $y(1)$ if $y(0) = \ln 2$?

$\int \frac{dy}{dx} = \int \frac{x}{1+x^2} = \frac{1}{2}\ln\left|1+x^2\right| + C$. Since $y(0)$=$\ln 2$ we have

$y(x) = \frac{1}{2}\ln\left|1+x^2\right| + \ln 2$ and $\therefore$ $y(1) = \frac{3}{2}\ln 2$

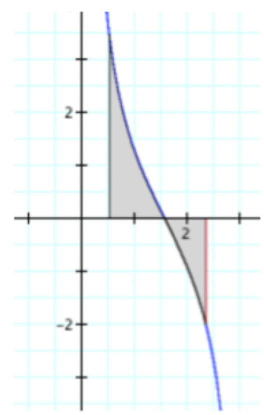
a. $\frac{3}{2}\ln 2$ **b.** $\ln 8$ **c.** $\frac{1}{2}\ln 2$ **d.** $\frac{3}{2}$

9. Find $\displaystyle\int_{x=\frac{\pi}{6}}^{x=\frac{3\pi}{4}} \frac{\sin 2x}{1-\cos^2 x}dx$ (Note: $\sin 2x = 2\sin x\cos x$)

$$\int_{x=\frac{\pi}{6}}^{x=\frac{3\pi}{4}} \frac{\sin 2x}{1-\cos^2 x}dx = \int_{x=\frac{\pi}{6}}^{x=\frac{3\pi}{4}} \frac{2\sin x\cos x}{1-\cos^2 x}dx = \ln\left|1-\cos^2 x\right| \Bigg|_{\frac{\pi}{6}}^{\frac{3\pi}{4}} = \ln 2$$

a. $\ln 2$ **b.** $2\sqrt{2}$ **c.** -1 **d.** $\dfrac{\sqrt{3}}{4}$

10. What is the area under $f(x) = \dfrac{\sin 2x}{1-\cos^2 x}$ from $x = \dfrac{\pi}{6}$ to $x = \dfrac{3\pi}{4}$?



a. $\ln 2$ **c.** $3\ln 2$

b. $\dfrac{\sqrt{3}}{2}$ **d.** 2

$$AREA = \int_{x=\frac{\pi}{6}}^{x=\frac{\pi}{2}} \frac{\sin 2x}{1-\cos^2 x}dx + \left|\int_{x=\frac{\pi}{2}}^{x=\frac{3\pi}{4}} \frac{\sin 2x}{1-\cos^2 x}dx\right| = 2\ln 2 + \ln 2 = 3\ln 2$$