Determine the number of terms of the series $\sum_{k=1}^{\infty} \frac{2(-1)^k}{k}$ that are needed to be computed in order for the sum to have an error less than 0.01.

$$|E_n| = S - S_n = a_{n+1} - (a_{n+2} - a_{n+3}) - (a_{n+4} - a_{n+5}) - \cdots < a_{n+1}$$

$\frac{1}{100} < \frac{2}{n+1} \Rightarrow n+1 < 200 \Rightarrow n = 199$ So for the error to be less than 0.01 we must have n be 199.

A series is *conditionally convergent* if the series itself, $\sum_{k=1}^{\infty} a_k$, converges but the series $\sum_{k=1}^{\infty} |a_k|$ diverges. A series *converges absolutely* of $\sum_{k=1}^{\infty} |a_k|$ converges. Determine whether each series converges absolutely, conditionally, or diverges.

a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

b) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+2}$

c) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k+4}$

d) $\sum_{k=1}^{\infty} \frac{(-1)^k k^4 3^k}{k!}$

e) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{3k+1}$

f) $\sum_{k=5}^{\infty} \frac{(-1)^{k+1} 2^k}{k^2}$

Some of the known sums and approximations are

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \qquad \sum_{n=1}^{\infty} \frac{1}{n^3} \approx 1.2020569$$

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} \qquad \sum_{n=1}^{\infty} \frac{1}{n^5} \approx 1.0369278$$

$$\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945} \qquad \sum_{n=1}^{\infty} \frac{1}{n^7} \approx 1.0083493$$

(**Telescoping Series**) Find the sum:

$$\sum_{k=0}^{\infty} \frac{1}{k^2+3k+2} = \sum_{k=0}^{\infty} \frac{1}{(k+2)(k+1)} = \sum_{k=0}^{\infty} \frac{1}{(k+1)} - \frac{1}{(k+2)}$$

$$= \left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{k}-\frac{1}{k+1}\right)+\left(\frac{1}{k+1}-\frac{1}{k+2}\right)$$

$$= \left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\cdots+\left(\cancel{\frac{1}{k}}-\cancel{\frac{1}{k+1}}\right)+\left(\cancel{\frac{1}{k+1}}-\frac{1}{k+2}\right)$$

$$= 1 - \frac{1}{k+2} \quad = \ 1$$

Find the sum -

a) $\displaystyle\sum_{k=\mathbf{1}}^{\infty} \frac{1}{k^2+3k+2}$

b) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k^2+4k+3}$

c) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k^2+k}$

d) $\displaystyle\sum_{k=0}^{\infty} \frac{3}{k^2+7k+12}$

(**Special Series**) Find the sum:

$$\sum_{k=0}^{\infty} \frac{1}{k!} = \boldsymbol{e}$$

$$\sum_{k=0}^{\infty} \frac{4(-1)^k}{2k+1} = \boldsymbol{\pi}$$

Taylor's (and Maclaurin's) Formula

$$f(x)=\sum_{k=0}^{\infty}\frac{f^{(k)}(a)}{k!}(x-a)^{k}$$

$$f(x)=\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}(x)^{k}$$