

Find the Maclaurin series for $\cos x$ and test for convergence.

$$f(x) = \sum_{k=0}^{\infty} \left(\frac{f^{(k)}(0)}{k!} \right) x^k$$

$$k=0: \rightarrow \cos x \rightarrow \cos 0 \rightarrow 1 \rightarrow \frac{1}{0!} \rightarrow 1 \cdot x^0 \rightarrow 1$$

$$k=1: \rightarrow -\sin x \rightarrow -\sin 0 \rightarrow 0 \rightarrow \frac{0}{1!} \rightarrow 0 \cdot x^1 \rightarrow 0$$

$$k=2: \rightarrow -\cos x \rightarrow -\cos 0 \rightarrow -1 \rightarrow \frac{-1}{2!} \rightarrow -\frac{1}{2!} \cdot x^2$$

$$k=3: \rightarrow \sin x \rightarrow \sin 0 \rightarrow 0 \rightarrow \frac{0}{3!} \rightarrow 0 \cdot x^3 \rightarrow 0$$

$$k=4: \rightarrow \cos x \rightarrow \cos 0 \rightarrow 1 \rightarrow \frac{1}{4!} \rightarrow \frac{1}{4!} \cdot x^4$$

$$k=5: \rightarrow -\sin x \rightarrow -\sin 0 \rightarrow 0 \rightarrow \frac{0}{5!} \rightarrow 0 \cdot x^5 \rightarrow 0$$

$$k=6: \rightarrow -\cos x \rightarrow -\cos 0 \rightarrow -1 \rightarrow \frac{-1}{6!} \rightarrow -\frac{1}{6!} \cdot x^6$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 \dots$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

Now check for convergence.

$$\lim_{k \rightarrow \infty} \left| \frac{\frac{x^{2k+2}}{(2k+2)!}}{\frac{x^{2k}}{(2k)!}} \right| = \lim_{k \rightarrow \infty} \left| \frac{x^{2k+2}}{(2k+2)!} \cdot \frac{(2k)!}{x^{2k}} \right| = \lim_{k \rightarrow \infty} \left| \frac{x^2}{(2k+1)(2k+2)} \right| = 0$$