Numerical series problems from the web ssite of Marta Hidegkuti.

(http://www.teaching.martahidegkuti.com/shared/lnotes/calculus.html)

Part I.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

2. $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

3. $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$

4. $\sum_{n=0}^{\infty} \frac{5^n+3^n}{4^n}$

5. $\sum_{n=0}^{\infty} \sqrt{n}$

6. $\sum_{n=1}^{\infty} \sqrt[n]{n}$

7. $\sum_{n=1}^{\infty} \frac{8}{n^2}$

8. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

9. $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

10. $\sum_{n=1}^{\infty} \frac{2}{e^n}$

11. $\sum_{n=1}^{\infty} \operatorname{sech} n$

12. $\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$

13. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

14. $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

Part
II.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

2. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

3. $\sum_{n=0}^{\infty} \frac{2^n+1}{3^n}$

4. $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$

5. $\sum_{n=3}^{\infty} \frac{1}{n^2-2n}$

6. $\sum_{n=0}^{\infty} \frac{5^n}{4^n+1}$

7. $\sum_{n=1}^{\infty} n^{-5/4}$

8. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$

9. $\sum_{n=1}^{\infty} \frac{8}{n^2+1}$

10. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

11. $\sum_{n=1}^{\infty} \frac{8\tan^{-1} n}{1+n^2}$

12. $\sum_{n=1}^{\infty} \frac{2}{e^n}$

13. $\sum_{n=1}^{\infty} \operatorname{sech} n$

14. $\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$

15. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

16. $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+2)!}$

17. $\sum_{n=1}^{\infty} \frac{n}{\left(1+\frac{1}{n}\right)^n}$

Part III.

1. In case of the following series, use the ratio test to determine convergence or divergence of the series.

a) $\sum_{n=0}^{\infty} \frac{n!}{3^n}$ c) $\sum_{n=0}^{\infty} \frac{2^{3n-1}+1}{3^n}$ e) $\sum_{n=0}^{\infty} n!\, 2^{1-n}$ g) $\sum_{n=0}^{\infty} \frac{(3n)!}{(n!)^3}$

b) $\sum_{n=0}^{\infty} \frac{5^n}{(2n)!}$ d) $\sum_{n=0}^{\infty} \frac{3^n-1}{5^n}$ f) $\sum_{n=0}^{\infty} \frac{3^{n-1}}{(3n+4)\,5^n}$ h) $\sum_{n=1}^{\infty} n^{100} 2^{-2n}$

2. In case of the following series, use the root test to determine convergence or divergence of the series.

a) $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n}\right)^n$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$ c) $\sum_{n=1}^{\infty} 5^n 2^{-2n}$ d) $\sum_{n=1}^{\infty} n^{100} 2^{-2n}$

3. Use any method we know so far (that is: direct limit of s_n, comparison test, integral test, ratio test, root test, nth term test, or grouping of the terms) to determine whether the following series converge or diverge.

a) $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ d) $\sum_{n=1}^{\infty} n^2 e^{-n}$ g) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$ j) $\sum_{n=1}^{\infty} \frac{n\, 2^n\, (n+1)!}{3^n n!}$

b) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ e) $\sum_{n=1}^{\infty} \frac{(-2)^n}{7^n}$ h) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$

c) $\sum_{n=1}^{\infty} \left(\frac{n+1}{3n+1}\right)^{2n+1}$ f) $\sum_{n=1}^{\infty} \frac{1}{n^{n+1}}$ i) $\sum_{n=1}^{\infty} \frac{n^n}{(2^n)^2}$

Part IV.

1. Determine convergence or divergence of the alternative series.

a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ d) $\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!}$ g) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+5}{n^2+4}$ e) $\sum_{n=1}^{\infty} (-1)^n \ln\left(1+\frac{1}{n}\right)$ h) $\sum_{n=1}^{\infty} (-2)^{-n}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2-1}$ f) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\tan^{-1} n}{n^2+1}$

2. Determine whether the following series converge absolutely, converge conditionally, or diverge.

a) $\sum_{n=1}^{\infty} \frac{(-100)^n}{n!}$ d) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$ f) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$

b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (n!)^2}{(2n)!}$ g) $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$

c) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$ f) $\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2\, 3^n}{(2n+1)!}$