

Euler's proof that God exists: $e^{\pi i} + 1 = 0$

i

$$i^2 = -1$$

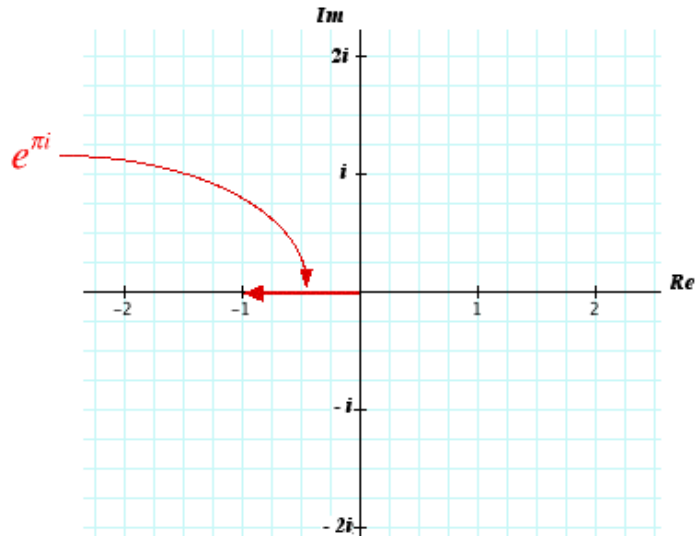
$$i^3 = -i$$

$$i^4 = +1$$

$$i^5 = i$$

$$i^6 = -1$$

$$i^7 = -i$$



$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \dots$$

So

$$\begin{aligned} e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \dots \\ &= 1 + ix - \frac{(x)^2}{2!} - \frac{i(x)^3}{3!} + \frac{(x)^4}{4!} + \frac{i(x)^5}{5!} - \frac{(x)^6}{6!} - \frac{i(x)^7}{7!} + \dots \\ &= \left(1 - \frac{(x)^2}{2!} + \frac{(x)^4}{4!} - \frac{(x)^6}{6!} + \dots \right) + i \left(x - \frac{(x)^3}{3!} + \frac{(x)^5}{5!} - \frac{(x)^7}{7!} + \dots \right) \\ &= \cos x + i \sin x \end{aligned}$$