

# Fundamental Theorem of Calculus

IF  $f$  IS A CONTINUOUS  
FUNCTION ON THE INTERVAL  $[a, b]$ , AND  $F$  IS ANY ANTIDERIVATIVE OF  $f$  ON  $[a, b]$ , THEN

$$\int_a^b f(x) dx = F(b) - F(a)$$

IF  $f$  IS CONTINUOUS,  $a$  IS IN ITS DOMAIN, AND  $F$  IS DEFINED BY

$$F(x) = \int_a^x f(t) dt$$

THEN  $F$  IS DIFFERENTIABLE, AND  $F'(x) = f(x)$ .

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$$\frac{d}{dx} \left[ \int_0^x \sqrt{t^2 + 4} \, dt \right] \text{ here } f(t) = \sqrt{t^2 + 4}$$

$$\text{so } \frac{d}{dx} \left[ \int_0^x f(t) \, dt \right] = \frac{d}{dx} \left[ F(t) \Big|_0^x \right] =$$

$$\frac{d}{dx} [ F(x) - F(0) ] = f(x) \frac{d}{dx}(x) - f(0) \frac{d}{dx}(0) = \sqrt{x^2 + 4} - 0 = \sqrt{x^2 + 4}$$

# Fundamental Theorem of Calculus

$$\frac{d}{dx} \left[ \int_x^5 \sqrt{t^3 + 5} dt \right] \text{ here } f(t) = \sqrt{t^3 + 5}$$

$$\text{so } \frac{d}{dx} \left[ \int_x^5 f(t) dt \right] = \frac{d}{dx} \left[ F(t) \Big|_x^5 \right] =$$

$$\frac{d}{dx} [ F(5) - F(x) ] = f(5) \frac{d}{dx}(5) - f(x) \frac{d}{dx}(x) = 0 - \sqrt{x^3 + 5} = -\sqrt{x^3 + 5}$$

# Fundamental Theorem of Calculus

$$\frac{d}{dx} \left[ \int_5^{x^2} \sqrt{t^3 - 4} dt \right] \text{ here } f(t) = \sqrt{t^3 - 4}$$

$$\text{so } \frac{d}{dx} \left[ \int_5^{x^2} f(t) dt \right] = \frac{d}{dx} \left[ F(t) \Big|_5^{x^2} \right] =$$

$$\frac{d}{dx} [ F(x^2) - F(5) ] = f(x^2) \frac{d}{dx}(x^2) - f(5) \frac{d}{dx}(5) = \sqrt{x^6 - 4} \cdot 2x - 0 = 2x \sqrt{x^6 - 4}$$