

Miscellaneous Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin \frac{1}{2}x = \pm \sqrt{\frac{1 - \cos x}{2}} \Rightarrow 1 - \cos x = 2 \sin^2 \frac{1}{2}x$$

$$\cos \frac{1}{2}x = \pm \sqrt{\frac{1 + \cos x}{2}} \Rightarrow 1 + \cos x = 2 \cos^2 \frac{1}{2}x$$

$$\tan \frac{1}{2}x = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$$

$$\sin(x + y) - \sin(x - y) = 2 \cos x \sin y$$

$$\cos(x + y) + \cos(x - y) = 2 \cos x \cos y$$

$$\cos(x + y) - \cos(x - y) = -2 \sin x \sin y$$

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$

$$\begin{aligned}\sin x + \sin y &= 2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y) \\ \sin x - \sin y &= 2 \cos \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y) \\ \cos x + \cos y &= 2 \cos \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y) \\ \cos x - \cos y &= -2 \sin \frac{1}{2}(x+y) \sin \frac{1}{2}(x-y)\end{aligned}$$

$$\arcsin x + \arccos x = \frac{1}{2}\pi$$

$$\sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x} \quad \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\operatorname{ctnh} x = \frac{1}{\tanh x} \quad \operatorname{sech} x = \frac{1}{\cosh x} \quad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\operatorname{ctnh}^2 x - 1 = \operatorname{sech}^2 x$$

$$\sinh(x+y) = \frac{1}{2}e^{x+y} - \frac{1}{2}e^{-x-y}$$

$$\cosh(x+y) = \frac{1}{2}e^{x+y} + \frac{1}{2}e^{-x-y}$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$\cosh^{-1} x = \ln(x \pm \sqrt{x^2 - 1})$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$$