MTH 105 A
Final Examination
Summer 2003 **Name** _____
(100 points)

Show your work ***and*** *circle your final answer(s).*

1. The concentration $c(t)$ in milligrams per cubic centimeter of a particular drug in a patient's bloodstream is given by $c(t) = \dfrac{0.16\,t}{t^2 + 4t + 4}$ where t is the number of hours after the drug is taken. How many hours after the drug is given will the concentration be maximum? What is the maximum?

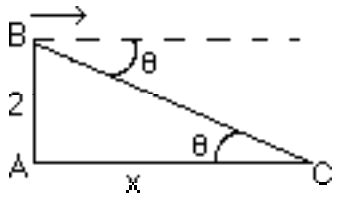
2. If the area of a healing wound changes at a rate given approximately by

$$\frac{dA}{dt} = -\frac{4}{t^3} \qquad 1 \le t \le 10$$

where t is in days and $A(1) = 2$ cm^2, what will be the area of the wound in 10 days?

3. Consider the curve defined by $x^2 + xy + y^2 = 27$. Write an expression for the slope of the curve at any point $< x, y >$.

4. A bomber B is flying 200 miles per hour on a level course 2 miles above the sea. The bombardier is sighting on a cruiser C directly ahead of the plane. The angle of depression between the plane's path and the line of sight is 30˚. How fast must the instrument be turning at this instant in order to keep the cruiser in view? (Neglect the speed of the cruiser)



5. Evaluate the following:

a) $\int \frac{x}{x+1}\,dx$

b) $\int_{-1}^{1} 2x^3(1+x^4)^3\,dx$

* **c)** $\int \sin^2 x\,dx$

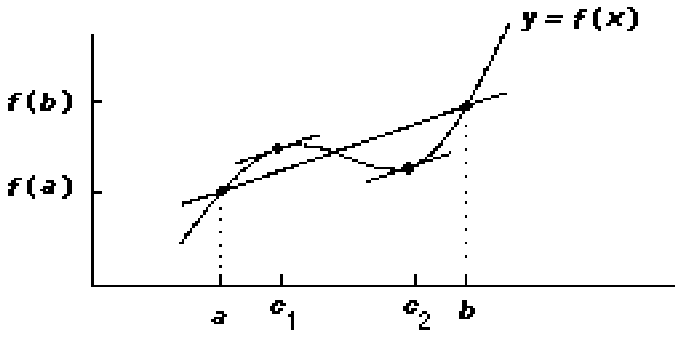
d) $\int x^2\sqrt{x^2-9}\,dx$

e) $\int \tan^3 x \sec^2 x\,dx$

f) $\lim_{x \to 0} \frac{3x - \sin x}{x}$

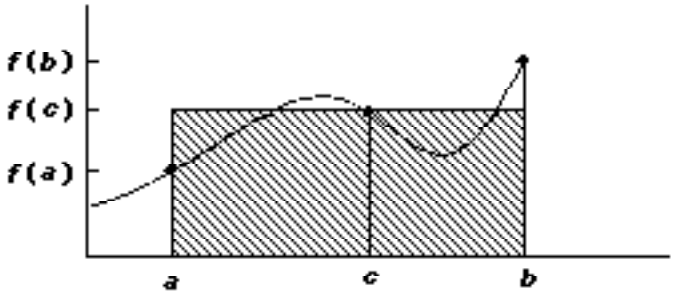
* Use the identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

6. The Mean Value Theorem for derivatives assures us that there is at least one point c in [a, b] where $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ (see the figure below). Find a value c in [0, 1] for $f(x) = \sqrt{x(1 - x)}$ which will satisfy the hypothesis of the Mean Value Theorem.



7. Find all local minima, local maxima and inflections for $f(x) = x^4 - 4x^3 + 10$.

8. Another interpretation of the Mean Value Theorem for Integrals is that we can find the average value ("height") of a function $f(x)$ on an interval $[\, a, b \,]$ when $f(c) = \dfrac{1}{b - a}\displaystyle\int_a^b f(x)dx$ for some c in that interval $[\, a, b \,]$. Find the average value $f(x) = xe^x$ on $[\, 0, 2 \,]$ **and** what value c will give us this value?



9. The length of a pendulum is slowly decreasing at the rate of 0.1 cm/sec. What is the rate of change of the period of the pendulum when the height is 16 cm, if the relation between the period and length is

$$T(\ell) = \pi\sqrt{\frac{\ell}{245}} \quad ?$$