

MTH 111 B
Examination #2
Summer 2003
(100 points)

Name _____

Show your work *and* circle your final answer.

1. Find the slope of the curve $\frac{2x}{x-1}$ at the point $\langle 2, 4 \rangle$.

2. A certain quantity of heat is required to raise the temperature of unit mass of a substance from a standard temperature, say 0°C ., to a temperature T . The temperature rate of change of Q is called the *specific heat* of the substance at temperature T . It is found by experiment that the quantity of heat Q required to raise the temperature of 1 gram of water from 0°C . to $T^\circ \text{C}$. (between 0° and 200°) is

$$Q = T + 2 \cdot 10^{-5} T^2 + 3 \cdot 10^{-7} T^3$$

Find the specific heat of water at $T = 35^\circ \text{C}$.

3. In the study of simple harmonic motion, under certain conditions the tangent of the angle by which the displacement lags the impress force is given by

$$\tan \varphi = \sec \sqrt{\omega} \quad \text{where } \omega, (\text{in hertz}) \text{ is the frequency of vibration.}$$

Find the rate of change of $\tan \varphi$ with respect to ω when $\omega = 3 \text{ Hz}$.

4. The distance d between ion layers of a crystalline solid such as table salt is given by $d = \sqrt[3]{\frac{M}{2N\rho}}$ where M is the molecular mass, N is called Avogadro's number and ρ is the density. Find the rate of change of d with respect to ρ . (M and N are constants)

5. The electric power (in watts) produced by a certain source is given by

$$P = \sec 2r \tan 2r \quad \text{where } r \text{ is the resistance in the circuit.}$$

Find the instantaneous rate of change of power with respect to resistance ?

6. Find the slope of the tangent line to the curve $y = 3 \sin 2x$ at $x = \frac{\pi}{8}$.

7. The speed in signaling in submarine cables varies as $f(x) = x^2 \ln \frac{1}{x}$, where x is the ratio of the radius of the core to the thickness of the covering of the cable. Find $f'(\frac{1}{\sqrt{e}})$.

8. The strength of electric current needed to excite a nerve x units long was found to be $S = \frac{0.25 e^x}{e^x - 1}$. Find the rate of change of S with respect to x .

9. Use the derivative to show that the function $f(x) = e^{-x^2}$ is decreasing on the interval $(0, \infty)$.

10. If $w = \ln(\ln z)$, find $\frac{dw}{dz} \Big|_{z=e^2}$.

11. Find $y''(\frac{1}{2}\pi)$ when $y = \sin x \cos x$.

12. By differentiating the trigonometric identity $\sin 2x = 2 \sin x \cos x$, obtain the corresponding identity for $\cos 2x$.