**MTH 111 B**
**Examination #1**
**Summer 2003**
**(100 points)**

**Name** _____

*Show your work* ***and*** *circle your final answer.*

**1.** Evaluate the following:

**a)** $\lim_{x \to 0} \dfrac{(3+x)^2 - 9}{x}$

**b)** $\lim_{x \to 0} \dfrac{1}{x}\left(\dfrac{1}{3+x} - \dfrac{1}{3}\right)$

**c)** $\lim_{x \to 9} \dfrac{x - 9}{3 - \sqrt{x}}$

**d)** $\lim_{x \to \infty} \left(3 + \dfrac{1}{x^2}\right)$

**2.** The illuminance $I$ is inversely proportional to the square of the distance $x$ from the source. (*ie*. $I = \dfrac{k}{x^2}$ for some $k > 0$ ) Find $\lim_{x \to 0} I$.

**3.** The following expression occurs in fracture mechanics:

$$K = \frac{cx^2}{(1+x)^2} \quad \text{where c is a constant.}$$

Find $\lim_{x \to \infty} K$.

**4.** The function $f(x) = \dfrac{3x + 4 \tan x}{x}$ is undefined at $x = 0$. What value must be assigned to $f(0)$ if $f(x)$ is to be continuous at this point? (You must confirm your answer algebraically )

**5.** Algebraically confirm that the value of $\dfrac{4 - 4 \cos x}{x^2}$ is near 2 if $x$ is near 0.

(Use the fact that $1 - \cos x = 2 \sin^2 \frac{1}{2}x$ )

**6.** The combined capacitance for two capacitors connected in series is given by

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

**a)** What happens to $C_T$ as $C_1$ gets closer to $C_2$?

**b)** What is $\lim_{C_1 \to 0} C_T$ ?

**7.** Pierre de Fermat formulated the derivative as a geometric entity. He argued that the derivative at any point is the limiting process of secant lines which approach that point. Answer the following using Fermat's formulation for the derivative.

**a)** Given $f(x) = \dfrac{x^2}{2x+1}$, find the slope of the tangent line at $x = 2$.

**b)** Find the equation of the tangent line at $x = 2$.

**8.** Sir Isaac Newton formulated the derivative as a physical entity. He argued that the derivative at any point is the limiting process of average rates of change which approach that point. Answer the following using Newton's formulation for the derivative.

**a)** Given $f(x) = \dfrac{1}{x^2}$, find the instantaneous rate of change at $x = 1$.

**b)** Find the equation of the tangent line at $x = 1$.