MTH 104 **Hamiltonian Paths & Circuits** **Mathematics for Elementary School Teachers II**

Spring 2007

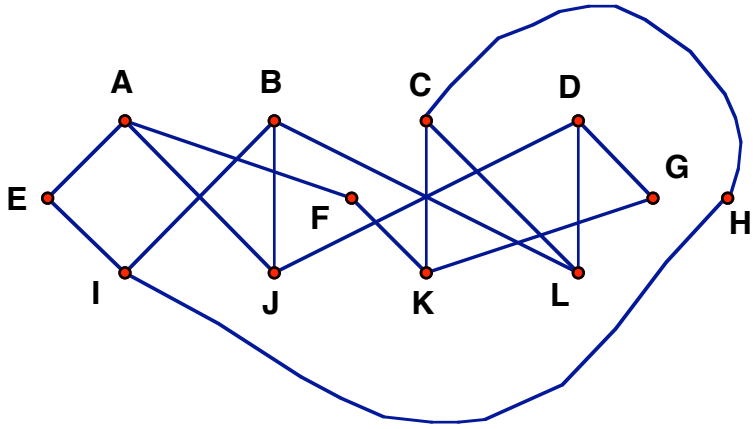
Name _____

NOTE: even though there is no simple algorithm that tells us how to find a Hamiltonian path if one exists, two observations are worth noting:

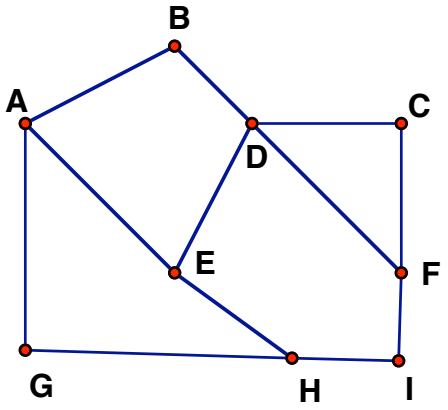
1. If there is a vertex of degree one, then any Hamiltonian path must start or end there.

2. If there is a vertex of degree two, then either the path must start or end there, or else both edges incident to the vertex must be traveled.

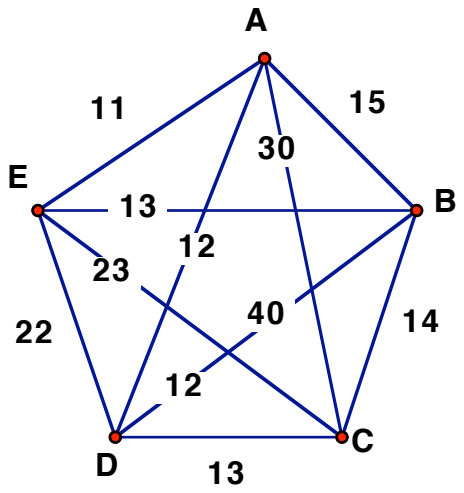
1. A sales man wishes to visit all the cities on the map bellow and return to the starting point. What route should he followed he does not wish to visit any city more than once?



2. Show that a vacationer wanting to visit each of the cities in the map below (exactly once) would not be able to end his airline tour at the same starting point.



3. Use the *Repetitive Nearest Neighbor* Algorithm and the *Cheapest Link* Algorithm to fine an optimal cost Hamiltonian circuit in the network below.



The Repetitive Nearest Neighbor Algorithm

- Pick a vertex as the starting point
- From the starting vertex go to its "nearest neighbor"
- Continue building the circuit, one vertex at a time, by always going from a vertex to its nearest neighbor *from among the vertices left to yet visit.*
- From the last vertex return to the starting point

↳ **Repeat the above steps for each vertex**

The Cheapest Link Algorithm

- Pick the edge with the smallest weight first
- Pick the next "cheapest" edge
- Continue picking the "cheapest" edge available except when
  - it closes a circuit
  - it results in three edges coming out of a single vertex
- When there are no more vertices to join, close the circuit