Counting

Sum Rule Principle

Suppose some event *E* can occur in *m* ways and a second event *F* can occur in *n* ways, and suppose the events cannot occur simultaneously. Then *E* and *F* can occur in *m* + *n* ways.

Product Rule Principle

Suppose some event *E* can occur in *m* ways and a second event *F* can occur in *n* ways. Then *E* followed by *F* can occur in *m* x *n* ways.

MacDonald's Problem

How many ways can you have a *Big Mac*?
(2 all beef patties, special sauce, lettuce, cheese, pickles, onions, on a sesame seed bun)

$$ {}_7C_0 + {}_7C_1 + {}_7C_2 + {}_7C_3 + {}_7C_4 + {}_7C_5 + {}_7C_6 + {}_7C_7 = 128 $$

OR

2 x 2 x 2 x 2 x 2 x 2 x 2 = 2^7

Permutations

A ***permutation*** of a group of objects is an ordered arrangement of the objects.

How many ways can *n* **distinct** items be arranged *n* at a time ?

$$_nP_n = n!$$

Permutations

How many ways can *n* **distinct** items be arranged *r* at a time ?

$$ {}_nP_r = n\,!\,/\,(n - r)\,! $$

Permutations

How many ways can 7 books be arranged if 2 books must remain together?

$$_6P_6 \times 2! = 1440$$

Permutations

How many ways can 7 books be arranged into 4 slots ?

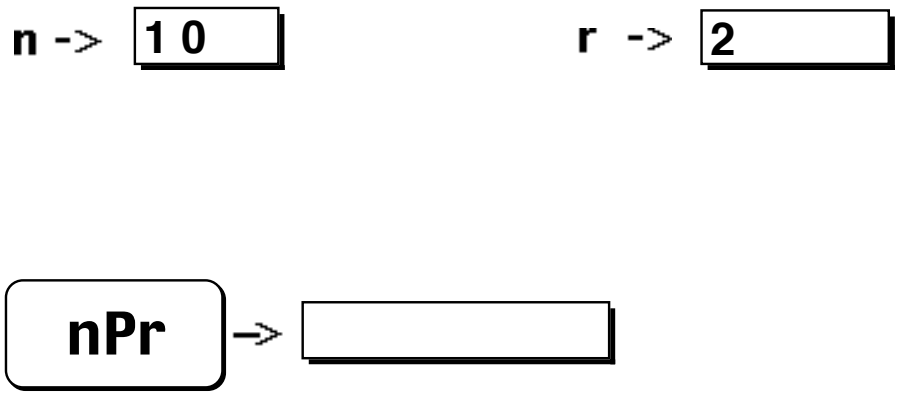
n -> 1 3

r -> 4

nPr ->

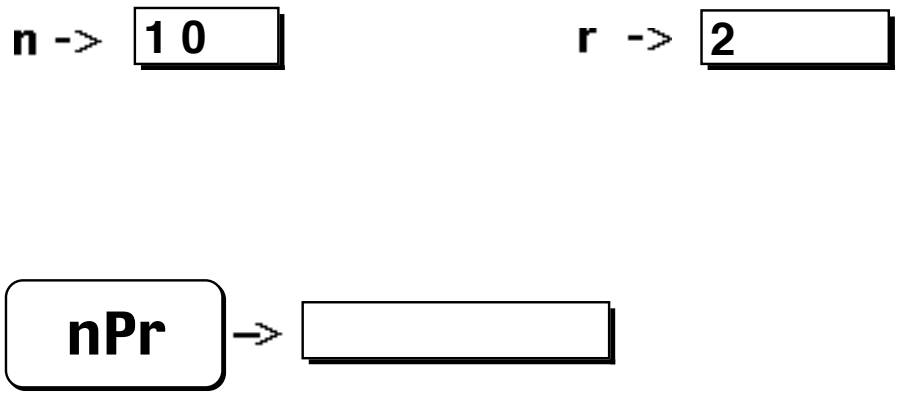
Permutations

How many ways can 10 students be seated in 2 chairs ?



Permutations

How many ways can 2 students be seated in 10 chairs ?



Permutations

How many ways can *n* ~~**distinct**~~ items be arranged *r* at a time ?

How many ways can the letters in the word root be arranged ? r **O** o t

$$_nP_n = n! / (n_1!)(n_2!)(n_3!)\ldots$$

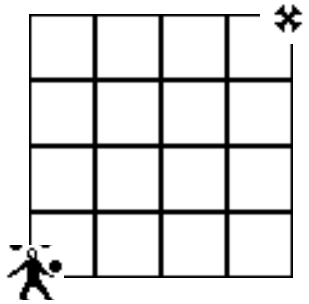
Permutations

" the MISSISSIPPI problem"

How many ways can the letters in the word

TATTOO be arranged ?

Go



Circular Permutations

How many ways can the letters in the word

ABCD be arranged in a circle ?

Go

Permutations

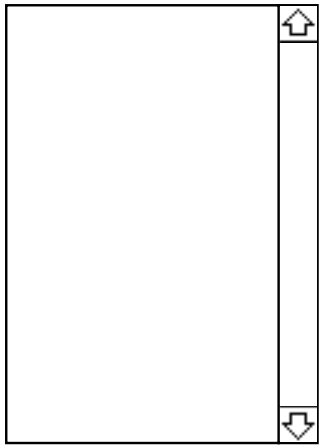
" the HALLOWEEN problem"

How many ways can the letters in the word

HALLOWEEN be arranged taken

4 at a time?

GO



Permutations

(exponential generating functions)

" the HALLOWEEN problem"

H's : 0, 1 $\longrightarrow (1 + x)$
A's : 0, 1 $\longrightarrow (1 + x)$
L's : 0, 1,2 $\longrightarrow (1 + x + \frac{1}{2}x^2)$
O's : 0, 1 $\longrightarrow (1 + x)$
W's : 0, 1 $\longrightarrow (1 + x)$
E's : 0, 1,2 $\longrightarrow (1 + x + \frac{1}{2}x^2)$
N's : 0, 1 $\longrightarrow (1 + x)$

$$\longrightarrow \begin{cases} (1 + x)^5 \\ (1 + x + \frac{1}{2}x^2)^2 \end{cases}$$

In[2]:=
```
Expand[(1 + x)^5 * (1 + x + x^2/2)^2]
```

Out[2]=
$$1 + 7x + 22x^2 + 41x^3 + \frac{201x^4}{4} + \frac{169x^5}{4} + \frac{49x^6}{2} + \frac{19x^7}{2} + \frac{9x^8}{4} + \frac{x^9}{4}$$

Combinations

A ***combination*** is a way of selecting several things out of a larger group, where (unlike permutations) order does not matter.

How many combinations of *n* items can be made if taken *n* at a time ?

$$_nC_n = 1$$

Combinations

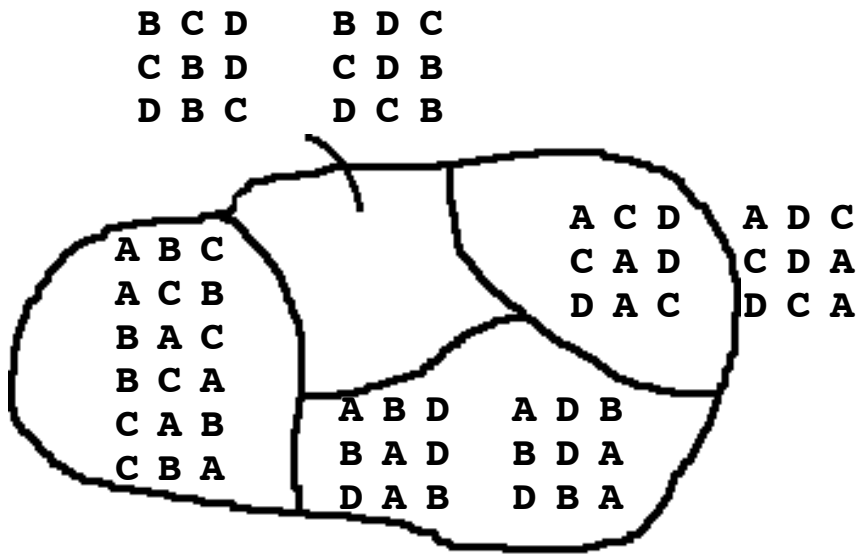
A ***combination*** is a way of selecting several things out of a larger group, where (unlike permutations) order does not matter.

How many combinations of *n* items can be made if taken *k* at a time ?

$$_nC_k = \frac{1}{k!}\left(_nP_k\right) = \frac{n!}{k!(n-k)!}$$

Combinations

How many ways can 4 students be chosen for 3 committees ?



nCk →

Combinations

How many ways can 7 students be chosen for 2 committees ?

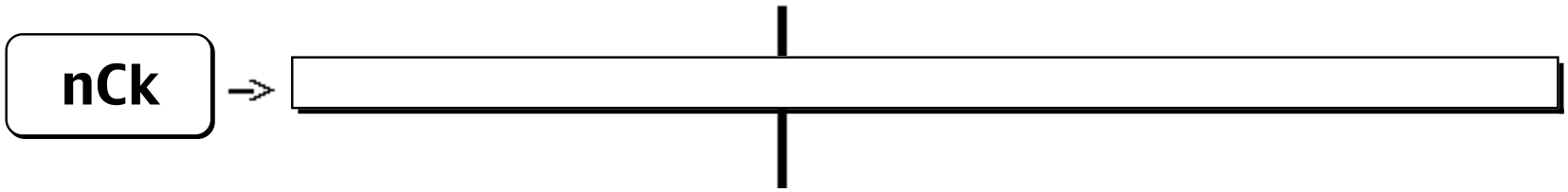
nCk →

Combinations

How many ways can 152 students be chosen for 50 committees ?

(152 !) / (50 ! * 102 !)

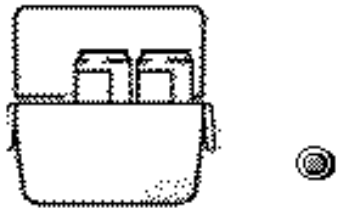
44 844 984 003 122 433 509 745 860 950 406 683 658 640



Soda Can Problem

How many ways can I select 15 cans of soda from a cooler containing large quantities of Coke, Pepsi, Diet Coke, Root Beer, and Sprite?

	Coke	Pepsi	Diet Coke	Root Beer	Sprite
A:	111	111	111	111	111
B:	11		111111	111111	1
C:		1111	1111111	1111	



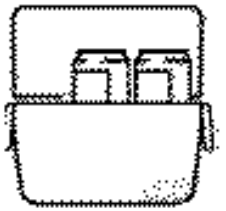
Soda Can Problem

How many ways can I select 15 cans of soda from a cooler containing large quantities of Coke, Pepsi, Diet Coke, Root Beer, and Sprite?

A: 111**0**111**0**111**0**111**0**111

B: 11**00**1111111**0**111111**0**1

C: **00**1111**0**1111111**0**1111



Soda Can Problem

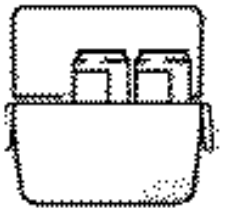
How many ways can I select 15 cans of soda from a cooler containing large quantities of Coke, Pepsi, Diet Coke, Root Beer, and Sprite?

A: 111**0**111**0**111**0**111**0**111

D: 111 111 111 111**0000**

C (15 + 4, 15) = C (19, 15) or C (19, 4)

= 3,876



Soda Can Problem

How many ways can I fill a box holding 100 pieces of candy from 30 different types of candy?

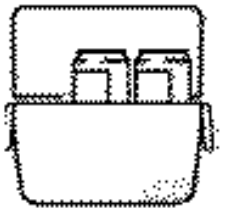
1111111111111111111111**0000000000**

1111111111111111111111**0000000000**

1111111111111111111111**0000000000**1111111111

$$C\,(100 + 29,\ 100) = \frac{129!}{100!29!} =$$

60,284,731,216,266,553,294,577,246,880



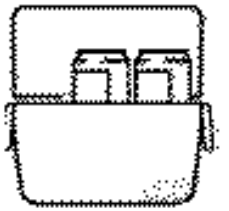
Soda Can Problem

How many ways can I fill a box holding 100 pieces of candy from 30 different types of candy? * (with at least one piece of each type)

1010101010101010101010
1010101010101010101010
1010101010101010101 1111111111 1111111111 1111111111
1111111111 1111111111 1111111111 1111111111

$$C\ (70 + 29,\ 70) = \frac{99!}{70!29!} =$$

8,811,701,946,483,283,447,189,128



How many ways can 10 balls be placed into three containers?

10 BALLS

64

7,0,3
7,2,1
2,6,2
8,0,2
9,0,1
4,0,6
1,8,1
0,8,2
5,4,1
5,1,4

GO

Two container(s) may be empty

10 BALLS 66

4,4,2
5,0,5
3,7,0
9,1,0
4,6,0
5,2,3
2,4,4
8,2,0
1,6,3
1,5,4

GO

Three container(s) may be empty

10 BALLS 3

4,1,5
7,1,2
2,3,5

GO Container(s) may NOT be empty

10 BALLS 63

8,1,1
5,0,5
5,1,4
5,4,1
3,3,4
2,7,1
9,0,1
8,2,0
9,1,0
7,3,0

GO One container may be empty

10 BALLS 1

6,2,2

GO

Container(s) may NOT be empty
and are indistinguishable

10 BALLS 10

6,0,4
9,0,1
2,8,0
5,0,5
1,5,4
2,6,2
10,0,0
7,2,1
3,6,1
3,2,5

GO

Two container(s) may be empty
and are distinguishable